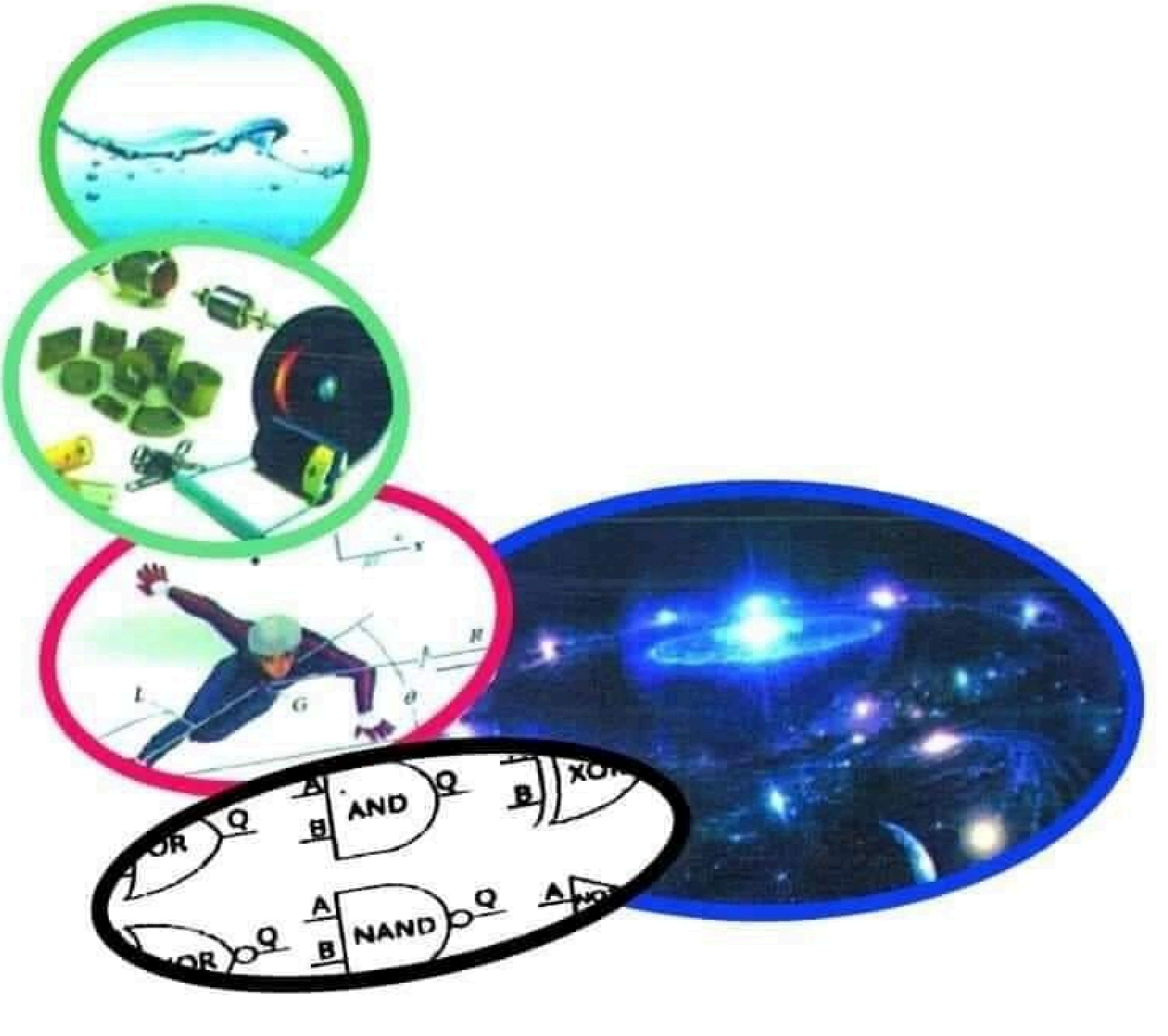
THE GOVERNMENT OF
THE REPUBLIC OF THE UNION OF MYANMAR

MINISTRY OF EDUCATION

TEXTBOOK

PHYSICS

Grade 11



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CHAPTER 1

MOTION IN A PLANE

In Grade 10, one dimensional motion such as linear motion and free fall motion was studied. It is the simplest type of motion which can be encountered in many cases.

Learning Outcomes

It is expected that students will

- examine two-dimensional motion; projectile and circular motion.
- examine angular speed and angular acceleration.
- solve two-dimensional motion problems.
- understand the proper use of quantities, notations and units for two-dimensional motion.

In this chapter we will consider motion in two dimensions (or) motion in a plane. There are several important cases in two-dimensional motion.

1.1 TWO-DIMENSIONAL MOTION

We consider some cases in which an object moves in a plane. The object may move in both the x and y direction simultaneously. The motion of the object is said to be a two-dimensional motion.

In order to describe the motion of an object in two dimensions, vector concept must be used. The object moves along a curved path between points P and Q as shown in Figure 1.1. The displacement vector of the object from P to Q is $\Delta\vec{r}$. The perpendicular components of $\Delta\vec{r}$ are $\Delta\vec{x}$ and $\Delta\vec{y}$.

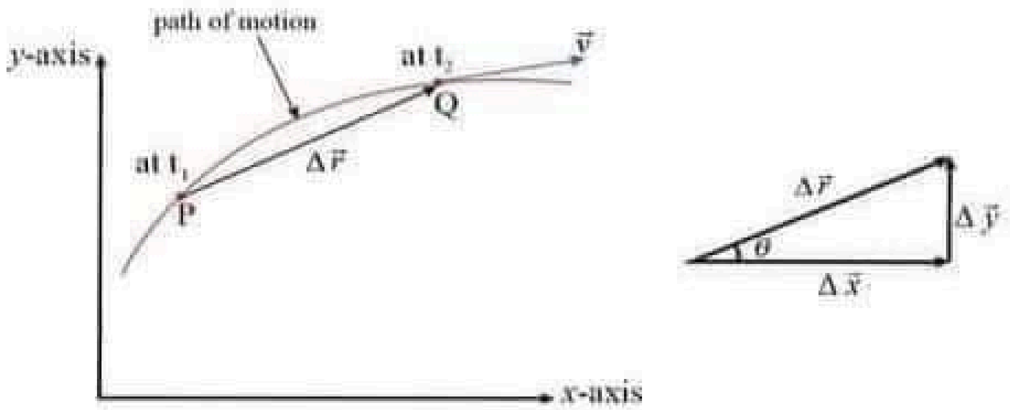


Figure 1.1 Motion of an object in two dimensions

In Figure 1.1 $\Delta\vec{r} = \Delta\vec{x} + \Delta\vec{y}$

The magnitude of $\Delta\vec{r}$, $\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

To find the direction of $\Delta\vec{r}$, $\tan\theta = \frac{\Delta y}{\Delta x}$

$$\theta = \tan^{-1}\frac{\Delta y}{\Delta x}$$

In previous level, the average velocity and the instantaneous velocity are given as follows:

Average velocity $\overline{\vec{v}} = \dfrac{\Delta \vec{r}}{\Delta t}$ (1.1)

Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$ (1.2)

Average acceleration and instantaneous acceleration are given by the following equations.

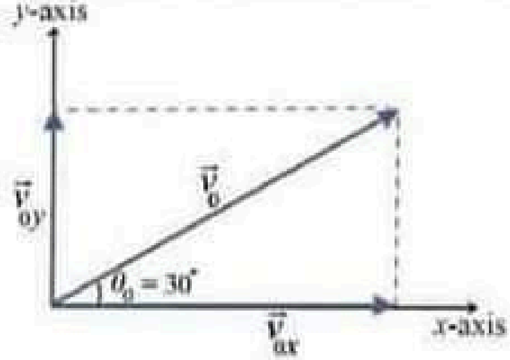
Average acceleration $\overline{\vec{a}} = \dfrac{\Delta \vec{v}}{\Delta t}$ (1.3)

Instantaneous acceleration $\vec{a} = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{d\vec{v}}{dt}$ (1.4)

An object moving with constant speed along a curved path is accelerating as the direction of the velocity is changing. The direction of the velocity of the object is tangential to its path. Hence, the object is accelerated whenever the velocity changes in magnitude, direction, (or) both.

Example 1.1 A stone is thrown from the cliff of a mountain upward at an angle of 30° to the horizontal with an initial speed of 20 m s^{-1}. Calculate the x and the y components of its initial velocity.

$v_0 = 20$ m s^{-1}, $\theta_0 = 30°$



Horizontal components of initial velocity

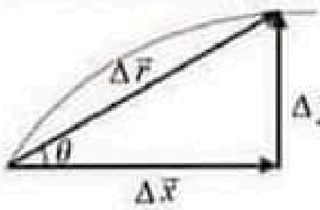
$v_{0x} = v_0 \cos \theta_0 = 20 \cos 30°$

$= 20 \times 0.866 = 17.32$ m s^{-1}

Vertical components of initial velocity

$v_{0y} = v_0 \sin \theta_0 = 20 \times \sin 30° = 20 \times 0.5 = 10$ m s^{-1}

Example 1.2 A soccer ball is kicked at an angle with the ground. The ball traverse the horizontal distance of 10 m and the vertical distance of 3 m in 3 s. Find the displacement and the average velocity of the ball in 3 s.



The horizontal distance $\Delta x = 10$ m

The vertical distance $\Delta y = 3$ m

Magnitude of the displacement $\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$$\Delta r = \sqrt{(10)^2 + (3)^2} = \sqrt{109} = 10.44 \text{ m}$$

Magnitude of the average velocity of the ball $\Delta v = \dfrac{\Delta r}{\Delta t} = \dfrac{10.44}{3} = 3.48$ m s^{-1}

The direction of the displacement is as below.

$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{3}{10} = 0.3$$

$$\theta = \tan^{-1} 0.3 = 16.7°$$

The direction of the displacement and the average velocity are the same which makes an angle 16.7° with the ground.

Reviewed Exercise

- Draw the diagram to show the direction of velocity of an object which is moving along a curved path and then draw the vector diagram to show the velocity components in x and y directions at a starting point P.

Key Words: curved path, displacement, tangential velocity, acceleration

1.2 PROJECTILE MOTION

A projectile is any object thrown into space upon which the only acting force is the gravity. The path followed by a projectile is known as a trajectory.

The motion of an object moving in both horizontal x direction and vertical y direction simultaneously is called the projectile motion. Examples for the projectile motion are water fountain, the motion of cannon ball, the motion of the football as shown in Figure 1.2.

Figure 1.2 Illustrations for projectile motion

In the projectile motion, the path of motion is a curve. If air resistance is neglected, an object moves along the horizontal x direction with a constant velocity and in the vertical y direction with constant downward acceleration. In such case, only gravitational force is acting on it. Therefore, the downward acceleration is the acceleration due to gravity g which is a constant over the range of motion. The vertical motion of projectile is a free fall motion.

In Figure 1.3, the initial velocity of projectile is v_0. If the initial velocity makes an angle θ_0 with the horizontal, the x and y components of the initial velocity are $v_{0x} = v_0 \cos\ \theta_0$ and $v_{0y} = v_0 \sin\ \theta_0$.

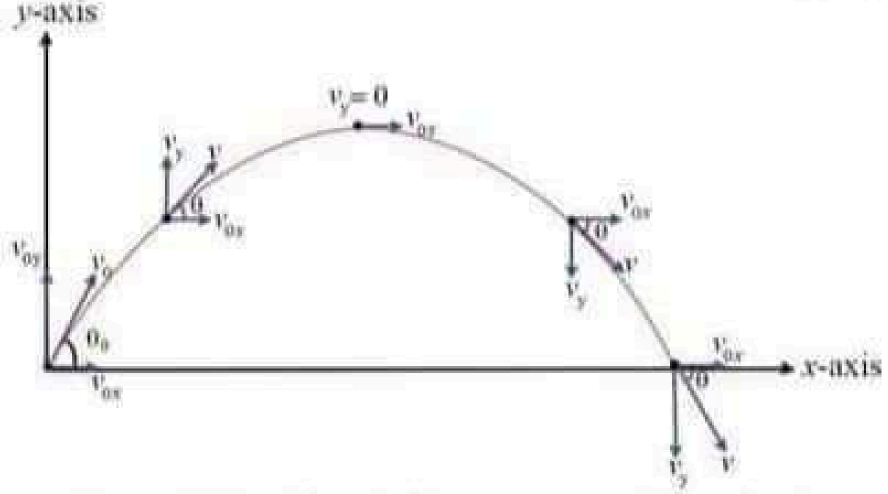


Figure 1.3 Velocities and their components of the projectile

In order to analyze the projectile motion, the motion can be considered into two parts, horizontal motion (x direction) and vertical motion (y direction).
In x direction, $a_x = 0$ and $v_x = v_{0x} = v_0 \cos\theta_0$ = constant.

The horizontal displacement of projectile can be written as a function of time; $x = v_{0x}\, t = (v_0 \cos\theta_0)t$.
In y direction, v_{0y} is the initial velocity and a_y is $-g$. Since the acceleration along the vertical y direction is downward direction, g can be taken as negative sign. The velocity and the displacement of projectile in the time t are given by the following equations.

$$v_y = v_{0y} + a_y t \tag{1.5}$$

$$v_y^2 = v_{0y}^2 + 2a_y y \tag{1.6}$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2 \tag{1.7}$$

The magnitude of the velocity of the projectile at any instant of time is, $v = \sqrt{v_x^2 + v_y^2}$ and the direction of the velocity of the projectile is $\tan\theta = \frac{v_y}{v_x}$.
The projectile reaches the highest point when its vertical component of velocity v_y is zero. If the time taken to reach this highest point is t_1 and the maximum height is H from the horizontal then we get

$$v_y = v_{0y} + a_y t$$

$$v_y = v_{0y} - gt$$

$$0 = v_0 \sin\theta_0 - gt_1 \quad \text{(or)} \quad t_1 = \frac{v_0 \sin\theta_0}{g}$$

The time of flight is twice the time to reach the maximum height. We note that the time of flight is T.

$$T = 2t_1 = \frac{2v_0 \sin\theta_0}{g} \tag{1.8}$$

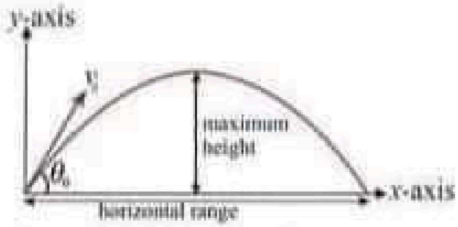


Figure 1.4 Maximum height and horizontal range of the projectile motion

Using the equation,

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$y = (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2$$

$$H = (v_0 \sin\theta_0)\,t_1 - \frac{1}{2}gt_1^2$$

We get,

$$H = v_0 \sin\theta_0 \left[\frac{v_0 \sin\theta_0}{g}\right] - \frac{1}{2}g\left[\frac{v_0 \sin\theta_0}{g}\right]^2$$

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} \tag{1.9}$$

The horizontal range R is the horizontal distance from the starting point to the point where the projectile returns to the same height as shown in Figure 1.4.

The horizontal range is obtained as $R = v_{0x}T = v_0 \cos\theta_0 \dfrac{2v_0 \sin\theta_0}{g}$.

Since, $2\sin\theta_0 \cos\theta_0 = \sin 2\theta_0$

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \qquad (1.10)$$

Another case of the projectile motion is an object thrown horizontally as shown in Figure 1.5.
There is no vertical component to its initial velocity; that is, $v_{0y} = 0$, $v_{0x} = v_0$, $x = v_0$

The velocity of the object in the time t is, $v_y = -gt$.

The vertical displacement in time t is, $y = -\frac{1}{2}gt^2$.

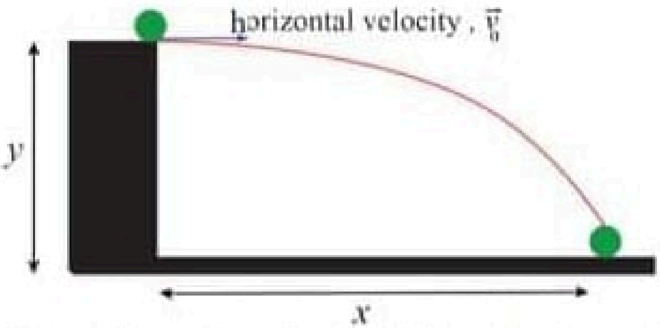


Figure 1.5 Projectile motion of an object thrown horizontally

Example 1.3 An object is projected upward with a 30° launch angle and an initial speed of 40 ms^{-1}. How long will it take for the object to reach the top of its trajectory? Find the maximum height of its trajectory.

$a_y = -g$ where $g = 9.8$ m s^{-2}

$v_0 = 40$ m s^{-1}, $\theta_0 = 30^\circ$, $v_{0y} = v_0 \sin 30^\circ = 40 \times 0.5 = 20$ m s^{-1}(upward direction),

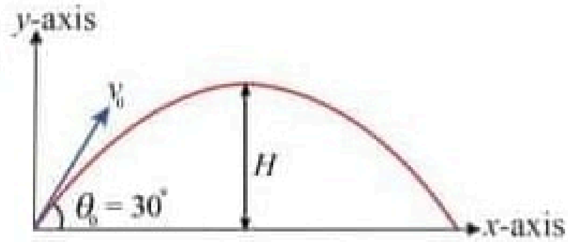
At the top of the trajectory $v_y = 0$

$$v_y = v_{0y} + a_y t$$
$$v_y = v_{0y} - gt$$
$$0 = 20 - (9.8)\,t$$
$$t = 2.04 \text{ s}$$

The time taken to reach the top of its trajectory is 2.04 s.

For maximum height of the ball

$$v_y^2 = v_{0y}^2 + 2a_y y$$
$$v_y^2 = v_{0y}^2 - 2gy$$
$$0 = 20^2 - 2(9.8)\,H$$
$$H = 20.41 \text{ m}$$

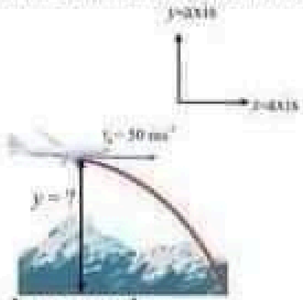


The maximum height of its trajectory is 20.41 m.

Example 1.4 A bomb is dropped from an airplane moving horizontally with its speed of 50 ms^{-1}. If the bomb will reach the ground in 5 s, find the altitude of the plane. The air resistance is negligible.

$v_{0y} = 0$, $v_{0x} = 50\ ms^{-1}$, $a_y = -g$ where $g = 9.8\ m\ s^{-2}$

$$y = v_{0y}t - \frac{1}{2}gt^2$$
$$y = 0 - \frac{1}{2}\times(9.8)\times 5^2$$
$$y = -122.5\ m$$



Minus sign means that the direction of y component displacement is downwards.
The altitude of the plane = 122.5 m

Example 1.5 A stone is thrown with a speed 20 m s^{-1} and at an angle 30° above the horizontal. Find (i) the horizontal range (ii) the maximum height reached (iii) the time of flight of the stone.

$v_0 = 20\ m\ s^{-1}$, $\theta_0 = 30$

(i) The horizontal range R

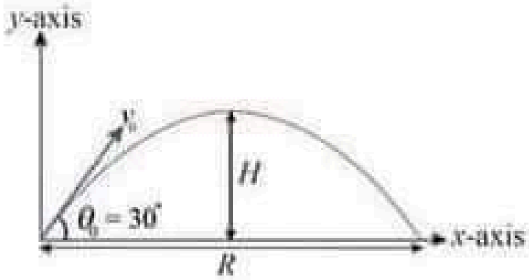
$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{20^2 \sin(2\times 30^\circ)}{9.8} = 35.35\ m$$

(ii) The maximum height reached H

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} = \frac{20^2(\sin 30^\circ)^2}{2\times 9.8} = 5.10\ m$$

(iii) The time of flight of the stone T

$$T = \frac{2v_0 \sin\theta_0}{g} = \frac{2\times 20\times \sin 30}{9.8} = 2.04\ s$$



Reviewed Exercise

1. Why is the horizontal motion of the projectile taken as uniform motion?
2. Under what conditions can you have the two-dimensional motion with a one-dimensional acceleration?

Key Words: projectile, free fall motion, uniform motion

1.3 CIRCULAR MOTION

Circular motion is a specific type of a two-dimensional motion. It is a movement of an object along the circumference of a circle (or) rotation along a circular path. If a ball is tied to the end of a string and whirl it above the head in a horizontal circle, the ball is undergoing the circular motion as shown in Figure 1.6 (a).

We can experience a pulling force (known as the tension) exerted on the ball by the string. Other examples of the circular motion are shown in Figure 1.6 (b),(c) and(d) ; a satellite orbiting the earth, motion of a wheel, turning of a car around curved path respectively. If an object (rigid body) rotates about an axis, each particle consisting of the object moves in a circular motion.

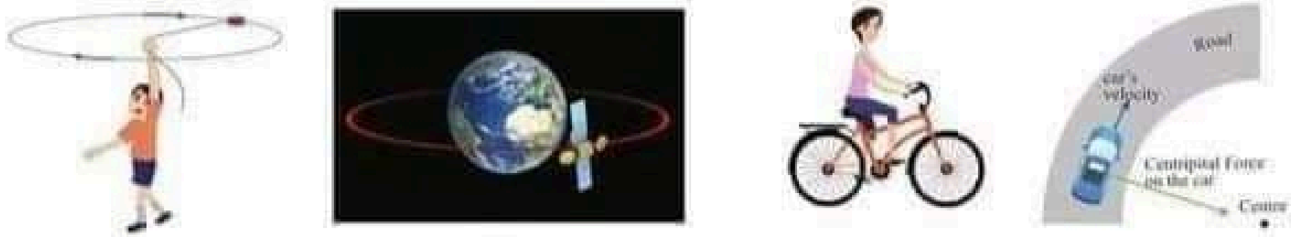


Figure 1.6 Illustrations of circular motion

Although an object moves in a circle at a constant speed, its velocity is not constant. The direction of the velocity is tangential to the path. Velocity is constantly changing because the direction of the velocity is changing continuously. The velocity in circular motion is also called the tangential velocity.

Figure 1.7 shows an object moves along a circular path. In fact, the object undergoes the circular motion about O. Since the object is on the reference line, its angular position is zero. After time t has elapsed, it has moved to a new position. In this time interval, it has rotated through an angle θ with respect to the reference line and through a distance s measured along the circumference of the circle, called an arc length. Change of the angular position is called the angular displacement θ.

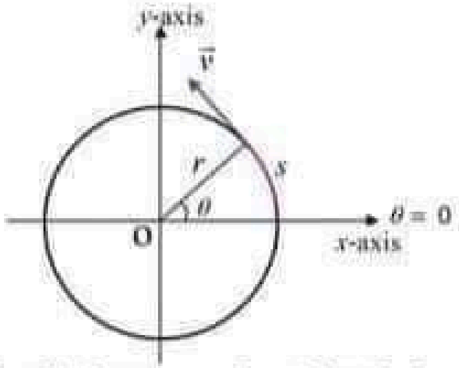


Figure 1.7 The angular displacement of an object in circular motion

The angular displacement is the ratio of the arc length to radius.

The magnitude of angular displacement (θ), measured in radian (rad), is given by

$$\theta = \frac{s}{r} \tag{1.11}$$

where θ = angular displacement
s = arc length
r = radius of circle

Angular displacement can also be expressed in degree and revolution (rev).

One radian is defined as the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

If $s = r$, $\theta = 1$ radian

$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$$

For example, $60^\circ = \frac{\pi}{180^\circ} \times 60^\circ = \frac{\pi}{3} rad$

$$30^\circ = \frac{\pi}{180^\circ} \times 30^\circ = \frac{\pi}{6} rad$$

Average Angular Velocity and Instantaneous Angular Velocity

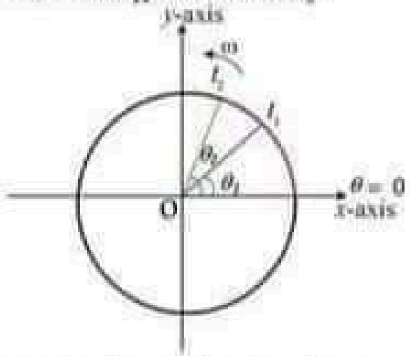


Figure 1.8 Change of angular displacement with time in circular motion

Figure 1.8 shows angular displacement of an object; θ_1 at time instant t_1 and θ_2 at time instant t_2. Average angular velocity is the ratio of the change in angular displacement to time taken.

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \quad (1.12)$$

where $\bar{\omega}$ is average angular velocity and $\Delta\theta$ is angular displacement in time interval Δt.

If the time interval Δt approaches zero, the instantaneous angular velocity can be written as

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad (1.13)$$

Instantaneous angular velocity is defined as the time rate of change of angular displacement.

At constant angular velocity, $\bar{\omega} = \omega$, $\omega = \frac{\theta}{t}$

Where ω is angular velocity, θ is angular displacement and t is time taken

Angular velocity is measured in radian per second (rad s^{-1}), revolution per second (rps) and revolution per minute (rpm or rev min^{-1}).

Average Angular Acceleration and Instantaneous Angular Acceleration

The average angular acceleration is the ratio of change of angular velocity to time taken.
The average angular acceleration is given by

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \quad (1.14)$$

where $\bar{\alpha}$ = average angular acceleration

ω_1 = angular velocity at time instant t_1

ω_2 = angular velocity at time instant t_2

If the time interval Δt approaches zero, the instantaneous angular acceleration can be defined as

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \quad (1.15)$$

The instantaneous angular acceleration is the value of the angular acceleration at a specific instant of time. Instantaneous angular acceleration is the time rate of change of angular velocity.

When a body is moving with a constant angular acceleration $\bar{\alpha} = \alpha$.

Eq.(1.15) can be written as $\bar{\alpha} = \alpha = \dfrac{\omega - \omega_0}{t}$

where α = angular accleration, ω = final angular velocity

ω_0 = initial angular velocity, t = time taken

Angular acceleration is measured in radian per second squared (rad s^{-2}) and revolution per second squared (rev s^{-2}).

Relation between Angular and Linear Quantities

The magnitude of the angular velocity (angular speed) is related to that of the tangential velocity (tangential speed) as follows.

$$v = \frac{ds}{dt} = \frac{d(r\theta)}{dt}$$

Since r is constant,

$$v = r\frac{d\theta}{dt}$$

$$v = r\omega \qquad (1.16)$$

Figure 1.9 shows the direction of the linear velocity and the angular velocity.

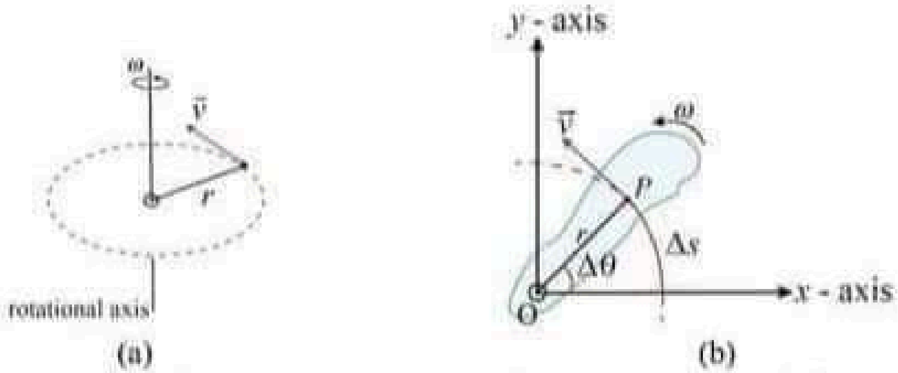


Figure 1.9 Relation between angular speed and tangential speed (linear speed)

The relation between the angular acceleration α and the tangential acceleration a is

$$a = \frac{dv}{dt} = \frac{d(r\omega)}{dt}$$

Since r is constant,

$$a = r\frac{d\omega}{dt}$$

$$a = r\alpha. \qquad (1.17)$$

Angular velocity ω and angular acceleration α are vector quantities. In Eq.(1.17), the direction of linear acceleration a is tangential to the path. It is called the tangential acceleration. In Eq.(1.16) and Eq.(1.17), unit of ω is rad s^{-1} and unit of α is rad s^{-2}.

Example 1.6 Express the angular velocity of a 45 rpm (revolutions per minute) record turntable in units of radians per second.

$$1 \text{ rev min}^{-1} = \frac{2\pi}{60} \text{ rad s}^{-1}$$

$$45 \text{ rev min}^{-1} = 45 \times \frac{2\pi}{60} = 1.5\,\pi \text{ rad s}^{-1}$$

Example 1.7 The angular velocity of a rotating disc increases from 2 rad s^{-1} to 5 rad s^{-1} in 10 s. What is the average angular acceleration?

$\omega_0 = 2$ rad s^{-1}, $\omega = 5$ rad s^{-1}, $t = 10$ s

The average angular acceleration, $\bar{\alpha} = \frac{\omega - \omega_0}{t} = \frac{5-2}{10} = 0.3$ rad s^{-2}

Example 1.8 A particle in disc rotating with a uniform angular speed of 2 rev s^{-1} is 0.2 m from the axis of rotation. What are (i) the tangential speed of the particle and (ii) the angle through which it rotates in 3 s?

Since a particle rotates with a uniform angular speed, the motion of particle is uniform circular motion.

$\omega = 2$ rev s^{-1} (rps) $= 2 \times 2\pi = 4\pi$ rad s^{-1}, $r = 0.2$ m

(i) The tangential speed,

$$v = r\omega$$
$$= 0.2 \times 4\,\pi = 0.8\,\pi \text{ m s}^{-1} = 0.8 \times 3.142 = 2.514 \text{ m s}^{-1}$$

(ii) The angle of rotation in $t = 3$ s,

$$\theta = \omega t = 4\,\pi \times 3 = 12\pi \text{ rad} = \frac{12\pi}{2\pi} \text{ rev} = 6 \text{ rev}$$

Reviewed Exercise

- Calculate the angular speed of the second hand and the minute hand of a clock in terms of rad s^{-1}.

Key Words: angular displacement, angular velocity, angular acceleration

SUMMARY

An object moves in both x and y direction simultaneously is said to be undergoing **projectile** motion.

Circular motion is a movement of an object along the circumference of a circle or rotation along a circular path.

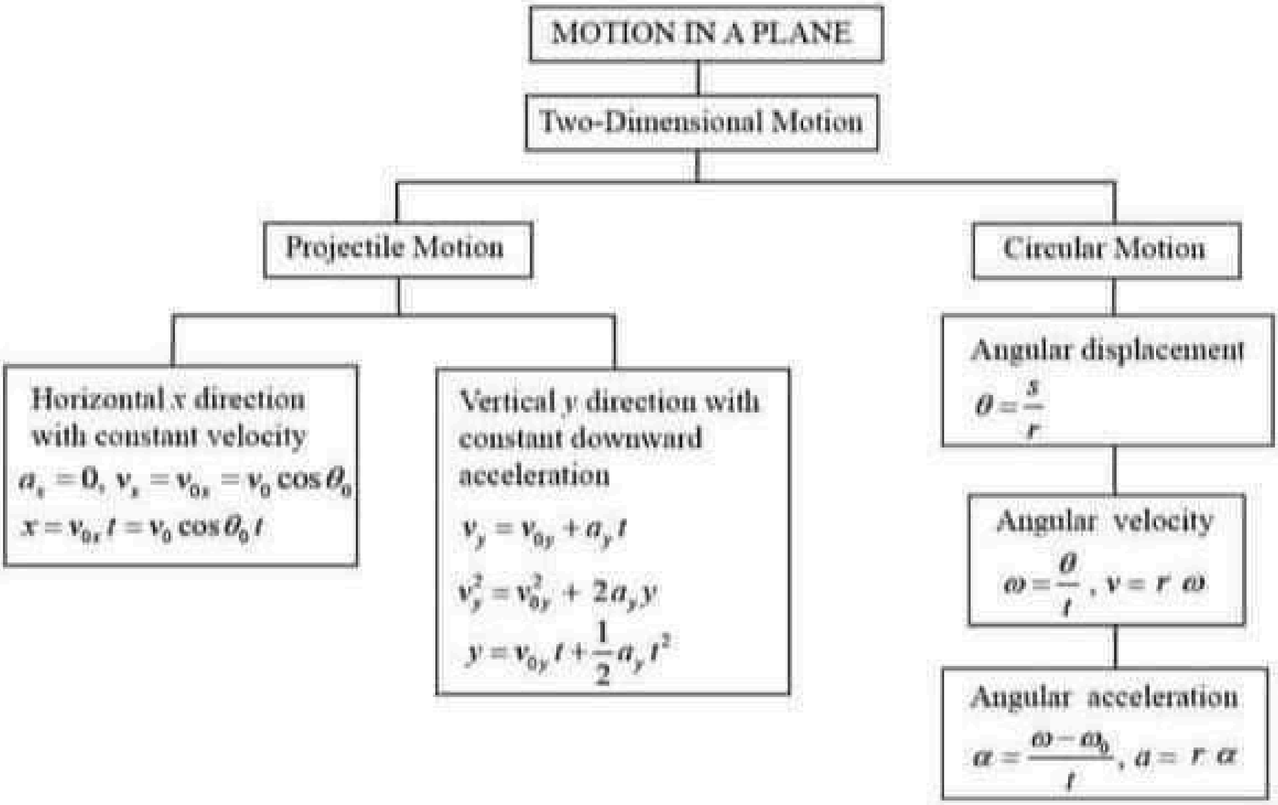
Instantaneous angular velocity is defined as the time rate of change of angular displacement.

Instantaneous angular acceleration is the time rate of change of angular velocity.

EXERCISES

1. A cannon ball is shot with initial velocity 141.4 m s^{-1} with 45° angle of inclination. Find the position and velocity of the cannon ball at 5 s.
2. A stone is thrown horizontally from a cliff 100 ft high. The initial velocity is 20 ft s^{-1}. How far from the base of the cliff does the stone strike the ground?
3. A ball is thrown horizontally with an initial speed of 10 m s^{-1} from an 80 m cliff. How long does it take to reach the ground?
4. A footballer kicked a ball with an initial velocity of 20 m s^{-1} at an angle of 60°.
   (i) How long is the ball in the air? (ii) What are the range and maximum height of the ball?
5. A circular disc rotates initially at rest experiences a uniform angular acceleration of 0.25 rad s^{-2}. What is the angular speed after rotating 10 s?
6. A disc rotating at angular speed of 10 rad s^{-1} is slowed down by a uniform angular acceleration to a speed of 4 rad s^{-1} in 3 s. What is the angular acceleration?
7. The tips of the blades in a food blender are moving with a speed of 21 m s^{-1} in a circle that has a radius of 0.053 m. How much time does take for the blades to make one revolution?

CONCEPT MAP



CHAPTER 2

ROTATIONAL DYNAMICS

The dynamics of translational motion involves describing the acceleration of an object in terms of its mass (inertia) and the forces that act on it. By analogy, the dynamics of the rotational motion involves describing the angular acceleration of an object in terms of its rotational inertia and the torques that act on it.

Learning Outcomes

It is expected that students will

- distinguish between the scalar product and the vector product of two vectors.
- examine the turning effect of a force.
- examine the relationship between torque and the moment of inertia.
- understand the moment of inertia for objects of different shape with different rotational axes.
- explain the concept of equilibrium.
- examine angular momentum and the law of conservation of angular momentum.
- solve problems of daily life events associated with rotational motion.

In this chapter, we will study the rotational motion and vector forms of its physical quantities. Therefore, the manipulation of vectors will be needed to study.

2.1 SCALAR PRODUCT AND VECTOR PRODUCT

Vectors can be multiplied in two different ways; the scalar product and the vector product.

Scalar Product (Dot Product)

In the scalar product, a scalar can be formed by multiplying two vectors. The scalar product of vectors $\vec{A}$ and $\vec{B}$ is defined as Eq. (2.1).

$$\vec{A} \cdot \vec{B} = AB \cos\theta \qquad (2.1)$$

where θ is the angle between $\vec{A}$ and $\vec{B}$, A and B are the magnitudes of $\vec{A}$ and $\vec{B}$ respectively. The scalar product is also called the dot product.

For example, work is the scalar product of force and displacement. Although the force and the displacement are vectors, the work is a scalar.

The scalar product has the commutative property as described by Eq.(2.2).

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \qquad (2.2)$$

Vector Product (Cross Product)

The vector product of vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A}\times\vec{B}=\vec{C} \qquad (2.3)$$

The magnitude of $\vec{C}$ is $C = A\,B\,sin\,\theta$ and the direction of $\vec{C}$ is determined by using the right-hand rule. The direction of $\vec{C}$ is perpendicular to the plane that contains $\vec{A}$ and $\vec{B}$ as shown in Figure 2.1.

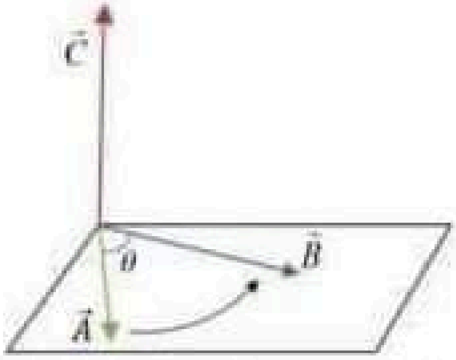


Figure 2.1 The direction of $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$

The result of this product is a vector quantity. The vector product between two vectors is denoted by a cross (×), hence this product is called the cross product.
The vector product does not obey the commutative property as Eq.(2.4).

$$\vec{A}\times\vec{B}=-(\vec{B}\times\vec{A}) \qquad (2.4)$$

The formulae for the torque and the angular velocity are determined by using the vector product.
For example, the tangential velocity in a circular motion is expressed in terms of the radius and the angular velocity as $\vec{v}=\vec{\omega}\times\vec{r}$.

Right-hand rule: To use the right-hand rule, first you have to hold up your right hand so that index finger and middle finger are perpendicular to thumb. Now rotate your hand such that your index finger points in the direction of $\vec{A}$ and your middle finger points in the direction of $\vec{B}$. Your thumb will point in the direction of the cross product $\vec{A}\times\vec{B}$ as shown in Figure 2.2.

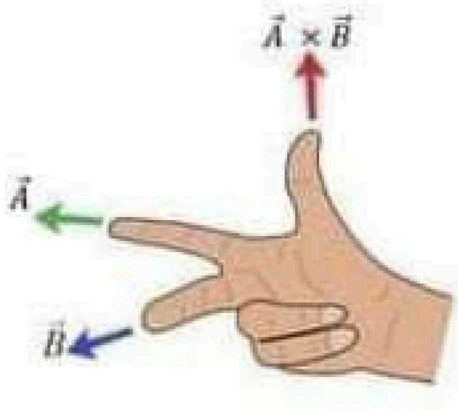


Figure 2.2 Right-hand rule

Example 2.1 The magnitude of $\vec{A}$ and $\vec{B}$ are 5 unit and 8 unit respectively. If the angle between $\vec{A}$ and $\vec{B}$ is 60°, calculate (i) $\vec{A}\cdot\vec{B}$ (ii) $\vec{A}\times\vec{B}$.

The magnitude of $\vec{A} = A = 5$ unit, The magnitude of $\vec{B} = B = 8$ unit, $\theta = 60°$

(i) $\vec{A}\cdot\vec{B} = AB\cos\theta = 5\times 8\cos 60° = 40\times 0.5 = 20$ unit

(ii) Let be $\vec{A}\times\vec{B} = \vec{C}$

The magnitude of $\vec{A}\times\vec{B} = C = AB\sin\theta = 5\times 8\sin 60° = 40\times 0.866 = 34.64$ unit

The direction of $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

Reviewed Exercise

- Why is work a scalar quantity when both force and the displacement are vector quantities?

Key Words: dot product, cross product

2.2 TURNING EFFECT OF FORCE

When a force is applied to an object, it can turn the object about a certain point known as the pivot (or) the fulcrum. Figure 2.3 shows the location of the pivot, the direction of the applied force and the turning effect of the force produced.

There are many examples around us where we use the turning effect of force. Kids playing seesaw, lifting the load by using crowbar, opening a door of a room and tightening (or) loosening a nut by turning a spanner are familiar to us in daily life. In all these cases, the objects experiencing the turning effects are pivoted either at the hinges (or) fulcrums.

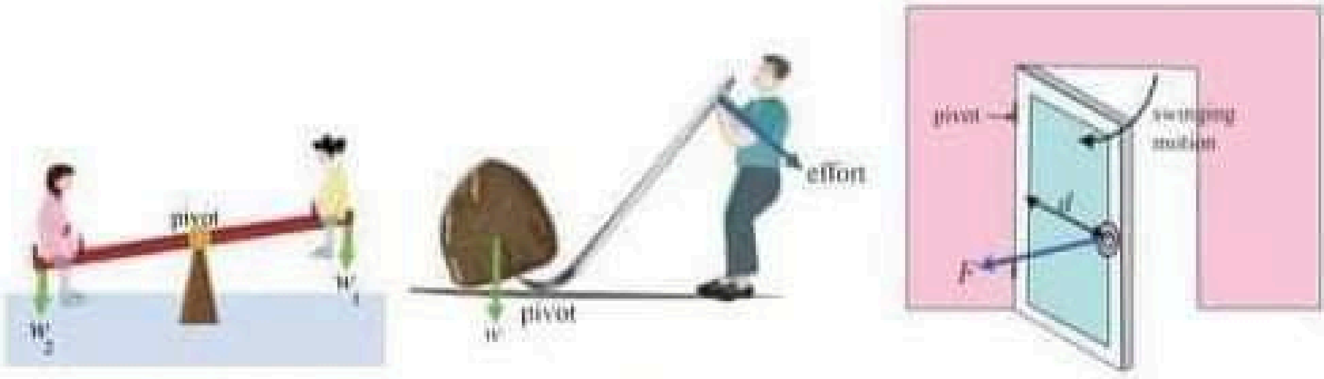


Figure 2.3 Illustrations for turning effect of the force [CREDIT: Source of Internet]

Moment of a Force (or) Torque

A force which acts on a pivoted body at a distance from the fulcrum tends to make that body rotate. The turning effect of a force about a particular fulcrum is measured by the moment of that force (or) torque.

Torque is a vector quantity and its vector form can be expressed as the Eq.(2.5).

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad (2.5)$$

The direction of torque is perpendicular to both $\vec{r}$ and $\vec{F}$ as shown in Figure 2.4. The distance from the fulcrum to the point of action of the force is r and θ is the angle between vectors $\vec{r}$ and $\vec{F}$ as shown in Figure 2.4 and 2.5 (a).

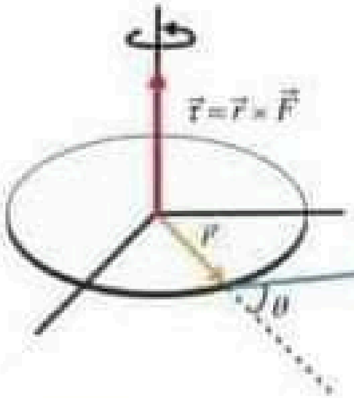


Figure 2.4 Torque as a vector product of $\vec{r}$ and $\vec{F}$

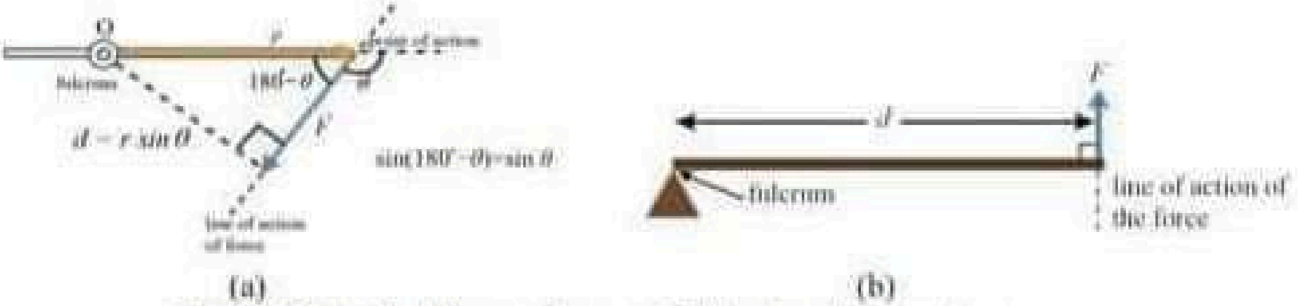


Figure 2.5 Applied force acting on rigid body and moment arm

The magnitude of the moment of force (or) torque τ is defined as the product of the force F and the perpendicular distance d from the line of action of the force to the fulcrum.

$$\tau = F r \sin\theta = Fd \qquad (2.6)$$

where τ = moment of force (or) torque

d = perpendicular distance from the line of action of the force to the fulcrum (moment arm)

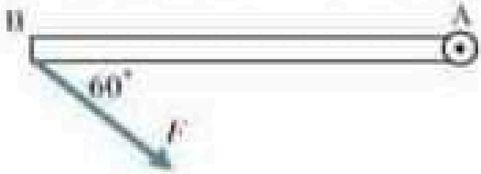
The perpendicular distance from the line of action of the force to the fulcrum ($d = r \sin\theta$) is also called the moment arm of a force.

The line of action of a force is a line along which a force is considered to act.

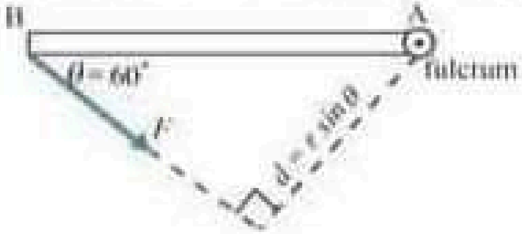
If a force acting on the object is at an angle of 90°, r equals d as shown in Figure 2.5(b).

In SI units, the moment of force (or) torque is measured in newton metre (N m, which is never written as joule).

Example 2.2 The length of a beam AB is 2 m and the force F acting at B is 10 N in given figure. Find the moment arm of the force and the magnitude of torque (the moment of force) about the point A.



To find the moment arm of the force , we must draw the given diagram as follows:



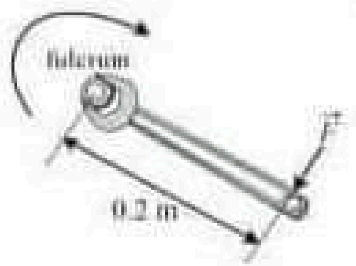
$F = 10\text{ N}$, $r = 2\text{ m}$

$d = r\sin\theta$

$= 2 \times \sin 60° = 2 \times 0.866 = 1.732\text{ m}$

$\tau = F\,d = 10 \times 1.732 = 17.32\text{ N m}$

Example 2.3 In a given figure a force of 10 N is applied to a spanner to tighten a nut. The length of the spanner is 0.2 m. What is the moment of a force exerted when the force acts at (i) the end and (ii) the middle of spanner?



For both case (i) and (ii), since angle between $\vec{F}$ and $\vec{r}$ is 90°,

(i) $d = r\sin\theta = 0.2 \times \sin 90° = 0.2\text{ m}$

The moment of force at the end of spanner,

$\tau = F\,d = 10 \times 0.2 = 2\text{ N m}$

(ii) $d = \frac{r}{2}\sin\theta = \frac{0.2}{2} \times \sin 90° = 0.1\text{ m}$

The moment of force at the middle of spanner,

$\tau = F\,d = 10 \times 0.1 = 1\text{ N m}$

Reviewed Exercise

- Which quantity is a measure of the turning effect of a force?

Key Words: fulcrum, pivot, torque

2.3 RELATION BETWEEN TORQUE AND MOMENT OF INERTIA

Moment of Inertia

Consider a particle of mass m at a distance r from the axis of rotation, being acted upon by a tangential force $\vec{F}$ as shown in Figure 2.6.

Let its angular velocity be ω and its angular acceleration be α.

Tangential acceleration of that particle rotating about an axis is $a = r\,\alpha$.

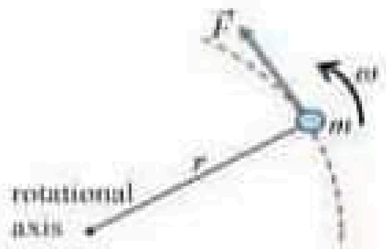


Figure 2.6 Rotation of a particle about a fixed axis with the angular velocity (ω)

According to Newton's second law, $F = ma = mr\alpha$

Torque due to tangential force; $\tau = Fr = (mr\alpha)r$

$$\tau = mr^2\alpha \qquad (2.7)$$

In Eq.(2.7) the terms mr^2 is the moment of inertia of that particle about the axis (I).

$$\tau = I\alpha$$

If an object is composed of the particles of masses $m_1, m_2, m_3, \ldots\ldots, m_i$ and the distances of each particle from the axis of rotation are $r_1, r_2, r_3, \ldots\ldots, r_i$ as shown in Figure 2.7.

The moment of inertia of that object is $m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots + m_ir_i^2$.

The moment of inertia I can be written as

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots + m_ir_i^2$$

$$I = \sum_i m_i r_i^2 \qquad (2.8)$$

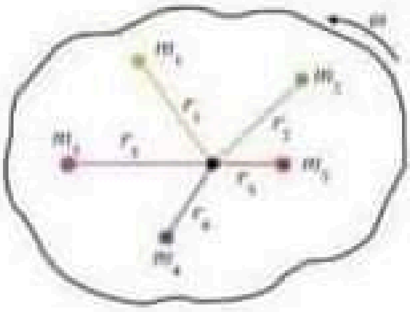


Figure 2.7 A rigid object rotating about an fixed axis with the angular velocity ω

The moment of inertia of an object is defined as the sum of the products obtained by multiplying the mass of each particle in a given object and the square of its distance from the axis. The moment of inertia is a scalar and the SI unit is kg m^2.

The moment of inertia must be specified with respect to a chosen axis of rotation.

Figure 2.8 shows the formulae of moment of inertia for some homogenous objects with respective axis.

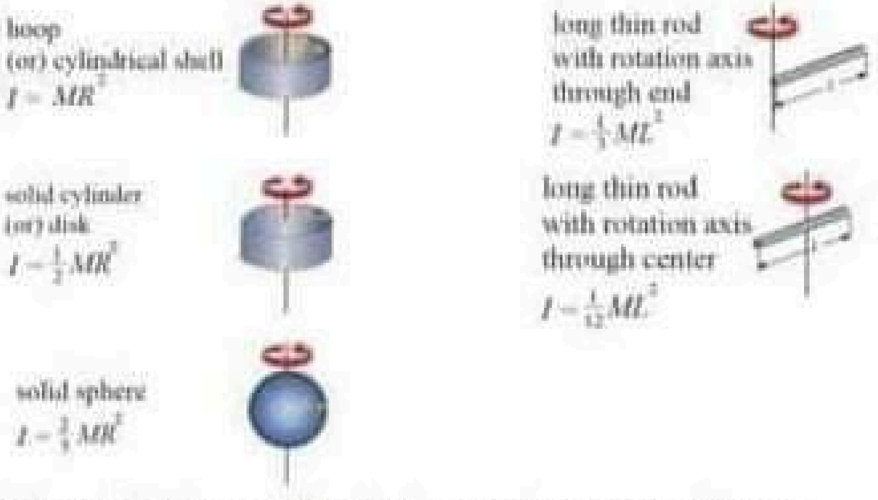
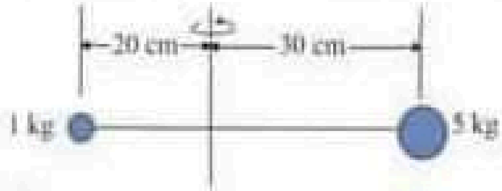


Figure 2.8 Moment of inertia for some homogenous objects

Example 2.4 Two balls are connected by a rigid rod of negligible mass. What is its moment of inertia with respect to the axis as shown in figure? The axis is perpendicular to the rod.



$m_1 = 1$ kg , $r_1 = 20$ cm $= 20 \times 10^{-2}$ m
$m_2 = 5$ kg , $r_2 = 30$ cm $= 30 \times 10^{-2}$ m
The moment of inertia

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 = 1 \times (20 \times 10^{-2})^2 + 5 \times (30 \times 10^{-2})^2$$

$$= 49 \times 10^{-2} = 0.49 \text{ kg m}^2$$

Example 2.5 Calculate the moment of inertia of the earth about its axis of rotation. Mass of the earth is 5.91×10^{24} kg and its radius is 6.38×10^6 m. The moment of inertia of solid sphere about its axis of rotation is $I = \frac{2}{5}MR^2$.

The earth can be assumed as a solid sphere.

mass of the earth $M = 5.91 \times 10^{24}$ kg , radius of the earth $R = 6.38 \times 10^6$ m

$I = \frac{2}{5}MR^2 = \frac{2}{5} \times 5.91 \times 10^{24} \times (6.38 \times 10^6)^2$

$I = 9.62 \times 10^{37}$ kg m^2

Centre of Gravity

The centre of gravity of a particular object is a point at which all its weight may be considered to act. For an object of regular shape and uniform density, the centre of gravity is at its geometrical center as shown in Figure 2.9. For example, the weight of a metre stick of uniform density is considered to be acting at the 50 cm mark (its mid-point) as shown in Figure 2.10.

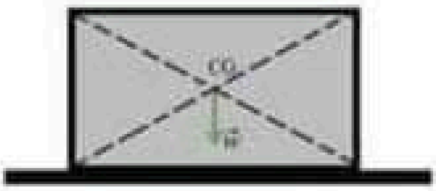


Figure 2.9 The center of gravity (CG) of a regular shaped object

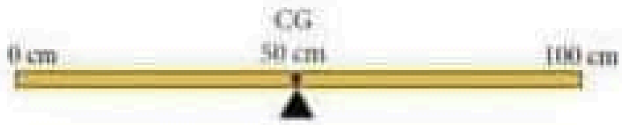


Figure 2.10 Balanced position of the metre rule

Reviewed Exercise

- If the axis is moved to one end of the rod passing through the 5 kg mass in example 2.4, what is the moment of inertia of the system with respect to the axis?

Key Words: torque, moment of inertia

2.4 EQUILIBRIUM

When the system is in static equilibrium, there is no linear motion and no rotational motion. Under this condition, the forces on it must be balanced and the moment of the forces on it must also be balanced.

The conditions of static equilibrium are (1) the resultant force on the system is zero, $\sum \vec{F} = 0$, and (2) the resultant torque on the system is zero, $\sum \vec{\tau} = 0$.

The Principle of Moments

The condition necessary for a pivoted object to be in balance is given by the principle of moments. This principle states that if an object such as a bar (or) a plank is to be in balance, the total clockwise moment about the fulcrum must equal the total anticlockwise moment.

The examples of the application of this principle are building site crane, beam balance etc. (Figure 2.11)

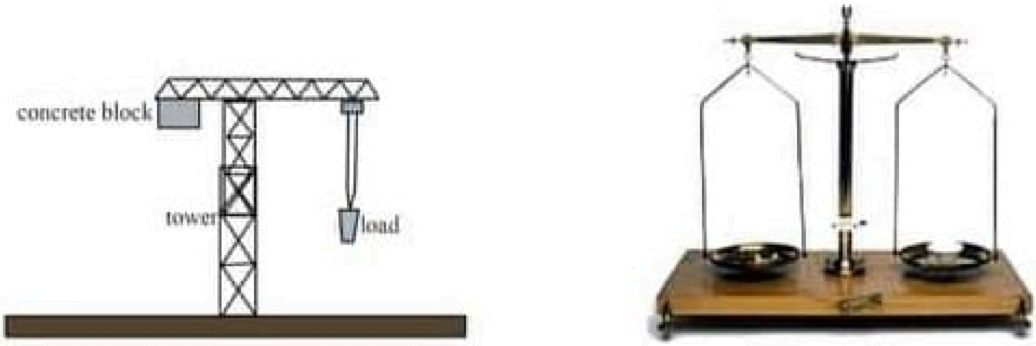
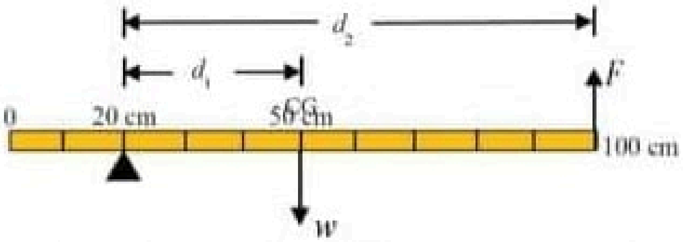


Figure 2.11 Application of principle of moment

Example 2.6 A uniform metre rule weighing 4 N, pivoted at the 20 cm mark, is suppor right-hand end at the 100 cm mark, by a vertical thread. Find the tension in the thread.

First, draw a diagram showing all the forces acting on the metre rule.



$d_1 = 50 - 20 = 30$ cm $= 0.3$ m, $d_2 = 100 - 20 = 80$ cm $= 0.8$ m, $w = 4$ N

In balance, by the principle of moment, taking moments about the pivot,

Total clockwise moment = Total anticlockwise moment

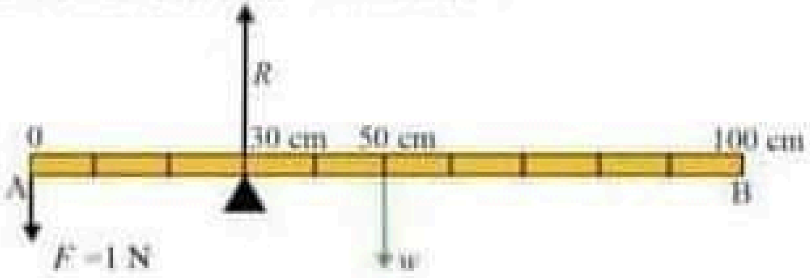
$$w\,d_1 = F\,d_2$$

$$4 \times 0.3 = F \times 0.8$$

$$F = 1.5\ \text{N}$$

The tension in the thread is 1.5 N.

Example 2.7 If AB is a uniform metre rule which is balanced as shown in the diagram, (i) what is the weight of the metre rule? (ii) what is the reaction R?



The perpendicular distance between the line of action of F and the pivot = d_1 = 30 – 0 = 30 cm = 0.3 m
The perpendicular distance between the line of action of w and the pivot = d_2 = 50 – 30 = 20 cm = 0.2 m

(i) By the principle of moment,

Total clockwise moment = Total anticlockwise moment

$$w\, d_2 = F\, d_1$$
$$w \times 0.2 = 1 \times 0.3$$
$$w = 1.5 \text{ N}$$

The weight of the metre rule is 1.5 N.

(ii) Total upward force = Total downward force

$$R = F + w$$
$$R = 1 + 1.5 = 2.5 \text{ N}$$

The reaction R is 2.5 N.

Reviewed Exercise

- State the conditions of static equilibrium when a body is acted by a number of parallel forces.

Key Words: static equilibrium, torque

2.5 ANGULAR MOMENTUM AND LAW OF CONSERVATION OF ANGULAR MOMENTUM

Angular Momentum

In rotational motion, the angular momentum of an object is defined as the product of its moment of inertia and the angular velocity. For linear motion, the momentum is equal to the product of the mass and the velocity.
The moment of inertia is analogous to mass and the angular velocity is analogous to the linear velocity.

$$\vec{L} = I\,\vec{\omega} \qquad (2.9)$$

where $\vec{L}$ = angular momentum, I = moment of inertia, $\vec{\omega}$ = angular velocity.

In SI units, since the unit of moment of inertia is kg m^2 and that of angular velocity is rad s^{-1}, the unit of angular momentum is kg m^2 s^{-1}.

Angular momentum is a vector quantity. The direction of the angular momentum is shown in Figure 2.12.

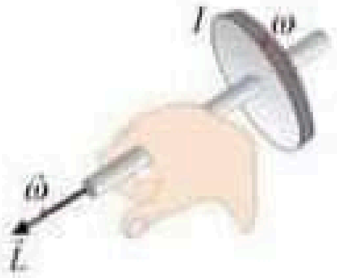


Figure 2.12 The direction of angular momentum $\vec{L}$ and angular velocity $\vec{\omega}$

Law of Conservation of Angular Momentum

The law of conservation of angular momentum is also one fundamental law of physics.

This law states that:

If there is no net external torque acting on an isolated system, the total angular momentum of the system is constant.

In symbols, if $\sum \vec{\tau} = 0$, then $\vec{L}_{total}$ = constant

An example of conservation of angular momentum is seen in an ice skater performing a spin as shown in Figure 2.13 (a). Her angular momentum is conserved because the net torque on her is very small (negligible net torque). Her rate of spin (angular speed) increases greatly as her moment of inertia decreases by pulling in her arms inwards. If she wants to slow down her rotation, she will stretch her arms outwards. When her moment of inertia increases, the rotational angular speed will decrease.

A springboard diver who is rotating when jumping off the board does not need to make any physical effort to continue rotating [Figure 2.13 (b)].

Therefore, we can easily see that the application of conservation of angular momentum to determine the angular velocity of a rotating system in which the moment of inertia is changing.

Figure 2.13 Illustrations for the conservation of angular momentum

Example 2.8 A child of mass of 30 kg stands at the edge of small merry-go-round that is rotating at a rate of 1 rad s^{-1}. The merry-go-round is a disc of radius of 2.5 m and mass of 100 kg. If the child walks in toward the center of the disc and stops 0.5 m from the center, what is the angular velocity of merry-go-round? (The friction can be ignored.)

Moment of inertia of disc $= \frac{1}{2}MR^2$

mass of child $= m = 30$ kg , mass of disc $= M = 100$ kg

radius of disc $= R = 2.5$ m

For child, initial distance from center of disc $r_i = 2.5$ m and

final distance from center of disc $= r_f = 0.5$ m

Since frictional force can be ignored,

$\sum \vec{\tau} = 0$, by conservation of angular momentum $\vec{L}_{total}$ = constant

Initial total angular momentum = Final total angular momentum

$$(L_i)_{disc} + (L_i)_{child} = (L_f)_{disc} + (L_f)_{child}$$

$$I_{disc}\omega_i + I_{child}\omega_i = I_{disc}\omega_f + I_{child}\omega_f$$

$$\left(\frac{1}{2}MR^2 + mr_i^2\right)\omega_i = \left(\frac{1}{2}MR^2 + mr_f^2\right)\omega_f$$

$$\left(\frac{1}{2}\times 100\times(2.5)^2 + 30(2.5)^2\right)1 = \left(\frac{1}{2}\times 100\times(2.5)^2 + 30(0.5)^2\right)\omega_f$$

$$\omega_f = 1.56 \text{ rad s}^{-1}$$

Reviewed Exercise

- What will happen to the angular velocity of an ice skater as she folds her arms and legs close together ?

Key Words: angular velocity, angular momentum, linear momentum

SUMMARY

Torque is a vector quantity and its vector form can be expressed as $\vec{\tau} = \vec{r} \times \vec{F}$.

The magnitude of the moment of force (or) torque is defined as the product of force and the perpendicular distance from the line of action of the force to the fulcrum.

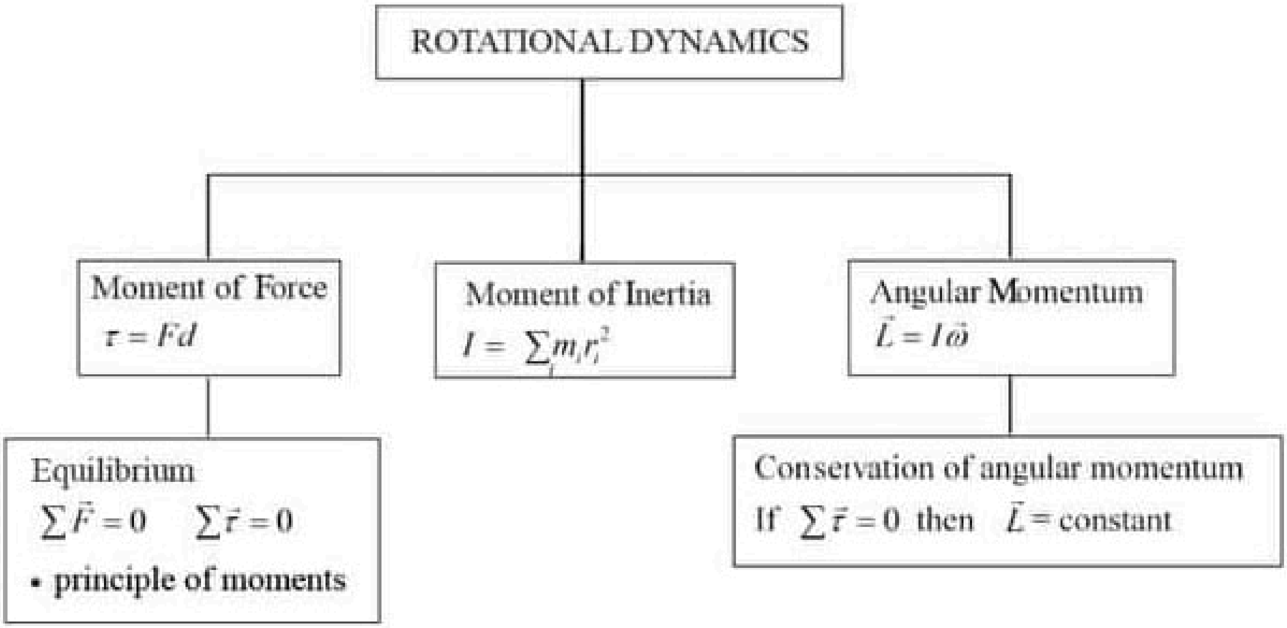
The moment of inertia of an object is defined as the sum of the products obtained by multiplying the mass of each particle in a given object and the square of its distance from the axis.

The **angular momentum** $\vec{L}$ of an object is defined as the product of its moment of inertia I and angular velocity $\vec{\omega}$.

EXERCISES

1. Is there a net torque acting on a rotating object with a constant angular velocity?
2. What is the physical meaning of the moment of inertia?
3. To get the larger turning effect of force, how should the applied force be acted?
4. A student pulls down with a force of 40 N on a rope that winds around a pulley of radius 0.2 m. Find the torque acting on the pulley.
5. A simple pendulum with the bob of 0.5 kg and its length of 80 cm is suspended from a rigid clamp. Find the torque produced by the weight of bob about the point of suspension when the pendulum is swung 20° from vertical line.
6. A student performs an experiment on turning effect of a force using a half metre rule of negligible mass. (i) He holds the half-metre rule at 40 cm mark in a horizontal position and hangs at 5 N weight at 15 cm mark. What is the moment of the weight about his hand? (ii) He then moves the weight to the 5 cm mark. He feels that it is more difficult to maintain the half-metre rule in the horizontal position. Explain why.
7. A bridge over a stream is made from a uniform wooden beam which weighs 4500 N and is 16 m long. Its ends A and B are supported on boulders. If a man weighing 800 N is standing on the bridge 4 m from A, what is the reaction at the boulder (i) under A (ii) under B?
8. A uniform beam of 1 m balances horizontally about a pivot at its midpoint when a weight of 1 N is suspended from the 15 cm mark and another weight w is suspended from the 90 cm mark. Calculate the weight w. In your calculation, why is the weight of beam not included?
9. A 45 kg girl and a 65 kg boy are sitting on a see-saw in equilibrium. If the boy is sitting 0.7 m from the fulcrum, where is the girl sitting?
10. A rod is rotated about a perpendicular axis through at its centre (or) through at one end. (i) Which condition will give the greater moment of inertia? (ii) How much greater will it be?
11. An object spins with an angular speed (ω). If its moment of inertia increases by a factor of 2 without applying an external torque, what will be the its new angular speed?

CONCEPT MAP


ROTATIONAL DYNAMICS
Moment of Force
$\tau = Fd$
Moment of Inertia
$I = \sum_i m_i r_i^2$
Angular Momentum
$\vec{L} = I\vec{\omega}$
Equilibrium
$\sum \vec{F} = 0 \quad \sum \vec{\tau} = 0$
• principle of moments
Conservation of angular momentum
If $\sum \vec{\tau} = 0$ then $\vec{L}$ = constant

CHAPTER 3

PRESSURE

Fluid can exert pressure by virtue of their weight. Fluid represents states of matter that take the shape of their containers. Fluid is either liquid (or) gas.

Learning Outcomes

It is expected that students will

- investigate atmospheric pressure and the use of the barometer.
- investigate pressure in a liquid.
- apply and explain the use of manometers.
- understand buoyancy and verify Archimedes' principle.
- explain and apply Pascal's law.
- apply basic knowledge of pressure in fluid to daily-life phenomena.

The earth is surrounded by the atmosphere up to a height of several miles. The atmosphere which consists largely of masses of gases has weight. Therefore, it is obvious that the atmosphere exerts pressure.

3.1 ATMOSPHERIC PRESSURE

The pressure exerted on a body by the atmosphere, due to the weight of the atmosphere is called atmospheric pressure. At the earth's surface the magnitude of the atmospheric pressure is about 100 kN m^{-2}. Atmospheric pressure which acts on human beings and animals on the surface of the earth is actually very high.

We do not normally feel the large atmospheric pressure because our body is full of air, blood vessels and body fluid, so the pressure inside our body is almost the same as the external pressure and so balance it. This is the reason why we are able to withstand the atmospheric pressure.

Nose bleeding which sometimes occurs at a place of low atmospheric pressure is due to the fact that the blood pressure is higher than the atmospheric pressure.

The atmospheric pressure changes according to locality and time. The atmospheric pressure at the plains is higher than that at the hilly regions.

Example 3.1 Find the force due to the atmosphere which is acting 3 m^2 area on the earth's surface. At the earth's surface, the magnitude of the atmospheric pressure is about 100 kN m^{-2}.

Area $A = 3\ m^2$, $p = 100\ kN\ m^{-2}$

$$p = \frac{F}{A}$$

The force acting due to the atmosphere, $F = p\,A = 100 \times 3 = 300$ kN

Reviewed Exercise

1. On which factors does the atmospheric pressure depend?
2. Why are you able to withstand atmospheric pressure?
3. Why does nose bleeding occur?

Key Words: pressure, atmospheric pressure

3.2 BAROMETER

The Italian scientist Evangelista Torricelli first noticed the variation of pressure due to height and time. He invented and constructed a barometer in 1644.

Barometer is a device for measuring atmospheric pressure. The simple mercury barometer is shown in Figure 3.1. It consists of a glass tube about 1 metre long sealed at one end and filled with mercury. The tube is then inverted and the open end is submerged in a reservoir of mercury; the mercury column is held up by the pressure of the atmosphere acting on the surface of mercury in the reservoir.

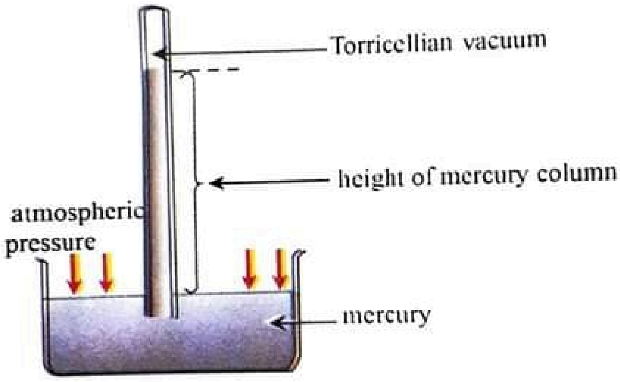


Figure 3.1 Simple mercury barometer

If the height of the mercury column is h, the cross-sectional area of the tube is A, then the volume of the mercury in the column is $h\,A$.

Mass of mercury column is $h\,A\,\rho$, where ρ is the density of mercury.

Weight of mercury column is $h\,A\,\rho\,g$.

In barometer, the force exerted by the atmosphere is balanced by the weight of the mercury column.

$$\text{Pressure exerted by the atmosphere} = \frac{\text{the force exerted by the atmosphere}}{\text{cross-sectional area of tube}}$$

$$p = \frac{\text{weight of mercury column}}{A}$$

$$p = \frac{h\,A\,\rho\,g}{A}$$

The atmospheric pressure in terms of height of mercury (liquid) column in barometer is

$$p = \rho\,g\,h \qquad (3.1)$$

Standard Atmospheric Pressure

The normal atmospheric pressure at sea level is the standard atmospheric pressure. A pressure of 760 mm Hg is known as standard atmospheric pressure (or) 1 atmosphere (1 atm).

The atmospheric pressure can be expressed in pascal as follows.
The density of mercury ρ is 13 590 kg m^{-3}, the acceleration due to gravity g is 9.8 m s^{-2}, height of the mercury column h at standard atmospheric pressure is 760 mm (or) 0.760 m.
Therefore, atmospheric pressure $= \rho g h = 13\,590 \times 9.8 \times 0.760 = 1.013 \times 10^5$ Pa

The standard atmospheric pressure is expressed in various units as shown below.
1 atm $= 1.013 \times 10^5$ Pa $=$ 1013 hPa $=$ 1.01 bar $=$ 14.7 lb in^{-2} (psi) $=$ 760 mm Hg $=$ 760 torr

It must be noted that the vertical height of the mercury column depends only on the pressure outside the tube as shown in Figure 3.2 (a). It does not depend on the tilt of the column. Figure 3.2 (b) shows the barometer being tilted but the vertical height h of mercury column remains unaffected, and independent of the diameter (width) of the tube as shown in Figure 3.2 (c).
The pressures are the same at each of the points marked x in Figures because the pressure in a liquid doesn't depend on the container angle (or) width. Of course if the tube is lowered below 760 mm, the mercury would completely fill the tube as shown in Figure 3.2 (d).

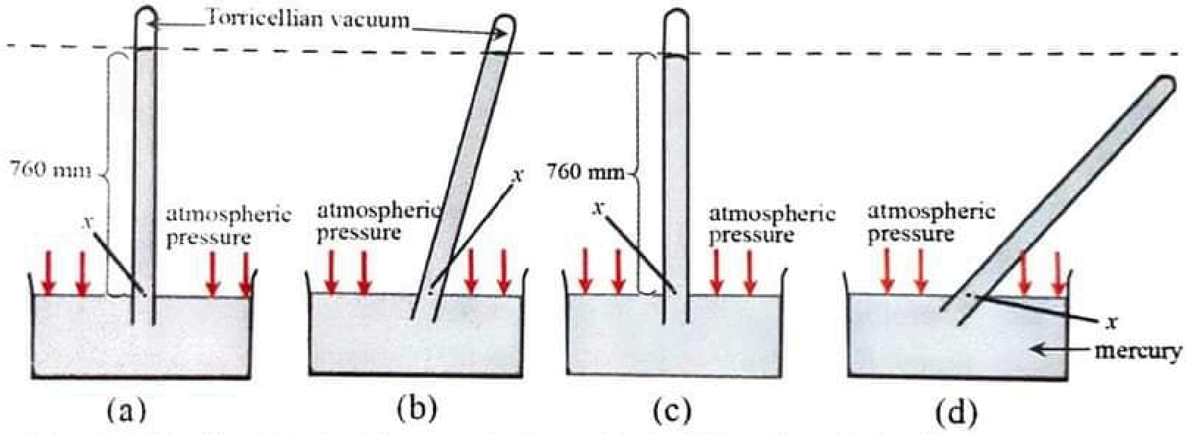


Figure 3.2 The height of mercury column is independent of the tilt and the diameter of the tube

Some Applications of the Atmospheric Pressure in Daily Life

(1) Sucking liquid by a drinking straw
The action of sucking increases the volume of the lungs, thereby reducing the air pressure in the lungs and the mouth. The atmospheric pressure acting on the surface of the liquid will then be greater than the pressure in the mouth, thus forcing the liquid to rise up the straw into the mouth. It is illustrated in Figure 3.3 (a).

(2) Drawing a liquid into a syringe
When the piston is pulled up, the pressure inside the cylinder decreases. Atmospheric pressure acting on the liquid drives the liquid into the cylinder through the nozzle as shown in Figure 3.3 (b).

(3) Pressing rubber sucker on a flat smooth surface
When pressing on a smooth surface, most of the air inside the rubber sucker is squeezed out and the pressure is reduced. The sucker is held in position by the atmospheric pressure on its outside surface as shown in Figure 3.3 (c).

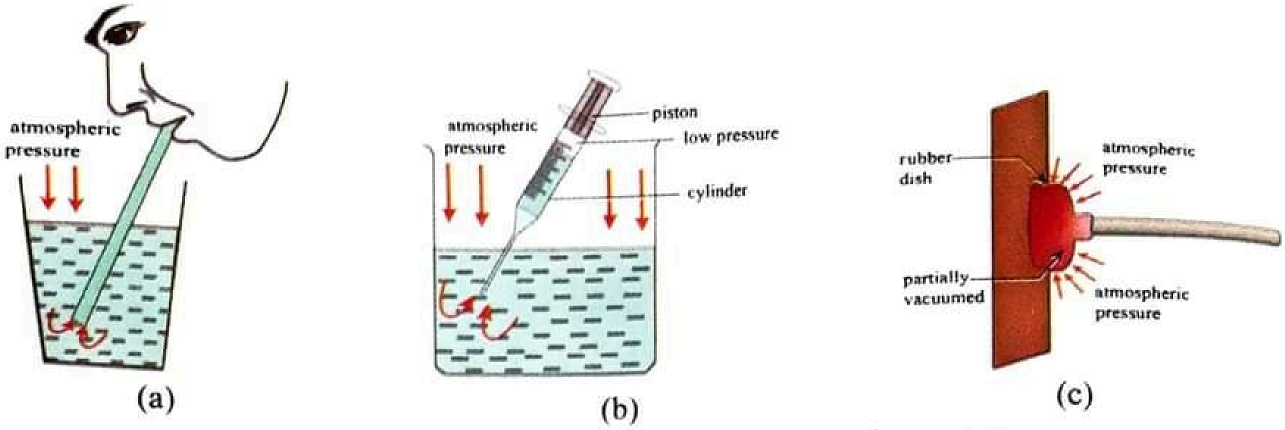


Figure 3.3 Some applications of the atmospheric pressure

Example 3.2 Express 2 atm pressure in mmHg and bars.

$$2 \text{ atm} = 2 \times 760 \text{ mm Hg}$$
$$= 1520 \text{ mm Hg}$$
$$2 \text{ atm} = 2 \times 1.01 \text{ b}$$
$$= 2.02 \text{ b}$$

Example 3.3 Compare the atmospheric pressures and forces acting on a man and a child who are standing side by side.

Let the force acting on the man be F_1, the force acting on the child be F_2, the body surface area of the man be A_1 and that of the child be A_2.

$$A_1 > A_2$$

Since the man and the child are standing side by side at the same place, atmospheric pressures p are the same.

Therefore, $p A_1 > p A_2$

Since $F = p A$, $F_1 > F_2$

Therefore, the force acting on the man is greater than the force acting on the child.

Reviewed Exercise

1. Why is mercury used in a barometer rather than water although mercury is a hazardous substance?
2. At sea level, the atmospheric pressure is 76 cm Hg. Assuming that pressure falls by 10 mm Hg per 120 m ascent. What is the height of a mountain where the barometer reads 70.5 cm Hg?

Key Words: normal atmospheric pressure, standard atmospheric pressure

3.3 PRESSURE IN A LIQUID

A liquid exerts pressure because of its weight as shown in Figure 3.4. The pressure depends on the depth under the surface of the liquid.

Let us fill a cylindrical container having bottom surface area A with a liquid of density ρ up to a height h. Since the volume of liquid in container is $V = A\,h$, the mass of liquid which fills the container is $m = \rho V = \rho A h$.

Therefore, the weight of the liquid will be $w = m g = \rho g A h$.

Force exerted by the liquid on the bottom of container is equal to weight of the liquid in the container.

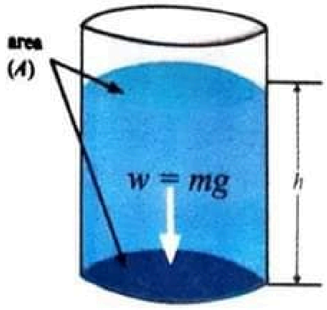


Figure 3.4 Liquid in a cylindrical container

Thus, the pressure exerted by the liquid at the bottom surface is

$$p = \frac{F}{A} = \frac{w}{A} = \frac{\rho g h A}{A}$$

$$p = \rho g h \qquad (3.2)$$

It is seen therefore that the pressure ($p = \rho g h$) exerted by the liquid is directly proportional to the height (or) depth of the liquid h and the density ρ. Although the weight of the liquid depends on its base area, the pressure exerted by the liquid is independent of the base area (cross-sectional area) in Eq.(3.2). The above result is true not only for a point at the bottom of the container but also for any depth inside the liquid.

The pressure p in the above discussion is only the liquid pressure. Actually, there is atmospheric pressure at the surface of the liquid in the container. Therefore, the true pressure at the depth h in the liquid will be,

$$p = p_{atm} + \rho g h \qquad (3.3)$$

where p_{atm} is the atmospheric pressure.

The deeper the point inside the liquid the greater is the pressure at that point. Since the weight of liquid becomes greater as the depth increases, the pressure also increases with depth as shown in Figure 3.5.

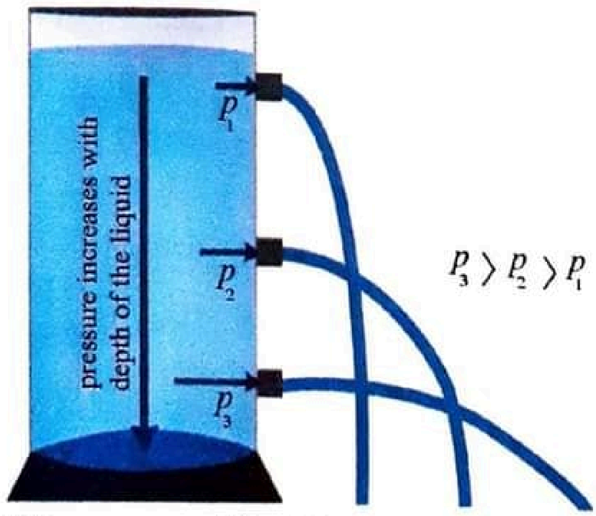


Figure 3.5 The pressure of liquid increases with depth of liquid

The pressure in a liquid at a particular point acts equally in all direction in Figure 3.6 (a). Pressure at any point inside a liquid is the same in all directions. Let a body be totally immersed in a liquid which is in a container. There will be pressure not only at the top of the body but also upward pressure at the bottom of the body and lateral pressures at the sides of the body in Figure 3.6 (b). Figure 3.6 (c) shows that pressure will be exerted from every direction on the body of spherical shape.

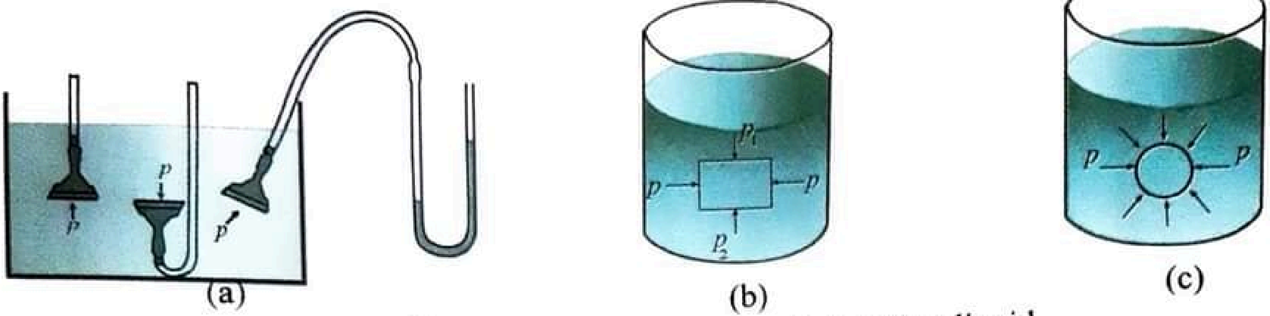


Figure 3.6 Pressure at any point inside a liquid

Figure 3.7 shows liquids of the same density in containers all having the same height. The pressure exerted on their bases would be the same even though their weights and shapes are different.

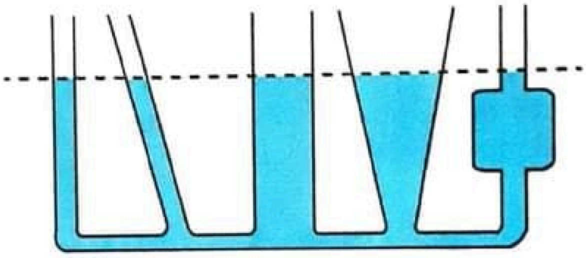


Figure 3.7 Pressure in different shape containers

Example 3.4 The total pressure at the bottom of a tank is 3 atm. To what height has the water been filled in the tank?

The pressure at the water surface in the tank is $p_{atm} = 1$ atm $= 1.01\times10^5$ Pa

The total pressure at the bottom of a tank is $p = 3$ atm $= 3\times1.01\times10^5$ Pa $= 3.03\times10^5$ Pa

Let height of water filled in the tank be h and density water is 1000 kg m^{-3}

The total pressure at the bottom of the tank is

$$p = p_{atm} + \rho g h$$

$$\rho g h = p - p_{atm}$$

$$1000 \times 9.8 \times h = (3-1) \times 1.01 \times 10^5$$

$$h = \frac{2\times1.01\times10^5}{10^3\times9.8}$$

$$= 20.61 \text{ m}$$

Example 3.5 The density of sea water is 1025 kg m^{-3}. How many times is the pressure at the depth of 2 km under the sea surface greater than the atmospheric pressure?

Density of sea water ρ = 1025 kg m^{-3} , $h = 2$ km $= 2 \times 10^3$ m
Since, the total pressure in a liquid, $p = p_{atm} + \rho gh$

The total pressure at the depth of 2 km under the sea surface is

$$p = p_{atm} + \rho g h$$

$$p_{2km} = p_{atm} + \rho g h$$

$$\frac{p_{2km}}{p_{atm}} = 1 + \frac{\rho g h}{p_{atm}}$$

$$= 1 + \frac{1025 \times 9.8 \times 2 \times 10^3}{1.01 \times 10^5}$$

$$= 199.9$$

Example 3.6 The pressure at the height of 1 m from the floor is the normal atmospheric pressure 1.01×10^5 Pa. If the temperature is 0 °C, what is the difference between the pressure on the floor and pressure at 1 m height? (density of air = 1.29 kg m^{-3})

$h = 1$ m, $p_{1m} = p_{atm} = 1.01 \times 10^5$ Pa
The total pressure on the floor is

$$p_{floor} = p_{atm} + \rho_{air}\, g\, h$$

$$p_{floor} = p_{1m} + \rho gh$$

$$p_{floor} - p_{1m} = \rho gh$$

$$p_{floor} - p_{1m} = 1.29 \times 9.8 \times 1$$

$$p_{floor} - p_{1m} = 12.64 \text{ Pa}$$

Reviewed Exercise

1. Why does the thickness of the dam increase downwards?
2. On which factors does the pressure in a liquid depend?

Key Words: liquid pressure, true pressure

3.4 MANOMETER

A glass tube open at both ends and bent into a U shape serves as a sensitive device for measuring pressure when filled with coloured water or light oil. Such a device is called a manometer as shown in Figure 3.8. Mercury can also be used as the filling liquid for a manometer.

When both sides of the U-tube are exposed to the atmosphere, the respective pressures exerted on the liquid columns in both sides are the same and the levels of the liquid in the two sides are the same in Figure 3.8 (a). If, however, the pressures on the two liquid columns are different, the levels will no longer be the same in Figure 3.8 (b).

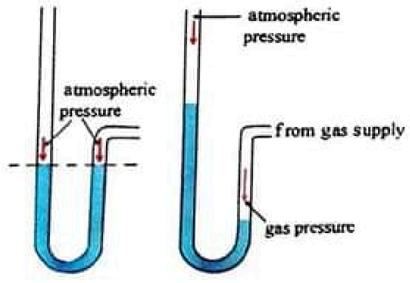


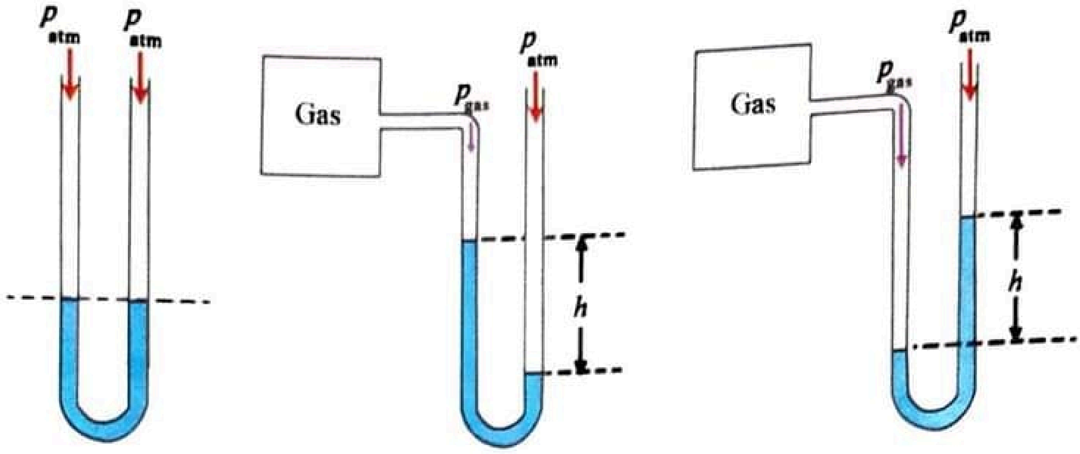
Figure 3.8 Manometer

Working Principle of Manometer

In Figure 3.9 (a), only the atmospheric pressure is exerted on the surface of liquid in two columns of the manometer.

In Figure 3.9 (b), the gas pressure p_{gas} is lower than the atmospheric pressure p_{atm}.

In Figure 3.9 (c), the gas pressure p_{gas} is higher than the atmospheric pressure p_{atm}.



(a)

$p_{gas} < p_{atm}$

$p_{gas} = p_{atm} - \rho g h$

(b)

$p_{gas} > p_{atm}$

$p_{gas} = p_{atm} + \rho g h$

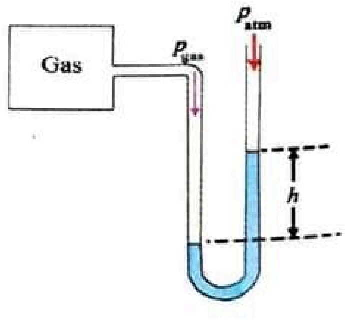
(c)

Figure 3.9 Pressure measured by the manometer

Manometers are very sensitive for measuring the pressure differences, especially when the filling liquid is water (or) light oil.

A manometer filled with mercury is not sensitive. A sphygmomanometer which is used to measure blood pressure is a one kind of mercury-filled manometer.

Example 3.7 A mercury manometer connected to a gas supply is shown in figure. If the difference in height of mercury column is 5 cm, calculate the gas pressure from the gas supply. Density of mercury is 13.6×10^3 kg m^{-3}.



$h = 5$ cm $= 5 \times 10^{-2}$ m , $\rho = 13.6 \times 10^3$ kg m^{-3}

Pressure of gas produced by gas supply

$$p_{gas} = p_{atm} + \rho g h$$

$$p_{gas} = 1.01 \times 10^5 + 13.6 \times 10^3 \times 9.8 \times 5 \times 10^{-2}$$

$$= 1.07 \times 10^5 \text{ Pa}$$

Reviewed Exercise

- Does the difference in height between two liquid levels in a manometer depend on the diameter of tube?

Key Words: manometer, pressure difference

3.5 ARCHIMEDES' PRINCIPLE

When bodies are immersed in a liquid there is loss in weight. This is because of a property of liquids called buoyancy. Since buoyant force is directed upward, it is called upward thrust.

Let us consider a block which is totally immersed in a liquid of density ρ as shown in Figure 3.10. Let the top of block be at the depth of h from the surface of the liquid, the thickness of the block be H, and top and bottom surface area be A.

The volume of the block is $V = A\,H$. Since the block is totally immersed in a liquid, the volume of the block V is equal to the volume of liquid displaced by the block.

The pressure on the top surface of the block $p_1 = p_{atm} + \rho g h$.

The pressure on the bottom surface of the block $p_2 = p_{atm} + \rho g (h + H)$.

Therefore, the downward force which is acting on a block is $F_1 = A\,p_1$ and upward force is $F_2 = A\,p_2$.

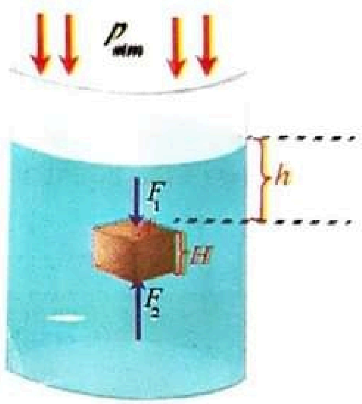


Figure 3.10 Forces on the surfaces of the block

The forces acting on the sides of the block cancel out. Then, the net force acting on the block in the upward direction is

$$F = F_2 - F_1$$

$$F = A\,(p_2 - p_1) = A\,[p_{atm} + \rho g\,(h+H) - (p_{atm} + \rho g h)]$$

$$F = A \rho g H$$

This force is called the buoyant force (or) upward thrust.

Since the volume of the block is $V = A\,H$, we have

$$F = V \rho g \qquad (3.4)$$

Therefore, it is found that

upward thrust = weight of liquid displaced.

Archimedes' principle states that the upward thrust acting on a body which is immersed in a liquid is equal to the weight of the liquid displaced by the body.

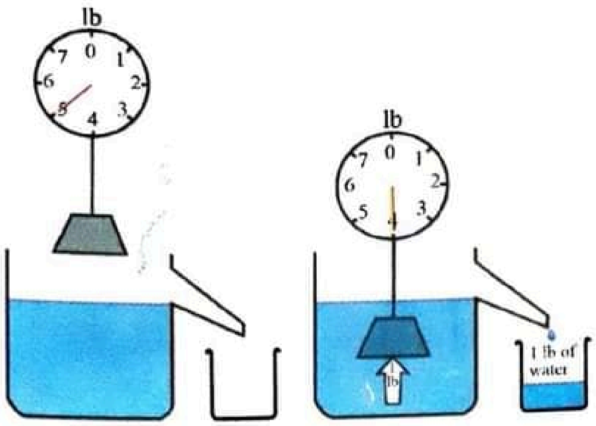


Figure 3.11 Upward thrust is equal to the weight of the liquid displaced

Archimedes' principle is true not only for liquids but also for gases. The densities of various substances can be obtained by using Archimedes' principle.

Apparent Weight of a Body

The weight of a body when it is immersed in a liquid is called apparent weight. It is less than the actual weight of the body because of upward thrust of the liquid acting on it.

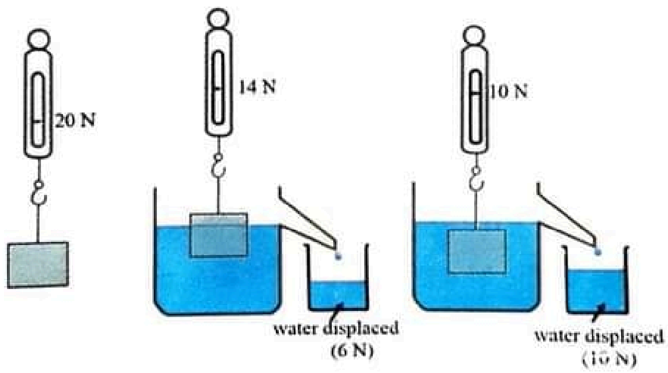


Figure 3.12 The weight of a body in a liquid

Let the volume of the body be V and its density be ρ.

Weight of the body before it is immersed in a liquid is its actual weight,

$$w_i = mg = \rho V g \quad (3.5)$$

Upward thrust of the liquid = $\rho_0 V g$

ρ_0 is density of the liquid.

Weight of the body immersed in the liquid (apparent weight) $w_f = w_i$ − upward thrust.

$$w_f = \rho V g - \rho_0 V g \quad (3.6)$$

Therefore,

$$\frac{w_f}{w_i} = \frac{(\rho - \rho_0)}{\rho} \quad (3.7)$$

Floating Body in a Liquid

If the weight of the body is greater than the upward thrust, the body will sink and if the weight is smaller than the upward thrust, the body will rise up to the surface. A body will float in a liquid (fluid) if the upward thrust acting on it is equal to its weight.

If the volume of the immersed portion of the body is V_s, the upward thrust is $\rho V_s g$, where ρ_0 is the density of liquid.

The weight of the body is $w = mg = \rho V g$

where ρ is density of the body, V is the volume of the body.

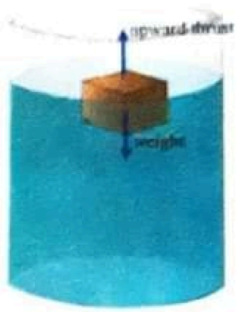


Figure 3.13 Balanced forces acting on the floating body

Since the body is in equilibrium, the net force acting on it is zero. The magnitude of upward thrust acting on it is equal to its weight.

weight = upward thrust

$$\rho V g = \rho_0 V_s g$$

$$\frac{\rho}{\rho_0} = \frac{V_s}{V} \qquad (3.8)$$

According to Eq.(3.8), the ratio of the densities is equal to the ratio of the immersed volume to the volume of the whole body.

Example 3.8 The weight of a body in air is 300 N and the weight is 200 N when it is immersed in water. Find the density and the volume of the body.

Actual weight of a body in its normal condition $w_i = 300$ N
Weight of the body in the liquid (apparent weight) $w_f = 200$ N
density of water $\rho_0 = 1000 \text{ kg m}^{-3}$

$$w_i = mg = \rho V g$$

$$w_f = \rho V g - \rho_0 V g$$

$$\frac{w_f}{w_i} = \frac{(\rho - \rho_0)}{\rho}$$

$$\frac{200}{300} = \frac{(\rho - 1000)}{\rho}$$

$$\rho = 3000 \text{ kg m}^{-3}$$

Let the volume of the body be V.

$$w_i = \rho V g$$

$$300 = 3000 \times V \times 10$$

$$V = 0.01 \text{ m}^3$$

Example 3.9 An iceberg is a large piece of freshwater ice, which has a density of 0.92×10^3 kg m^{-3} at 0°C. Ocean water has a density of about 1.025×10^3 kg m^{-3}. What fraction of an iceberg lies below the surface?

Density of ice ρ = 0.92×10^3 kg m^{-3}
Density of ocean water ρ_0 = 1.025×10^3 kg m^{-3}
Volume of iceberg = V
Immersed volume of iceberg = V_s

The fraction of iceberg which is immersed is $\frac{V_s}{V} = \frac{\rho}{\rho_0}$

$$\frac{V_s}{V} = \frac{0.92\times10^3}{1.025\times10^3} = 0.898$$

89.8% of the iceberg will lie below the surface.

Example 3.10 A helium balloon is designed to support a load of 1000 kg. If the balloon is filled with helium, what should its volume be? The mass of helium is not included in the net load 1000 kg. (Density of air = 1.29 kg m^{-3}, Density of helium = 0.18 kg m^{-3})

Density of air ρ_{air} = 1.29 kg m^{-3}
Density of helium ρ_{He} = 0.18 kg m^{-3}
mass of load = 1000 kg
Weight of load = m g
Weight of helium = $\rho_{He} Vg$
Buoyant force = $\rho_{air} Vg$

Since balloon is in equilibrium while supporting the load, the net force acting on it is zero.

Weight of load + Weight of helium = Buoyant force

$$mg + \rho_{He} Vg = \rho_{air} Vg$$
$$1000 + 0.18\,V = 1.29\,V$$
$$V = 900.9 \text{ m}^3$$

Reviewed Exercise

1. An ocean-liner was loaded at the port of Yangon. Would the ocean-liner sink deeper or not when it reached the ocean? (Density of seawater is greater than that of freshwater.)
2. Why is the weight of body lost when it is immersed in a liquid?
3. Under what condition can a body float in a liquid?
4. A steel block floats in mercury but sinks in water. So how does a steel ship manage to float in water?

Key Words: upward thrust, apparent weight, buoyancy

3.6 PASCAL'S LAW

Any increase in pressure at the surface of an enclosed fluid must be transmitted to every other point in the fluid. This concept was first recognized by French scientist Balise Pascal in 1650.

When a fluid completely fills a vessel and a pressure is applied to it at any part of the surface, that pressure is transmitted equally throughout the whole of the enclosed fluid. This is known as Pascal's law.

The concept of Pascal's law is very useful in practice in the working principle of the hydraulic brakes and hydraulic presses.

Hydraulic Brakes

Hydraulic brakes are used in cars and other vehicles. Figure 3.14 shows how the car hydraulic disc brake system works.

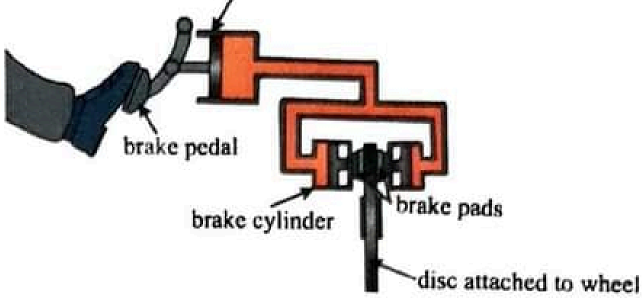


Figure 3.14 Hydraulic disc brake system

Hydraulic Press

A hydraulic press is a very useful machine. It is used for baling jute, hay, cotton and also for the pressing of the automobile bodies and for shaping steel and metal sheets.

Figure 3.15 shows how a small effort applied on a hydraulic press is turned into a large force. Therefore it is named force multiplier.

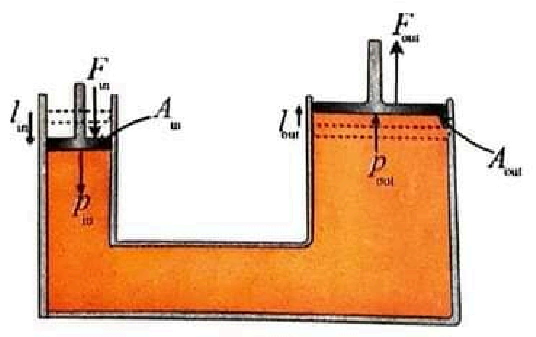


Figure 3.15 Schematic diagram of hydraulic press

The pressure obtained by applying effort F_{in} on area A_{in} of small piston is $p_{in} = \frac{F_{in}}{A_{in}}$.

According to Pascal's law, the pressure p_{in} is equal to the pressure p_{out} exerted on area A_{out} of large piston.

$$p_{in} = p_{out} \qquad (3.9)$$

The upward force F_{out} on area A_{out} is

$$F_{out} = p_{in} \times A_{out}$$

$$F_{out} = \frac{F_{in}}{A_{in}} \times A_{out} \qquad (3.10)$$

Assuming there is no friction in the hydraulic press system, the work done by effort equals the work done by the upward force.

Therefore, $$F_{in} l_{in} = F_{out} l_{out} \quad (3.11)$$

where l_{in} = the distance travelled by small piston

l_{out} = the distance travelled by large piston

Hydraulic Lift

Another useful machine based on Pascal's law is a hydraulic lift as shown in Figure 3.16.

By applying a small force on the small piston produces a large upward force on the large piston which can lift a large load.

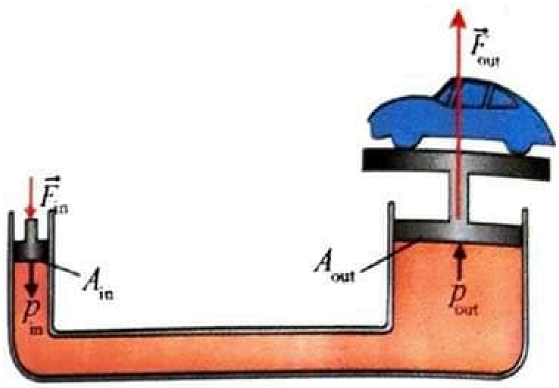


Figure 3.16 Hydraulic lift

Example 3.11 The small piston of a hydraulic lift has a cross-sectional area of $3\times10^{-4}\,m^2$ and its large piston has a cross-sectional area of $2\times10^{-2}\,m^2$. What must the downward force be applied to a small piston for the lift to raise a load whose weight is 15 kN?

$A_{in} = 3\times10^{-4}\,m^2$, $A_{out} = 2\times10^{-2}\,m^2$

Weight of load = upward thrust on the large piston

Therefore, $F_{out} = 15\text{ kN} = 15\times10^3\text{ N}$

According to Pascal's law, $p_{in} = p_{out}$

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$$

$$F_{in} = \frac{F_{out}}{A_{out}} \times A_{in} = \frac{15\times10^3}{2\times10^{-2}} \times 3\times10^{-4}$$

$$F_{in} = 225\text{ N}$$

Reviewed Exercise

- Which machines are based on Pascal's law in daily life?

Key Words: hydraulic brake, hydraulic press

SUMMARY

The pressure exerted on a body by the atmosphere, due to the weight of the atmosphere is called **atmospheric pressure**.

When bodies are immersed in a liquid there is loss in weight. This is because of a property of liquids called **buoyancy**.

EXERCISES

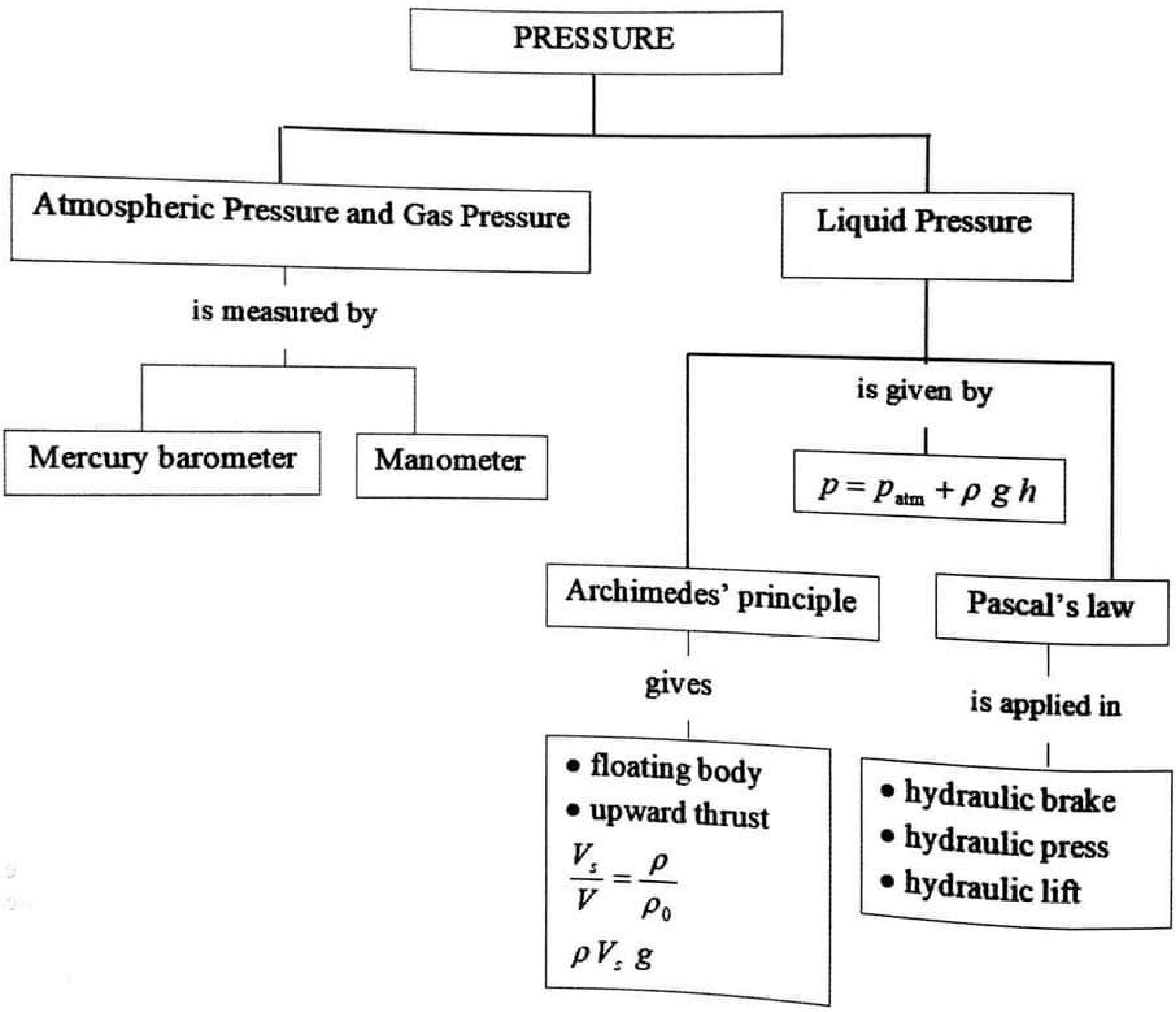
1. What will be the effect, if any, on the mercury column if the glass tube used has (i) a smaller internal diameter (ii) a slightly bigger internal diameter?
2. Calculate the height of a column of water which could be supported by the atmosphere at sea level. (Density of water is 1000 kg m^{-3})
3. What will be the new height of the column, if water is used instead of mercury? (mercury is 13.6 times heavier than water)
4. What is the height of a column of turpentine that would exert the same pressure as 5.0 cm of the mercury? (Density of turpentine = 840 kg m^{-3}, Density of mercury = 13 600 kg m^{-3})
5. Find the pressure on a diver who is at a depth of 5 m below surface of the water.
6. An object of density 2×10^3 kg m^{-3} weighs 100 N less when it is weighed while completely submerged in water than when it is weighed in air. What is the actual weight of this object?
7. The weighted rod floats with 6 cm of its length under water (Density 1000 kg m^{-3}). What length is under the surface when the rod floats in brine? (Density of brine 1200 kg m^{-3})
8. Why is it easier to float in the sea than in a swimming pool?
9. A fish rests on the bottom of a bucket of water while the bucket is being weighed on a scale. When the fish begins to swim around, does the scale reading change? Explain.
10. A beaker containing water and placed on a pan is balanced by the weight which is in the other pan of the balance. Explain what will happen if a man immerses his finger in the water without touching the beaker.
11. The density of lead block is 11.5 g cm^{-3} and it is floating in mercury of density 13.6 g cm^{-3}. (i) What portion of the lead block is immersed in mercury? (ii) What force is needed to press the block to immerse it totally if the mass of the lead block is 2 kg?
12. A plastic cube 30 cm on each side and with a mass of 20 kg floats in water. What fraction of the volume of cube is above the surface of the water? (Density of water is 1000 kg m^{-3})
13. A 30 kg balloon is filled with 100 m^3 hydrogen. What force is needed to hold the balloon to prevent it from rising up? (Density of hydrogen is 0.09 kg m^{-3})

14. The areas of the pistons of a hydraulic press are 2 in^2 and 10 in^2. How much effort should be applied on the small piston to produce an upward force of 500 lb on the large piston?

15. A hydraulic (water power) press consists of 1 cm and 5 cm diameter pistons. (i) What force must be applied on the small piston so that the large piston will be able to raise 10 N load? (ii) To what height would the load be raised when the small piston has moved 0.1 m?

16. A 15 N force is exerted on one piston of a simple hydraulic press. Its area is 0.025 m^2 and area of another piston is 0.50 m^2. Calculate (i) the pressure exerted on the liquid by the small piston (ii) pressure exerted on the large piston (iii) force exerted on the large piston (iv) maximum weight of load that can be lifted, if weight of large piston is 50 N.

CONCEPT MAP



CHAPTER 4

POWER AND EFFICIENCY

In physics, work is done only when an object moves under the influence of a force. It is often useful to consider the rate at which work is done.

Learning Outcomes

It is expected that students will

- calculate power and properly use the correct units of measurement.
- investigate efficiency.
- explain elastic potential energy and Hooke's law.
- apply basic knowledge of power and efficiency to daily life phenomena.
- know the sources of energy.

Whenever a force acting on an object produces movement, work is done by the force on the object and produces change in energy. The importance of the concept of work has also been described.

The rate of work (or) the rate of energy change is also important in practice.

4.1 POWER AND ITS UNITS

Work may be defined as the scalar product of the force applied and the displacement.

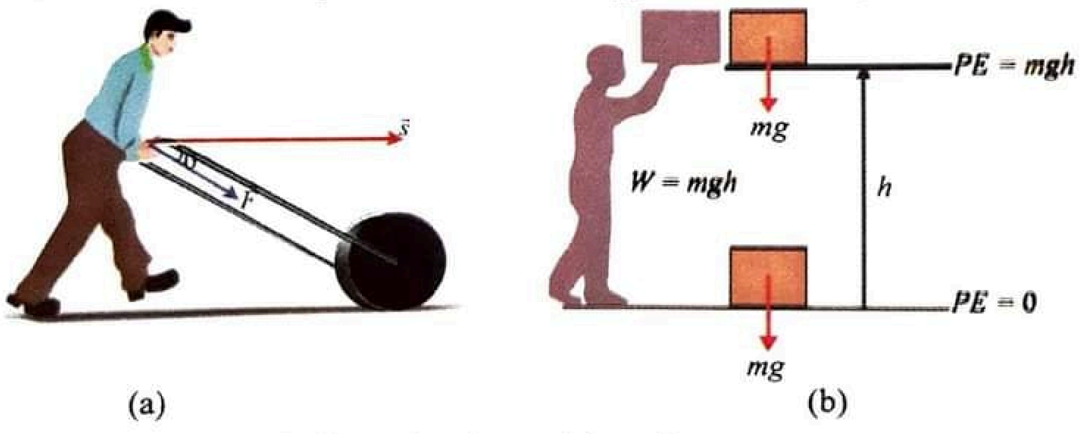


Figure 4.1 Illustration for work by a force

In Figure 4.1(a), if the force exerted by the man on roller is $\vec{F}$, the distance moved along the horizontal direction is s and the angle between $\vec{F}$ and $\vec{s}$ is θ, the work by the force is $W = \vec{F}.\vec{s} = Fs\cos\theta$.

In Figure 4.1(b), if a man lifts an object to a certain height, the potential energy of object will change. The change in potential energy is equal to the work done on the object. In this case $W = \Delta PE = mgh - 0 = mgh$. We notice that work is equal to change in energy.

Power

There are many cases where it is necessary to know the magnitude of the work done, but for some other cases it is more important to know the rate of doing work rather than the total amount of the work done.

The rate of doing work is defined as power.

Let W be the work done in time period t.

Then the power P is

$$P = \frac{W}{t} \tag{4.1}$$

The unit for power in SI units is the watt (W). If the work done is 1 joule in 1 second, the power is 1 watt.

$$1\ \text{W} = 1\ \text{J s}^{-1}$$

The units of power which are larger than watt are kilowatt (kW) and megawatt (MW).

$$1\ \text{kW} = 1000\ \text{W} = 10^3\ \text{W}$$
$$1\ \text{MW} = 1\ 000\ 000\ \text{W} = 10^6\ \text{W}$$

In the CGS system, the unit of power is erg s^{-1}. If the work done is 1 erg in 1 second the power is 1 erg s^{-1} (erg s^{-1} has no other name).

In the British system, the unit of power is foot-pound per second (ft-lb s^{-1}) and in British engineering system, the unit of power is horse power (hp).

If the work done in 1 second is 550 ft-lb, the power is 550 ft-lb s^{-1} (or) 1 horse power.
The relationships between different units of power are

$$1\ \text{W} = 1\ \text{J s}^{-1} = 10^7\ \text{erg s}^{-1}$$
$$1\ \text{hp} = 550\ \text{ft-lb s}^{-1} = 746\ \text{W}$$
$$1\ \text{hp} = 746 \times 10^7\ \text{erg s}^{-1}$$

If s is the displacement produced by a constant force F acting for the time t, the work done is Fs. Hence, the rate of doing work or power is

$$P = \frac{Fs}{t}$$

$$P = F\frac{s}{t} = F\overline{v}$$

where $\overline{v}$ is average velocity. For uniform motion $\overline{v} = v$

$$P = Fv \tag{4.2}$$

Hence, power is equal to the product of force and velocity. Although it is not a fundamental concept of physics, it is very useful in practice.

> Power is not a fundamental concept like energy but it is a very important concept for engineering works. For example, car engines, water pumps, refrigerators, air conditioner and electric bulbs, etc., are specified according to their power consumption.

Example 4.1 A 70 kg man is running up the stairs 3 m high in 2 s. (i) How much work is done by the man? (ii) What is the power exerted by the man ?

(i) Since the work done is the change in the potential energy of the man

$$\begin{aligned} W &= m\,g\,h \\ &= 70 \times 9.8 \times 3 \\ &= 2058\ \text{J} \end{aligned}$$

(ii) The power exerted by the man is

$$P = \frac{W}{t} = \frac{2058}{2} = 1029 \text{ W}$$

(The value of the power is very large. A man is able to produce such a power only for a short duration.)

Example 4.2 A water pump can raise 200 kg water to a height of 6 m in 10 s. Find the power of the water-pump.

$m = 200$ kg, $h = 6$ m, $t = 10$ s

The work done by water pump in 10 s is

$$W = m\,g\,h$$
$$= 200 \times 9.8 \times 6 \text{ J}$$

The power of the water pump is

$$P = \frac{W}{t}$$

$$P = \frac{200 \times 9.8 \times 6}{10} = 1176 \text{ W}$$

Example 4.3 A crane is lifting a 500 lb piano with a velocity of 2 ft s^{-1}. Express the power of the crane in hp.

Weight of the piano = applied force $F = 500$ lb, $v = 2$ ft s^{-1}

Since the force and the velocity are in the same direction,

$$P = F v = 500 \times 2 = 1000 \text{ ft-lb s}^{-1}$$

Since 1 hp = 550 ft-lb s^{-1}

$$P = \frac{1000}{550} = 1.82 \text{ hp}$$

Reviewed Exercise

1. Two students climb up the stairs of a building and reach the top at the same time. Their weights are different. Which student will expend more power?
2. Which is more advantageous: to pay wages according to the amount of work done (or) according to power?

Key Words: power, work

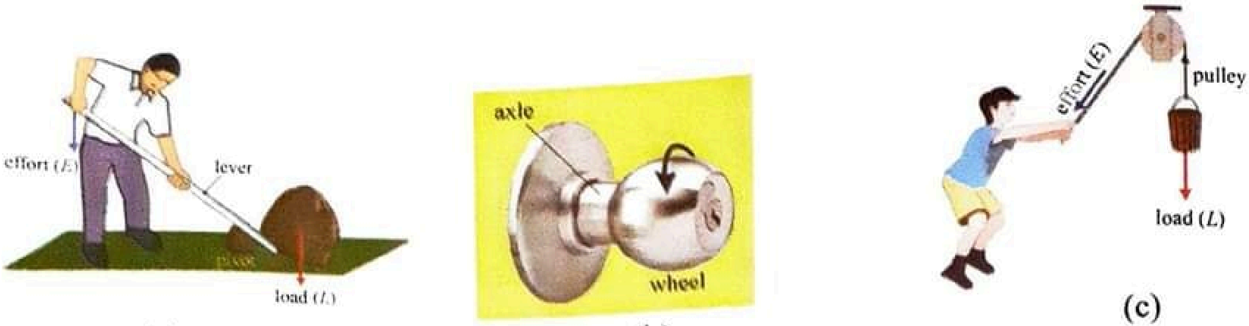
4.2 EFFICIENCY

Efficiency is another technical term that is derived from everyday usage. In physics and engineering efficiency has a precise meaning. This term is used in association with machines and devices.

Simple Machine

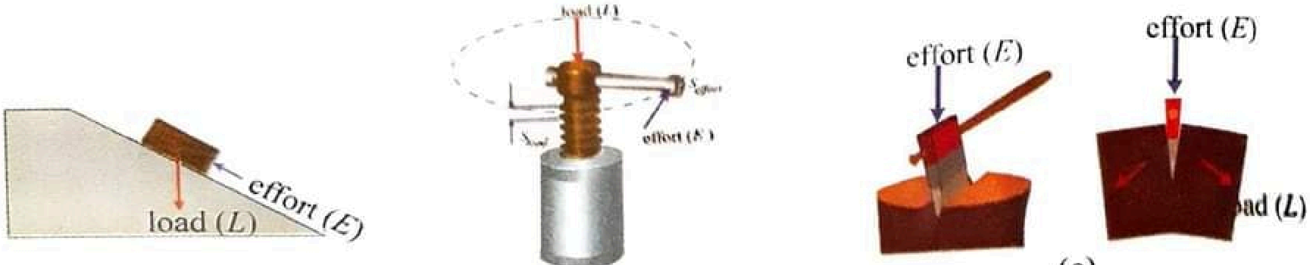
A simple machine is a mechanical device that amplifies the magnitude of a force and changes its position.

Simple machines are used to make work easier. Nowadays, machines are just combinations (or) more complicated forms of the six simple machines as shown in Figure 4.2 and Figure 4.3.
The six simple machines are lever, wheel and axle, pulley, inclined plane, screw and wedge.



(a) (b) (c)

Figure 4.2 (a) The lever (b) wheel and axle (c) pulley **[CREDIT: Source from the Internet]**



(a) (b) (c)

Figure 4.3 (a) Inclined plane (b) screw (c) wedge **[CREDIT: Source from the Internet]**

Mechanical Advantage

The term mechanical advantage is used to describe how effectively a machine works. If a load L is raised steadily by a machine when an effort E is applied, the mechanical advantage of the machine is defined as the ratio of load to effort.

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

$$MA = \frac{L}{E} \qquad (4.3)$$

Suppose an effort E of 25 N is applied at one end of a crowbar and just overcomes the resistance L of 100 N at the cover of a manhole.

Then mechanical advantage, $MA = \frac{L}{E} = \frac{100}{25} = 4$

In practice, not all of the effort is used up in lifting the load, some of it is spent in overcoming frictional forces present. It should, therefore, be remembered that the MA of a machine depends on the friction present.

Velocity Ratio

In lifting a large load with a machine, the small effort applied will have to move through a large distance for the heavy load to move through a small distance in the same time interval.

The ratio of the distance moved by the effort to that of the load in the same time is called the velocity ratio of the machine.

$$\text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load in the same time}}$$

$$VR = \frac{s_{\text{effort}}}{s_{\text{load}}} \qquad (4.4)$$

where s_{effort} = distance moved by the effort

s_{load} = distance moved by the load

Suppose that in lifting a load with a pulley system, the effort moves through 250 cm while the load moves through 50 cm in the same time interval.

For this case, velocity ratio $VR = \dfrac{250}{50} = 5$

The velocity ratio of a machine *VR* is usually much greater than 1.

Efficiency

In lifting a load with a machine, work is done on the load; this work obtained is called the output work. Also work is done by the effort during the same time interval, this work supplied is called the input work. The ratio of output work to input work is defined as the efficiency of the machine. Efficiency is generally expressed in the percentage form. Thus

$$\text{efficiency} = \frac{\text{output work}}{\text{input work}} \times 100\%$$

$$\text{efficiency} = \frac{W_{out}}{W_{in}} \times 100\% \qquad (4.5)$$

Efficiency Related to Mechanical Advantage and Velocity Ratio

We have already defined mechanical advantage and velocity ratio of a machine. We will express the relation between mechanical advantage, velocity ratio and efficiency of the machine.

Efficiency is related to *MA* and *VR* as follows.

$$\text{efficiency} = \frac{\text{output work}}{\text{input work}} \times 100\%$$

$$\text{efficiency} = \frac{\text{load} \times \text{distance by load}}{\text{effort} \times \text{distance by effort}} \times 100\%$$

$$\text{efficiency} = \frac{MA}{VR} \times 100\,\% \qquad (4.6)$$

It is impossible, in practice, to build a perfect machine for which output work is equal to input work; input work always exceeds output work. Therefore, the efficiency of a machine must always be less than 100 %.

Example 4.4 A machine with a velocity ratio of 8 requires 1000 J of work to raise a load of 500 N through a vertical distance of 1 m. Find the efficiency and mechanical advantage of the machine.

$W_{in} = 1000$ J, $L = 500$ N, $s_{load} = 1$ m

$W_{out} = L \times s_{load} = 500 \times 1 = 500$ J

$$\text{efficiency} = \frac{W_{out}}{W_{in}} \times 100\%$$

$$= \frac{500}{1000} \times 100\% = 50\%$$

Since,

$$\text{efficiency} = \frac{MA}{VR} \times 100\%$$

$$50\% = \frac{MA}{8} \times 100\%$$

$$MA = 4$$

Example 4.5 A crane lifts a 100 kg block of concrete through a vertical height of 16 m in 20 s. If the power supplied to the motor driving the crane is 1 kW, what is the efficiency of the motor?

$m = 100$ kg, $h = 16$ m, $t = 20$ s, $P_{in} = 1$ kW $= 10^3$ W

The work done on the concrete = output work of motor, $W_{out} = m\,g\,h$

$$= 100 \times 9.8 \times 16$$

$$= 15\,680 \text{ J}$$

Output power of the motor, $P_{out} = \dfrac{W_{out}}{t} = \dfrac{15\,680}{20} = 784$ W

$$\text{efficiency of the motor} = \frac{W_{out}}{W_{in}} \times 100\%$$

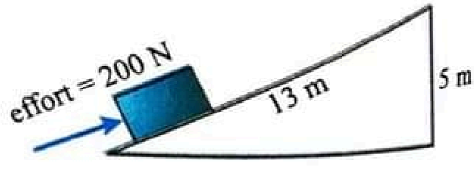
$$= \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{784}{10^3} \times 100\% = 78.4\%$$

Example 4.6 A box of mass 40 kg is pushed up an inclined plane of length 13.0 m and height 5.0 m above the ground level. If the applied force on the box is 200 N parallel to inclined plane, find (i) *MA* (ii) *VR* (iii) output work and input work (iv) efficiency for this case.

$s_{effort} = 13$ m, $s_{load} = 5$ m, effort = 200 N, $m = 40$ kg

Load = weight of box = $mg = 40 \times 9.8 = 392$ N



(i) $MA = \dfrac{L}{E} = \dfrac{392}{200} = 1.96$

(ii) $VR = \dfrac{s_{effort}}{s_{load}} = \dfrac{13}{5} = 2.6$

(iii) $W_{out} = m\,g\,h = 40 \times 9.8 \times 5 = 1960$ J

$W_{in} = E \times s_{effort} = 200 \times 13 = 2600$ J

(iv) efficiency $= \dfrac{MA}{VR} \times 100\,\% = \dfrac{1.96}{2.6} \times 100\% = 75.38\%$

Reviewed Exercise

- Is it possible to build a machine having 100% efficiency? Explain.

Key Words: efficiency, mechanical advantage, velocity ratio

4.3 THE STRETCHING OF COILED SPRING

Consider a spring suspended as shown in Figure 4.4. If a small load is attached to the free end of this spring, the spring will be stretched or elongated. When the load is taken off, the spring will return to its original length and form.

If a bigger load is hung at the free end, the spring will again be elongated, but this time elongation will be larger. Thus, if we increase the load, the elongation will also be increased.
Also, whenever the load is removed the spring returns to its original length l_0 and form.

l_0 = length of unstretched spring
l = length of stretched spring

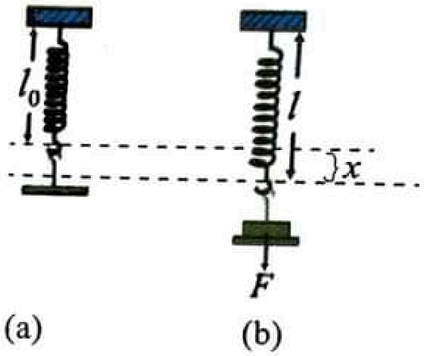


Figure 4.4 (a) Unstretched spring (b) Elongated stretched spring

Ability to retain the original form when the applied force is removed is called elasticity. Not only springs but also other objects such as threads and rubber bands have elastic property.

Elastic limit is a limit, beyond which if an elastic object is stretched, it will not return to its original form. It is different for different elastic bodies.

Hooke's Law

Robert Hooke, the English scientist, noted that when an elastic body such as a spring is stretched by a weight (or) a force, the amount of elongation of the spring is proportional to the applied force that produces it so long as the elastic limit is not exceeded. The amount of elongation is also called extension.

Hooke's law is formally stated as follows.
As long as the elastic limit of a body is not exceeded, the strain produced is proportional to stress causing it.

In symbols $F \propto x$ (or) $F = kx$ (4.7)

where F is the applied force (or) stress, x is the elongation (or) strain and k is a constant.
For a spring, k is also called spring constant. The unit of k is N m^{-1}.

Hooke's law expresses the elastic behavior of materials. The directly proportional relationship between applied force and elongation, is shown graphically by a straight line in Figure 4.5 (a).

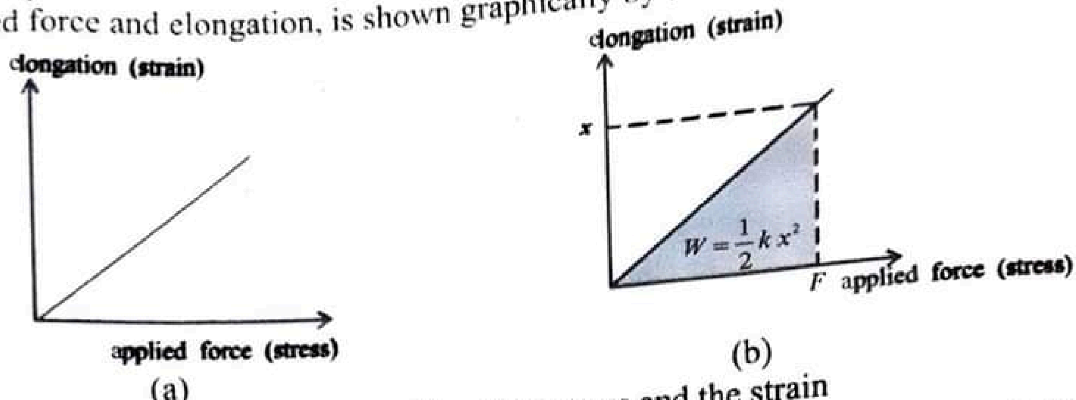


Figure 4.5 Relationship of the stress and the strain

If a spring is compressed (or) stretched to an extension x, the work is done by the applied force. This work done is stored as the potential energy of the spring. The work done is given by the area under the curve as shown in Figure 4.5 (b). Since this area is $\frac{1}{2}kx^2$, the elastic potential energy stored by the stretched spring is also $\frac{1}{2}kx^2$.

$$W = \Delta PE = \frac{1}{2}kx^2 \qquad (4.8)$$

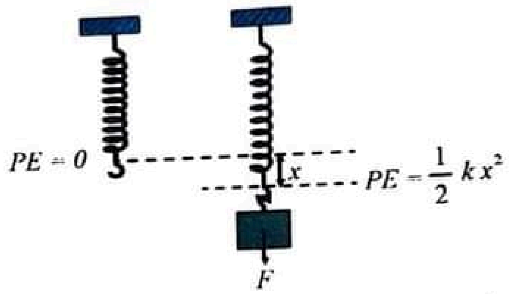


Figure 4.6 Elastic potential energy stored by a spring

Example 4.7 A spring requires a load of 25 N to increase its length by 4 cm. The spring obeys Hooke's law. What is the spring constant? How much load will give it to extend 12 cm?

$F = 25$ N, $x = 4$ cm $= 4\times10^{-2}$ m

$x' = 12$ cm $= 12\times10^{-2}$ m

By using Hooke's law, $F = kx$

The spring constant $k = \frac{F}{x} = \frac{25}{4\times10^{-2}} = 6.25\times10^{2}\ \text{N m}^{-1}$

The load needed to extend 12 cm, $F' = kx' = 6.25\times10^2\times12\times10^{-2} = 75$ N

Example 4.8 If the force stretches a spring with spring constant 100 N m^{-1}, how much work is needed to stretch the spring 0.4 m from rest position?

$k = 100$ N m^{-1}, $x = 0.4$ m

work needed to stretch the spring $W = \Delta PE = \frac{1}{2}kx^2 - 0$

$$W = \frac{1}{2}\times100\times(0.4)^2 = 8\ \text{J}$$

Reviewed Exercise

- Are elastic limits of different elastic bodies different (or) same? Can the threads and rubber bands have elasticity?

Key Words: strain, stress, elasticity, elastic limit

4.4 SOURCES OF ENERGY

Energy is the ability to do work or the total power derived from the natural resources we use for transportation, domestic appliances and the manufacture of all kinds of products. The energy exists in many forms and can be converted from one form to another. Energy sources can be classified as two major sources: conventional sources and non-conventional sources as shown in Figure 4.7

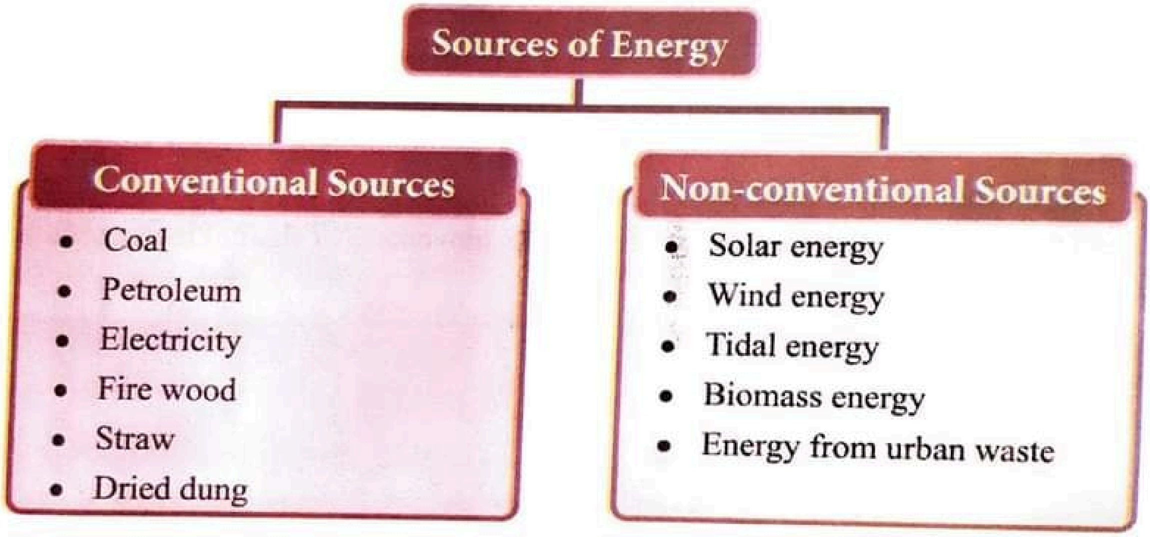


Figure 4.7 Conventional sources and non-conventional sources

Conventional energy sources are non-renewable and non-conventional energy sources are renewable energy sources.

Renewable Energy Sources

Renewable energy sources are sustainable energy sources. A renewable energy source is generated from unlimited natural sources (e.g. sun, wind, hydropower) and is always replenished and is safe to the environment. The most popular renewable energy sources currently are solar energy, wind energy, hydro energy, tidal energy, geothermal energy and biomass energy.

Solar Energy

Sun is the primary source of energy. Sunlight is a clean, renewable source of energy. It is a sustainable resource, meaning it does not run out because sun shines almost every day. With the help of photovoltaic technology, solar energy can be used to generate electricity. Through solar photovoltaic cells, solar radiation gets converted into DC electricity directly as shown in Figure 4.8 (a) and (b). The generated electricity can either be used as it is or can be stored in the battery.

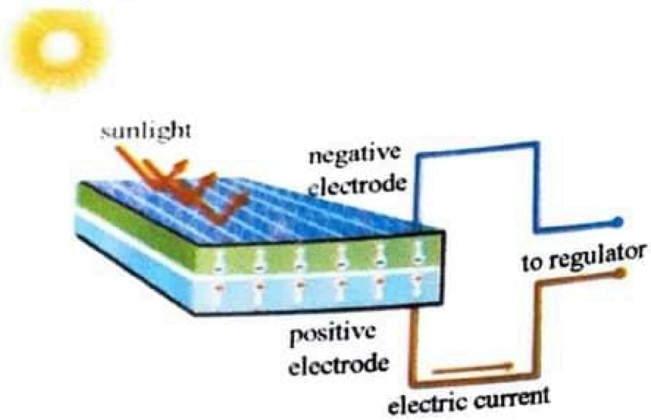


Figure 4.8 (a) Solar cell
[CREDIT: Source from the Internet]

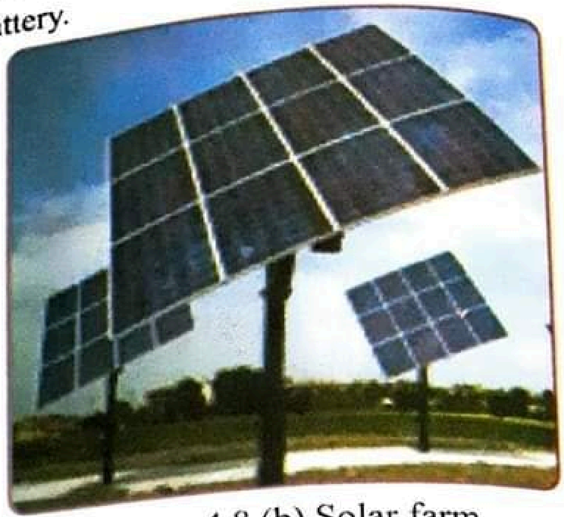


Figure 4.8 (b) Solar farm
[CREDIT: Source from the Internet]

Wind Energy

Wind is the natural movement of air across the land or sea. The wind when used to turn the blades of a windmill turns the shaft to which they are attached. This movement of shaft of the generator produces electricity as shown in Figure 4.9.

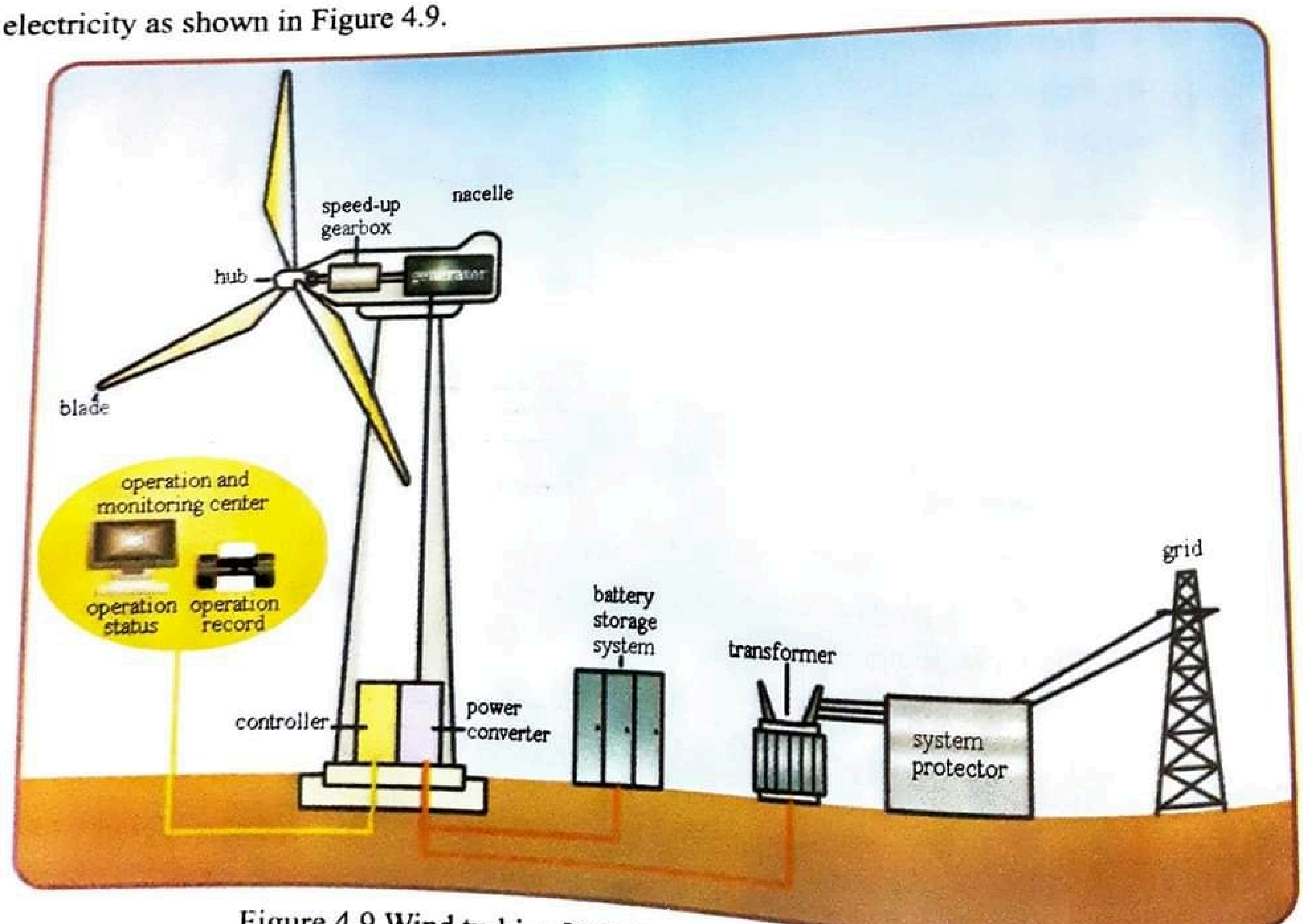


Figure 4.9 Wind turbine [CREDIT: Source from the Internet]

Hydro Energy

The tides in the sea, the flowing water in rivers or from the dams are sources of hydro energy. Hydro energy uses flowing water to power machinery and generate electricity as shown in Figure 4.10. For example; grain grinding machinery on farms and hydroelectric energy plants to power cities.

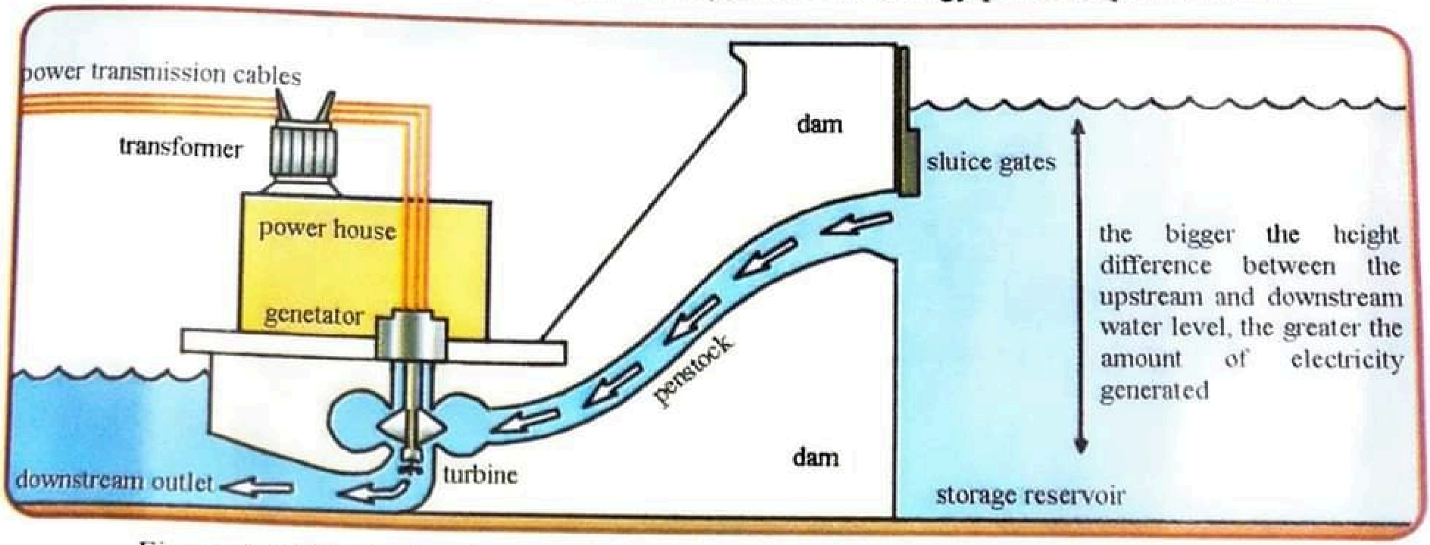


Figure 4.10 Hydroelectric power generation [CREDIT: Source from the Internet]

Tidal Energy

Tidal energy is another form of hydro energy that uses tidal currents twice-daily to drive turbine generator [Figure 4.11 (a) and (b)]. Although tidal flow unlike some other hydro energy sources is not constant, it is highly predictable.

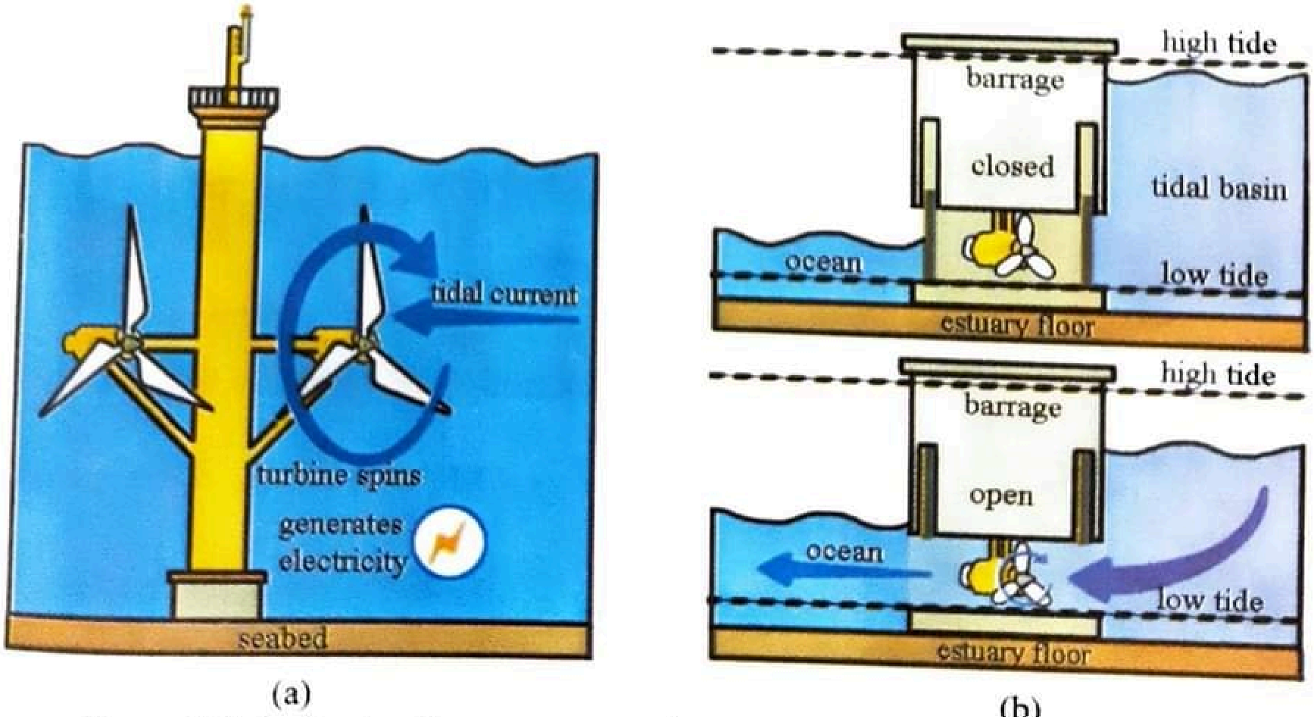


Figure 4.11 Hydroelectric power generation (a) tides in the sea (b) tidal current [CREDIT: Source from the Internet]

Geothermal Energy

Geothermal literally means heat generated by earth. Geothermal energy is heat stored on the earth crust and being used for electric generation (Figure 4.12) and also for direct heat application. Geothermal energy harnesses the earth's underground hot water and steam for heat and electricity. Geothermal energy is more readily available in areas affected by volcanism.

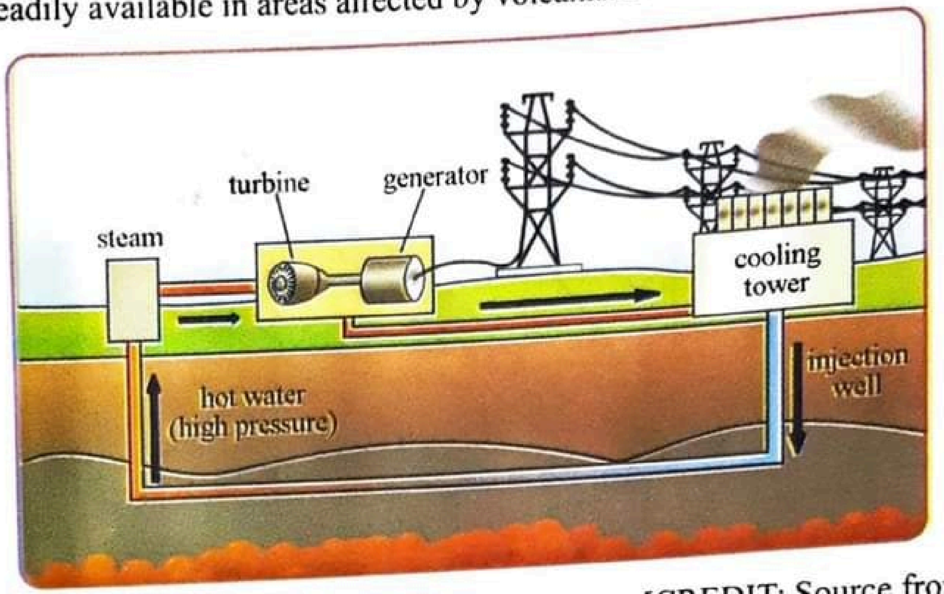
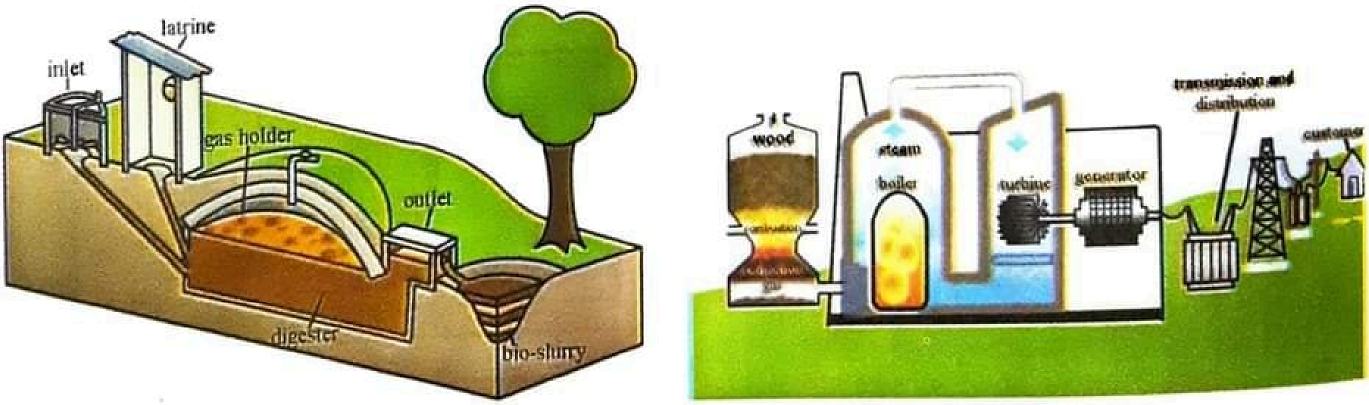


Figure 4.12 Geothermal energy used for electric generation [CREDIT: Source from the Internet]

Biomass energy

Biomass is plant or animal material used as fuel to produce electricity or heat (Figure 4.13). The agricultural waste, wood, charcoal or dried dung are used as biomass. Biomass can also be converted into other forms of energy such as methane gas, ethanol and biodiesel.



(a) (b)

Figure 4.13 Production and usage of biomass energy [CREDIT: Source from the Internet]

Roles of a Renewable Energy

The four major roles of a renewable energy are (i) electricity generation, (ii) air and water heating (or) cooling, (iii) fuel for transportation and (iv) rural energy services (off-grid). The benefits of using renewable energy instead of fossil fuels are free source, sustainable and emit zero or minimal amounts of greenhouse gases (GHG).

Non-renewable energy sources

Non-renewable energy comes from sources that will run out or will not be replenished for thousands or even millions of years. They are finite amount in existence. Four main types of non-renewable energy sources are oil, natural gas, coal and nuclear energy in Figure 4.14. Oil, natural gas and coal are collectively called fossil fuels. Fossil fuels were formed within the earth from dead plants and animals over millions of years. Non-renewable energy can be used for electricity, heating, manufacturing and transportation.

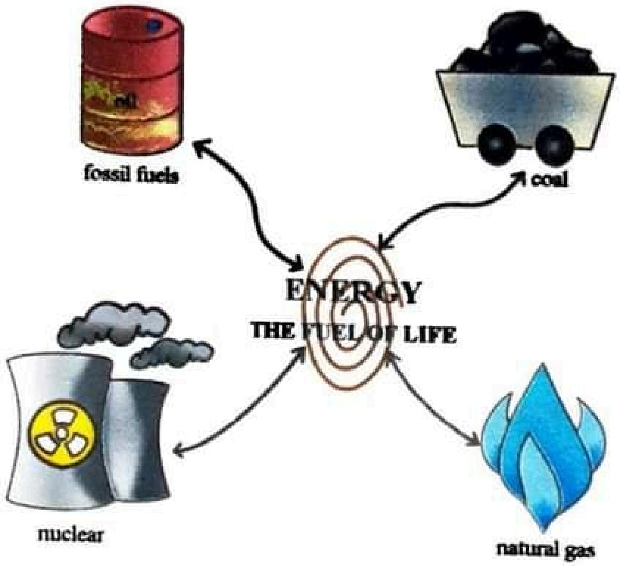
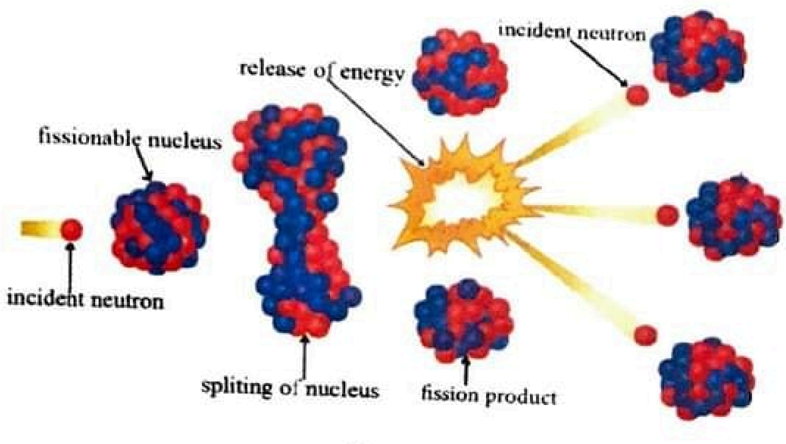


Figure 4.14 Non-renewable energy sources [CREDIT: Source from the Internet]

Nuclear energy

Nuclear energy is produced by using elements like uranium and thorium, which cannot be replenished. Nuclear energy can be used to generate electricity by means of nuclear fission as shown in Figure 4.15 (a) and (b). In a nuclear reactor, the nuclear fuel is used to carry out sustained fission chain reaction to produce electricity at a controlled rate.

(a)



(b)

Figure 4.15 (a) Nuclear reactor (b) nuclear fission chain reaction [CREDIT: Source from the Internet]

Reviewed Exercise

1. What are renewable and non-renewable energy?
2. Write down the names of the sources of renewable energy.

Key Words: sustainable energy, renewable energy sources, non-renewable energy sources

SUMMARY

The rate of doing work is defined as **power**.

The ratio of load to effort is defined as the **mechanical advantage** of the machine.

The ratio of output work to input work is defined as the **efficiency** of the machine.

The ratio of the distance moved by the effort to that of the load in the same time is called the **velocity ratio** of the machine.

Ability to retain the original form when the applied force is removed is called **elasticity**.

A renewable energy source is generated from unlimited natural sources (e.g. sun, wind, hydropower) and is always replenished and are safe to the environment.

EXERCISES

1. A woman of 40 kg mass climbs up by pulling a rope 8 m long with a constant velocity for 15 s. Find the power output of the woman.
2. The power output of the motor of a crane is 2 kW. With what speed can the machines lift a 1000 kg load?
3. A water pump is pumping up water from a well which is 200 m deep. (i) How much work must be done by the pump to raise 1 kg of water? (ii) What is the power output of the pump if it pumps up water at rate of 10 kg min^{-1}?
4. A steam engine generating 5 hp is lifting a 2000 lb load. How high will the load be raised in 10 s?
5. If a car of mass 1200 kg is moving with a constant velocity 37.5 m s^{-1} and experiences a resistive force of 450 N, what is power of car engine?
6. An electric motor in a washing machine has a power output of 1 kW. Find the work done in 20 min.
7. In a tug-of-war A-team is leading B-team. The rope is moving towards A-team at a regular rate of 0.01 m s^{-1}. If the tension of the rope is 4000 N what is the power output of A-team?
8. The rate of doing work for the first worker is twice that of the second worker. But the working hours per day of the second is two and a half times that of the first. Who is a better worker?
9. A system of levers with a velocity ratio of 25 overcomes a resistance of 3300 N when an effort of 165 N is applied to it, calculate (i) the mechanical advantage of the system (ii) its efficiency.
10. By using a block-and-tackle a man can raise a load of 720 N by an effort of 200 N. Find the mechanical advantage of the method.
11. The length of an unstretched spring is 12.0 cm. What load is needed to stretch the spring to a length of 15 cm? The spring constant is 100 N m^{-1}.

CONCEPT MAP

POWER AND EFFICIENCY

Power

$P = \frac{W}{t}$

Efficiency

$\text{efficiency} = \frac{W_{out}}{W_{in}} \times 100\%$

$\text{effciency} = \frac{MA}{VR} \times 100\%$

- mechanical advantage
- velocity ratio

$MA = \frac{L}{E}$

$VR = \frac{s_{effort}}{s_{load}}$

Elasticity

Hooke's law

$F = k\,x$

Work done by elastic force

$W = \frac{1}{2}kx^2$

Sources of Energy

Renewable energy sources

- solar
- wind
- hydro
- tidal
- geothermal
- biomass

Non- renewable energy sources

- nuclear
- oil
- natural
- coal

CHAPTER 5
HEAT AND THERMAL PHENOMENA

In this chapter, the relation between heat transfer and temperature change will be discussed. If heat is added to a substance, the state of that substance can also change.

Learning Outcomes

It is expected that students will

- examine the relationship between heat and internal energy.
- explain the concepts of thermal capacity, specific heat capacity and the measurement of heat.
- state and explain the law of heat exchange.
- identify the relationship between change of state and latent heat.
- examine the process of vaporization and its relationship to latent heat of vaporization.
- examine the relationship of fusion to latent heat of fusion.
- investigate and describe the dependence of melting point and boiling point on atmospheric pressure.

5.1 UNITS OF HEAT

Heat is the amount of energy transferred from one object to another because of a difference in temperature.

As heat is a form of energy, heat can be measured in energy unit.

The unit of heat is joule (J) in SI unit. Other heat units are calorie (cal) in the CGS system and kilocalorie (kcal) in the MKS system. The heat unit in British system is the British thermal unit (Btu).

Heat required to change the temperature of 1 kilogram mass of water by 1kelvin is called 1 kilocalorie.

The German doctor, Robert Mayer, found first that the energy which represents 1 cal is 4.184 J. The relations between heat units are

$$1 \text{ kcal} = 4184 \text{ J} = 10^3 \text{ cal}$$

$$1 \text{ Btu} = 1055 \text{ J}$$

$$1 \text{ ft-lb} = 1.356 \text{ J}$$

Example 5.1 How many kilocalories are equal to 2 Btu?

$$1 \text{ kcal} = 4184 \text{ J}$$

$$1 \text{ Btu} = 1055 \text{ J}$$

$$2 \text{ Btu} = 1055 \times 2 = 2110 \text{ J} = \frac{2110}{4184} = 0.504 \text{ kcal}$$

Reviewed Exercise

- Why can the energy unit be used as the heat unit?

Key Words: heat, temperature

5.2 HEAT AND INTERNAL ENERGY

The internal energy of a body is the sum of the total kinetic energy and potential energy of the molecules in the substance.

The kinetic energy of molecules is energy due to their motions. The kinetic energy is directly related to temperature. The rise in temperature of a substance is due to an increase in the average kinetic energy of the molecules.

So, whenever there is a rise in temperature of a substance, there must be an increase in the internal energy of the substance shown in Figure 5.1.

The potential energy is energy due to inter-molecular forces between the molecules. The magnitude of potential energy stored depends on the distance between the molecules (or) the particles in that substance.

The internal energy of the substance is known as its thermal energy. The transfer of thermal energy from one substance to another is referred to as heat.

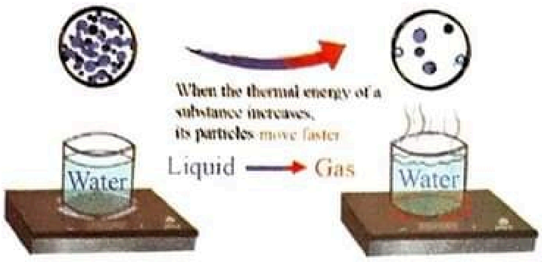


Figure 5.1 Relation between internal energy and temperature

Example 5.2 An egg contains 8×10^4 cal of the average energy value of food. If that calorie is the thermal unit used in the physical science, express the energy value of food of the egg in joules. (1 cal = 4.184 J)

The average energy value of food of the egg = 8×10^4 cal = $8 \times 10^4 \times 4.184 = 3.35 \times 10^5$ J

Reviewed Exercise

1. Why does the temperature of a substance increase?
2. What happen to the internal energy of a gas when it is heated?

Key Words: internal energy, inter-molecular forces

5.3 THERMAL CAPACITY

When an object at a certain temperature is placed in contact with another object at a higher temperature, the thermal energy (heat) is transferred from the object at a higher temperature to one at a lower temperature. Since the object at the lower temperature receives additional heat energy, its temperature increases.

The amount of thermal energy required to change the temperature of an object by one degree is called the thermal capacity (or) heat capacity of the object.

The thermal capacity of an object $C = \dfrac{\Delta Q}{\Delta T}$ (5.1)

where $\Delta Q =$ the thermal energy required

$\Delta T =$ the temperature change ($\Delta T = T_1 \sim T_2$)

$C =$ thermal capacity

In SI unit, the unit of thermal capacity is joule per kelvin ($J\ K^{-1}$).

The thermal capacity of an object depends on mass and a type of the material it is made of. The thermal capacity is an important property of materials. An object having lower thermal capacity can be heated up easily.

Water has a relatively high thermal capacity. Because of high thermal capacity, water is used to cool engines. In cold countries, water is used to store heat in solar heating system of the houses. Another common application is the use of hot water bags to keep warm. This relies on the ability of hot water to store a large amount of energy. The high thermal capacity of water also affects the weather.

Suppose that the internal energy of n moles of a substance changes by ΔQ due to the temperature change ΔT. The thermal capacity for 1 mole is molar thermal capacity.

$$C = \frac{1}{n}\frac{\Delta Q}{\Delta T} \qquad (5.2)$$

SI units of the molar thermal capacity is joule per mole per kelvin (J mol^{-1} K^{-1}).

Example 5.3 When a piece of iron is cooled from 70 °C to 40 °C, the thermal energy released is 700 J. What is its thermal capacity?

$T_1 = 70$ °C, $T_2 = 40$ °C, $\Delta Q = 700$ J

$$\Delta T = T_1 \sim T_2$$

$$\Delta T = 70 - 40 = 30 \text{ °C (or) } 30 \text{ K},$$

The thermal capacity,

$$C = \frac{\Delta Q}{\Delta T} = \frac{700}{30}$$

$$C = 23.33 \text{ J K}^{-1}$$

Reviewed Exercise

- How is thermal energy related to temperature change?

Key Words: thermal capacity, molar thermal capacity

5.4 SPECIFIC HEAT CAPACITY

As mention above, the thermal capacity is an important property of a substance. It depends on mass and type of material.

The thermal capacity per unit mass of a substance is the specific heat capacity. Therefore, the specific heat capacity of a substance is the heat needed to change the temperature of a unit mass of that substance by one degree.

The specific heat capacity

$$c = \frac{C}{m} \qquad (5.3)$$

(or)

$$c = \frac{\Delta Q}{m \Delta T} \qquad (5.4)$$

where c = specific heat capacity, C = thermal capacity

m = mass of substance, ΔT = temperature change

If the mass of a substance is m and its specific heat capacity is c, the amount of heat required to raise temperature ΔT is $m\,c\,\Delta T$.

By rewriting Eq. (5.4), the amount of heat is

$$\Delta Q = m\,c\,\Delta T \qquad (5.5)$$

ΔT is the temperature difference between the final and the initial temperature.

In the SI units, the unit of the specific heat capacity is joule per kilogram per kelvin (J $kg^{-1}K^{-1}$). The other units used for the specific heat capacity are kcal kg^{-1} °C^{-1} and cal g^{-1} °C^{-1}.
The specific heat capacities of various substances are given in Table 5.1.

Table 5.1 Specific heat capacities of various substances

Substance	Specific heat capacity	
	J $kg^{-1}K^{-1}$	kcal $kg^{-1}K^{-1}$
Aluminium	898	0.215
Glass	837	0.200
Diamond	518	0.124
Steel	447	0.107
Iron	443	0.105
Copper	385	0.092
Lead	130	0.031
Ice (-10 °C to 0 °C)	2089	0.500
Water	4184	1.000
Steam (100 °C to 200 °C)	1963	0.470
Hydrogen (gas)	14250	3.410
Helium (gas)	5180	1.240
Nitrogen (gas)	1040	0.249
Oxygen (gas)	915	0.219

Example 5.4 How much heat does it take to raise the temperature of 220 g of water from 25 °C to 100 °C? Specific heat capacity of water is 4184 J $kg^{-1}K^{-1}$.

$m = 220\text{ g} = 220 \times 10^{-3}\text{ kg} = 0.22\text{ kg}$, $\Delta T = T_1 \sim T_2 = (100 - 25) = 75$ °C (or) 75 K,

$c = 4184\text{ J kg}^{-1}\text{K}^{-1}$

The amount of heat $\Delta Q = m\,c\,\Delta T = 0.22 \times 4184 \times 75 = 69\,036$ J

Reviewed Exercise

- How does the specific heat capacity of water moderate the climate in a region near a large lake?

Key Words: specific heat capacity

5.5 LAW OF HEAT EXCHANGE

When heat is transferred from one object to another object the total heat lost by one object is equal to the total heat gained by the other object.

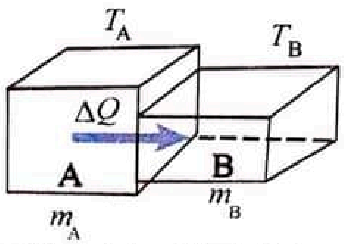


Figure 5.2 Heat transfer from object A to object B due to different temperature

Suppose that heat is transferred from an object A at temperature T_A to another object B at temperature T_B in Figure 5.2. In this case, the system consisting of two objects A and B must be regarded as an isolated system. When final temperature T of the objects are the same, no heat is transferred.

Heat lost by object A, $\Delta Q_{lost} = m_A c_A (T_A - T) = m_A c_A (\Delta T)_A$

Heat gained by object B, $\Delta Q_{gained} = m_B c_B (T - T_B) = m_B c_B (\Delta T)_B$ (5.6)

According to the law of heat exchange; $\Delta Q_{lost} = \Delta Q_{gained}$ (5.7)

$$m_A c_A (\Delta T)_A = m_B c_B (\Delta T)_B$$

Since heat is a form of energy, the law of heat exchange is one particular statement of the law of conservation of energy.
The specific heat capacity of a substance can be determined using the law of heat exchange.

Calorimeter

A calorimeter is a device used to measure the heat flow of a chemical reaction or physical change. It mainly consists of a metallic vessel made of copper (or) aluminium.

The metallic vessel with a stirrer for stirring the contents in the vessel is kept in an insulating jacket to prevent heat lost to the environment. There is a hole on the lid through which a thermometer is inserted as shown in Figure 5.3.

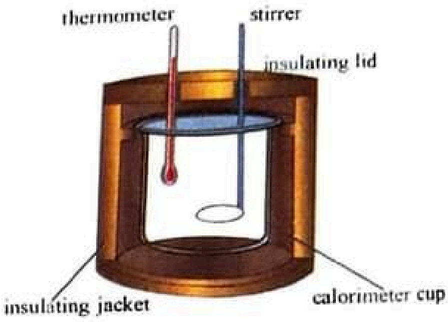


Figure 5.3 Calorimeter

Determination of Specific Heat Capacity of a Sample

The sample whose specific heat capacity is to be determined is placed in a calorimeter. When heating the calorimeter and the sample their temperature will change by ΔT.

Heat gained by the sample = $m\, c\, \Delta T$

where m = mass of the sample

c = specific heat capacity of the sample

Heat gained by the calorimeter = $m_c c_c \Delta T$

where m_c = mass of calorimeter

c_c = the specific heat capacity of calorimeter

Total heat gained by the sample and calorimeter:

$$\Delta Q_{gained} = m\, c\, \Delta T + m_c c_c \Delta T$$

Heat lost by the heat supply $\Delta Q_{lost} = \Delta Q$

By the law of heat exchange,

$$\Delta Q_{gained} = \Delta Q_{lost}$$

$$\Delta Q = m\, c\, \Delta T + m_c c_c \Delta T$$

$$c = \frac{\Delta Q - m_c c_c \Delta T}{m\, \Delta T} \quad (5.8)$$

Example 5.5 The specific heat capacity of 0.4 kg mass of a calorimeter is 627.6 J $kg^{-1}K^{-1}$. A 0.55 kg substance is in that calorimeter. The temperature of the calorimeter increases by 4 K when 2450 J of energy is added to it. Find the specific heat capacity of the substance in the calorimeter.

$m_c = 0.4$ kg, $c_c = 627.6$ J $kg^{-1}K^{-1}$, $m = 0.55$ kg, $\Delta Q = 2450$ J, $\Delta T = 4$ K

By using Eq. (5.8), $c = \dfrac{\Delta Q - m_c\, c_c\, \Delta T}{m\, \Delta T}$

$$c = \frac{2450 - (0.4 \times 627.6 \times 4)}{(0.55 \times 4)} = 657.2 \text{ J kg}^{-1}\text{K}^{-1}$$

Specific heat capacity of the substance is 657.2 J $kg^{-1}K^{-1}$

Reviewed Exercise

- Samples of copper and iron having equal mass heated with the same amount of heat will achieve different temperatures. Which sample will reach the higher temperature?

Key Words: heat gained, heat lost, calorimeter, specific heat capacity

5.6 CHANGE OF STATE AND LATENT HEAT

Most substances can exist in three states, i.e., solid, liquid and gas. Water, for example, may exist in the form of ice, water and steam (Figure 5.4).

A transition from one of the three states (solid, liquid and gas) to another is called a phase change. The change from one phase to another takes place very abruptly at a definite temperature.

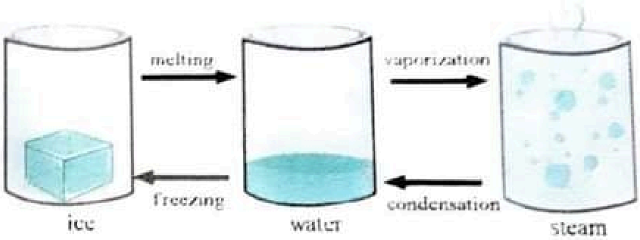


Figure 5.4 Change of states of water

The temperature at which a phase change occurs depends on pressure.

Latent Heat

The energy absorbed (or) liberated by a substance in a phase change is called the latent heat.

The energy absorbed (or) liberated by a unit mass of substance in a phase change is called the specific latent heat.

If L is the specific latent heat of an object, the heat needed to change the phase of that object of mass m is $\Delta Q = L\,m$.

The SI unit of specific latent heat is joule per kilogram (J kg^{-1}).

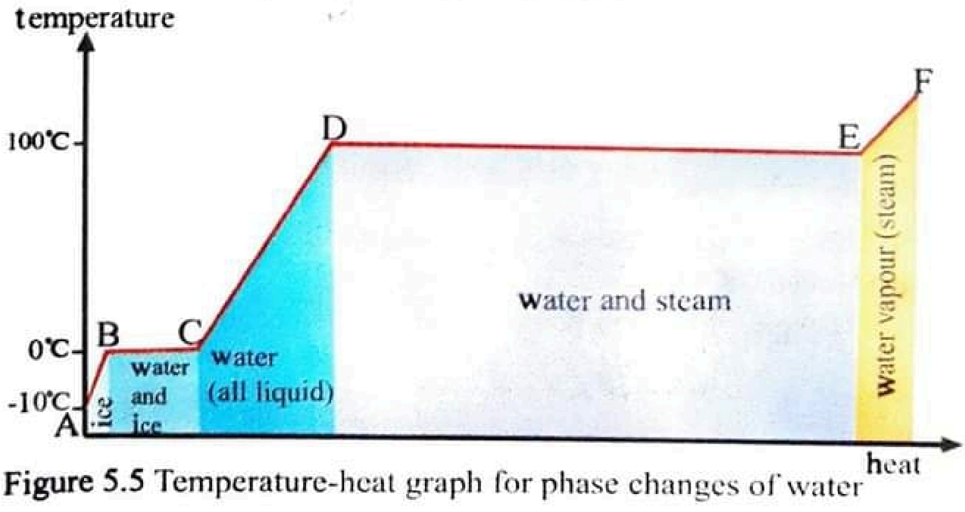


Figure 5.5 Temperature-heat graph for phase changes of water

Phase changes for water are illustrated by the temperature-heat graph in Figure 5.5. At point A, water can exist only as ice. If the pressure is kept fixed throughout and heat is added to ice the temperature rises until reaches to point B. As more heat is added the temperature does not rise, but the ice gradually melts into water. The temperature remains constant until all the ice has melted at point C.

Then, as more heat is added, the temperature of water steadily increases until reaches to point D. Again, the temperature remains constant until all the water has been changed into vapour (steam). Then, additional heat will increase the temperature of the vapour.

It is found that the temperature does not change during phase change.

If the above procedure is repeated at a low enough pressure, ice changes directly into vapour (from point A to point E) without passing through the liquid phase. Such direct change from the solid phase to the gas phase is called sublimation.

Sublimation is used in the freeze-drying process which does not damage food and preserves its quality and taste.

Reviewed Exercise

- Can heat be extracted from water at 0 °C?

Key Words: solid, liquid, gas, latent heat, specific latent heat, sublimation

5.7 VAPORIZATION AND SPECIFIC LATENT HEAT OF VAPORIZATION

If water in a container is heated the temperature will increase until it reaches 100 °C. At this temperature the water boils and begins to vaporize. If heat is added, the water keeps vaporizing. But the temperature of the boiling water remains constant. If heat is continually added, all the water turns completely into steam. After that if more heat is added, the temperature of steam will increase.

The process of changing liquid into vapour is called vaporization.

The temperature at which a liquid vapourizes under normal pressure is called its boiling point.

Specific Latent Heat of Vaporization

Heat that must be supplied to change 1 kg of liquid at its boiling point from liquid phase to vapour phase is called the specific latent heat of vaporization L_v of the liquid.

Different liquids have different boiling points. The boiling point of mercury is 357 °C and that of water is 100 °C. The exact value of the specific latent heat of vaporization of water is 2 255 176 J kg^{-1}. But 2.255×10^6 J kg^{-1} will be assumed for simplicity in calculations.

If steam at 100 °C is cooled, it condenses back to water. In this process, although the steam is losing heat, the temperature remains at 100 °C.

The change of vapour (or) gas phase into liquid phase is called condensation.

In condensation, the substance releases heat. In vaporization, the substance absorbs heat.

Example 5.6 How much heat is needed to change 5 kg of water at 100 °C to steam? (specific latent heat of vaporization $L_v = 2.255\times10^6$ J kg^{-1})

$m = 5$ kg, $L_v = 2.255\times10^6$ J kg^{-1}

The heat needed, $\Delta Q = m\,L_v = 5\times2.255\times10^6 = 1.13\times10^7$ J

Reviewed Exercise

- Why is getting burnt by steam at 100 °C worse than that by hot water of the same mass at 100 °C?

Key Words: vaporization, condensation, boiling point, specific latent heat of vaporization

5.8 FUSION AND SPECIFIC LATENT HEAT OF FUSION

If pieces of ice in a tray are heated, the temperature will increase. After some time, it will be seen that some pieces of ice change into water. The temperature of mixture of water and ice will found to be 0°C. The ice is melting continuously until all the ice turns to water without changing its temperature. After all the ice has melted, the temperature of water will rise if more heat is added.

The temperature at which a solid melts under normal pressure is called the melting point.
The melting point for ice under normal pressure is 0 °C.

Specific Latent Heat of Fusion

Heat required to melt 1 kg of a solid at its melting point is called the specific latent heat of fusion L_f

The exact value of the specific latent heat of fusion of ice is 333 464.8 J kg^{-1}. But 3.335×10^5 J kg^{-1} will be assumed for simplicity in calculations.

The specific latent heat of fusion, specific latent heat of vaporization, melting point and boiling point for various substances are given in Table 5.2.

Table 5.2 Specific latent heat of fusion and vaporization of various substances

Substance	Melting point (°C)	Specific latent heat of fusion (kJ kg^{-1})	Boiling point (°C)	Specific latent heat of vaporization (kJ kg^{-1})
Nitrogen	– 209.9	25.481	– 195.8	2.008×10^2
Ethyl alcohol	–114	1.042×10^2	78	8.535×10^2
Mercury	– 39	11.799	357	2.720×10^2
Water	0	3.335×10^2	100	2.255×10^3
Silver	960	88.282	2193	2.335×10^3
Lead	327	24.518	1750	8.70×10^2
Gold	1063	64.434	2660	1.577×10^3

Example 5.7 How much heat is required to melt 5 kg of ice at 0 °C? (specific latent heat of fusion of ice $L_f = 3.335\times10^5$ J kg^{-1})

$m = 5$ kg, $L_f = 3.335\times10^5$ J kg^{-1}

$$\Delta Q = L_f\,m = 3.335\times10^5\times5 = 1.67\times10^6 \text{ J}$$

The heat required is 1.67×10^6 J.

Example 5.8 Ice cubes at –10 °C with a total mass of 0.045 kg are mixed with 0.3 kg tea at 30 °C. What is the final equilibrium temperature? (The specific heat capacity of tea is the same as that of water, the specific heat capacity of ice $c_{ice} = 2089\ J\ kg^{-1}\ K^{-1}$.)

$m_{ice} = 0.045$ kg, $m_{tea} = 0.3$ kg, $c_{ice} = 2089\ J\ kg^{-1}\ K^{-1}$

If all the ice melts, the final temperature must be higher than 0 °C.

Let the final temperature be T.

The heat needed to bring the ice at – 10 °C to 0 °C is

$$\Delta Q_1 = m_{ice} c_{ice} \Delta T$$
$$= 0.045 \times 2089 \times \{0-(-10)\}$$
$$= 940.05\ J$$

The heat needed to melt the ice is

$$\Delta Q_2 = L_f m_{ice}$$
$$= 3.335 \times 10^5 \times 0.045$$
$$= 15\ 007.5\ J$$

The heat needed to warm the melted ice from 0 °C to T is

$$\Delta Q_3 = m_{ice}\ c\ \Delta T$$
$$= 0.045 \times 4184 \times (T-0) = 188.28\ T$$

The heat lost by the tea, originally at 30 °C, to T is

$$\Delta Q_4 = m_{tea} c_{tea} \Delta T$$
$$= 0.3 \times 4184 \times (30 - T)$$
$$= (37\ 656 - 1255.2\ T)$$

Since the heat lost must equal the heat gained,

$$\Delta Q_4 = \Delta Q_1 + \Delta Q_2 + \Delta Q_3$$
$$37\ 656 - 12\ 55.2\ T = 940.05 + 15\ 007.5 + 188.28\ T$$
$$1443.48\ T = 21\ 708.45$$
$$T = 15.04\ °C$$

Reviewed Exercise

- A lead bullet at 327 °C melts after striking a steel plate. With what velocity does the bullet strike the steel plate? (Latent heat of fusion of lead = $24.52 \times 10^3\ J\ kg^{-1}$) (Hint: all of the kinetic energy of the bullet changes into heat energy)

Key Words: melting point, specific latent heat of fusion

5.9 DEPENDENCE OF MELTING POINT AND BOILING POINT ON PRESSURE

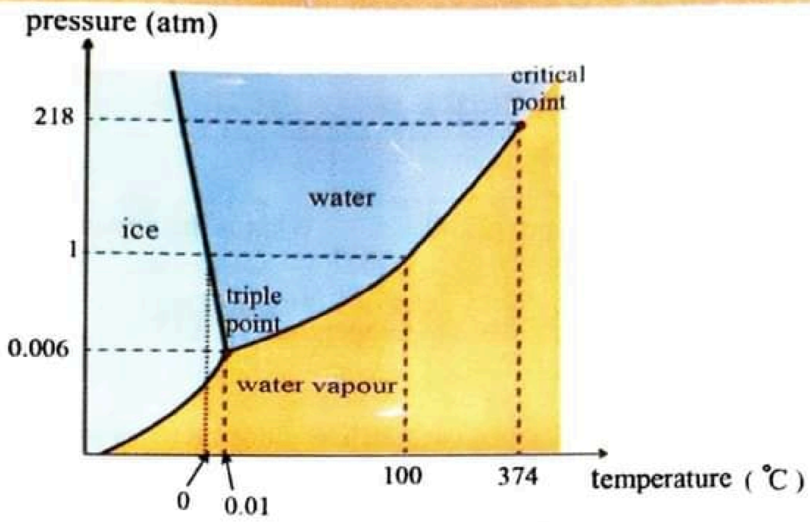


Figure 5.6 Pressure-temperature graph for phase change of water (The axes are not according to scale.)

The melting point and boiling point vary with pressure. Two significant points are the triple point and the critical point as shown in Figure 5.6.

At the triple point, liquid, solid and vapour phase may all exist together. At the critical point both the liquid and gas phases of a substance have the same density, and are therefore indistinguishable.

The melting point of ice varies with atmospheric pressure. The melting point of ice decreases as the pressure increases.

For an ice skater, the pressure of ice skates lower the melting point of ice and melts the ice under them. Hence, the blades can cut across ice because of the very thin layer of water is formed on top of the ice. However, the pressure exerts on the ice surface would return to normal after the blade passes. The thin layer of water freezes again and the ice rink won't become a pool of water.

The boiling point of water also varies with the atmospheric pressure. An increase in external pressure will raise the boiling point of water. This is used in pressure cookers. When a pressure cooker is in operation, the pressure inside it is twice the normal atmospheric pressure. At this pressure, water boils at a higher temperature of 120 °C and thus food can be cooked in a much shorter time.

Reviewed Exercise

- How does the melting point of ice and boiling point of water change with pressure?

Key Words: pressure, triple point, critical point

SUMMARY

The amount of thermal energy required to change the temperature of an object by one degree is called the **thermal capacity (or) heat capacity** of that object.

The **specific heat capacity** of a substance is the heat needed to change the temperature of a unit mass of that substance by one degree.

Heat that must be supplied to change 1kg of liquid at its boiling point from liquid phase to vapour phase is called the **specific latent heat of vaporization** of that liquid.

Heat required to melt 1kg of a solid at its melting point is called the **specific latent heat of fusion** of that solid.

EXERCISES

1. The heat capacity of a piece of copper is 200 J °C $^{-1}$. What is the amount of heat required to raise its temperature from 25 °C to 80 °C?

2. How much heat must be added to change the temperature of 0.15 kg helium from 30 °C to 80 °C without changing the volume? (Specific heat capacity of helium is 5.18×10^3 J kg^{-1} K^{-1})

3. A calorimeter at 10 °C contains 0.1 kg of carbon. The calorimeter is made of aluminium and has mass 0.02 kg. When 1000 J of energy is added to calorimeter and carbon the temperature increases to 30 °C. Specific heat capacity of aluminium is 898 J kg^{-1} K^{-1}. Find the specific heat capacity of carbon.

4. 1 litre of water at 100 °C is added 4 litre of water at 20 °C. What will be the final temperature of water? (Specific heat capacity of water is 4184 J kg^{-1}K^{-1})

5. 0.2 kg coffee at 80 °C is poured into 0.5 kg of glass at 20 °C. What is the final temperature of coffee?
(Specific heat capacity of water = 4184 J kg^{-1}K^{-1}, Specific heat capacity of glass = 837 J kg^{-1}K^{-1})

6. 0.5 kg of water at 30 °C is placed in refrigerator which can remove heat at an average rate 25 J s^{-1}. How long will it take to cool the water to 5 °C?
(Specific heat capacity of water = 4184 J kg^{-1}K^{-1})

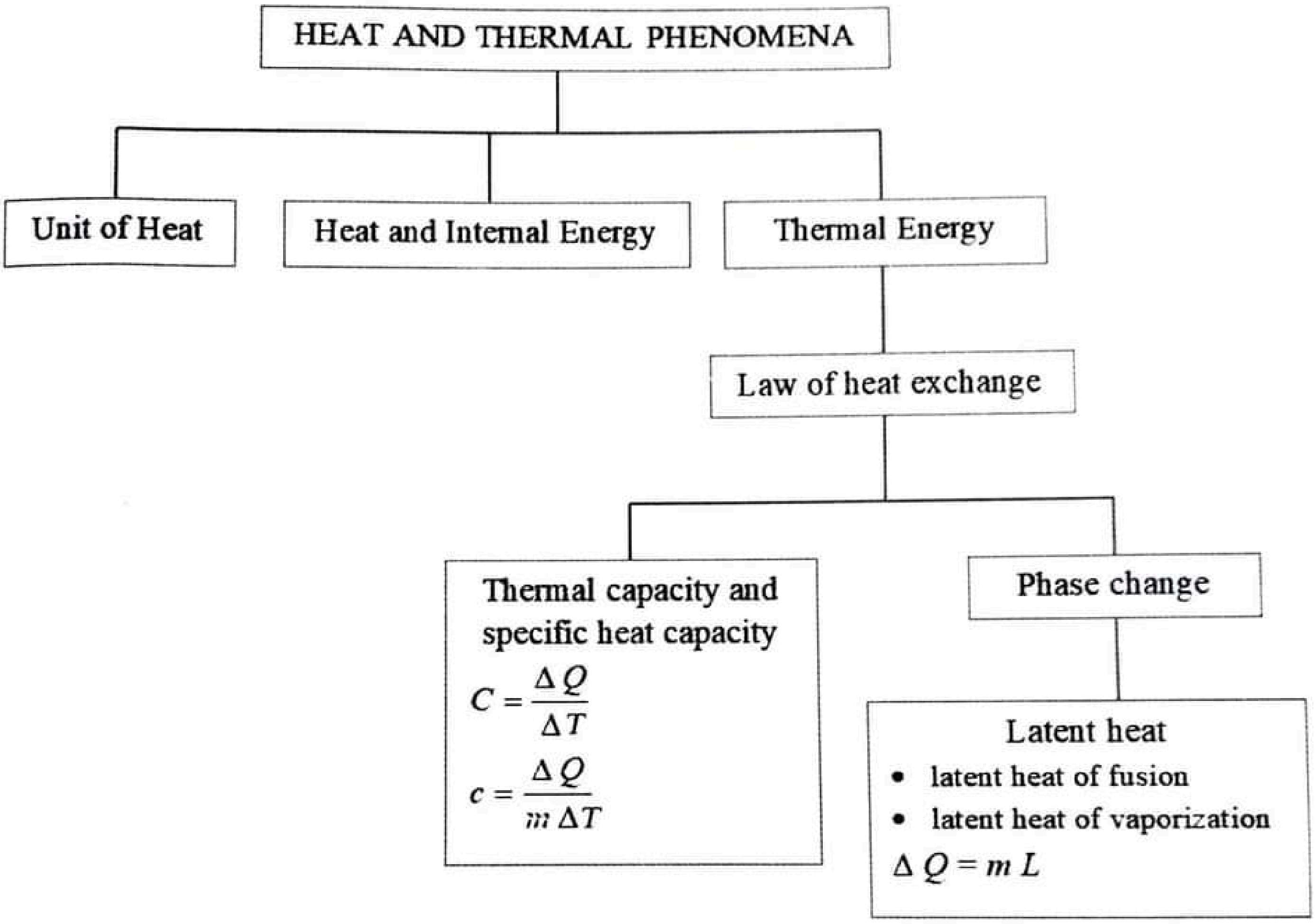
7. How much heat is needed to melt 10 kg of ice at −10 °C?
(Specific latent heat of fusion of ice = 3.335×10^5 J kg^{-1}, Specific capacity of ice = 2089 J kg^{-1} K^{-1})

8. Does all the ice melt when 0.15 kg of ice at 0 °C is put into 0.25 kg of water at 20 °C? What is the final temperature?

9. 0.2 kg of water at 0 °C is poured into a container having liquid nitrogen at −196 °C by mistake. How much nitrogen vaporizes? The boiling point of nitrogen is −196 °C and its latent heat of vaporization is 200 800 J kg^{-1}.

10. A 10 kg mass of copper block is dropped from a height of 50 m. Assume all of the potential energy change to heat, how much has the temperature increased when the copper block strikes the ground? (Specific heat capacity of copper = 385 J kg^{-1}K^{-1}) ($g = 9.8$ m s^{-2})

CONCEPT MAP


HEAT AND THERMAL PHENOMENA
Unit of Heat
Heat and Internal Energy
Thermal Energy
Law of heat exchange
Thermal capacity and specific heat capacity
$C = \frac{\Delta Q}{\Delta T}$
$c = \frac{\Delta Q}{m \Delta T}$
Phase change
Latent heat
• latent heat of fusion
• latent heat of vaporization
$\Delta Q = m L$

CHAPTER 6

VIBRATION OF STRINGS, RESONANCE AND VIBRATION OF AIR COLUMNS

Most musical instruments produce sound due to vibration of the string and air column. These vibrations give rise to waves known as stationary waves in the string (or) air column and cause progressive waves to spread out from musical instrument.

Learning Outcomes

It is expected that students will

- analyse the characteristics of stationary waves.
- investigate vibrating strings.
- examine sound produced by resonance columns and organ pipes.
- explain intensity of waves.
- acquire basic knowledge of generation and propagation of waves.

Waves are classified as progressive waves and stationary waves. Progressive waves spread out from the region in which they are produced. Unlike progressive waves, stationary waves do not spread out but remain in the region in which they are produced. So they are also called standing waves. Sound waves which travel in air when we speak and water waves which travel on the water surface when a stone is dropped are progressive waves. Progressive waves carry energy through the medium. The waves produced in hollow tubes such as flutes and in string instruments such as violins and mandolins are stationary waves.

6.1 STATIONARY WAVES

A stationary wave is the resultant wave by the superposition of two waves of the same type having equal amplitudes and velocities traveling in opposite directions.

The formation of a stationary wave can be demonstrated as follows. One end of a string is fastened to the vibrating arm of an electric vibrator and the other end is held by the hand. When the electric vibrator is activated while the string is held tightly, the string vibrates due to the electric vibrator. The incident wave travels from the vibrator to the hand and the reflected wave travels from the hand to the vibrator. The resultant wave obtained from the superposition of the incident and the reflected wave is a stationary wave as shown Figure 6.1.

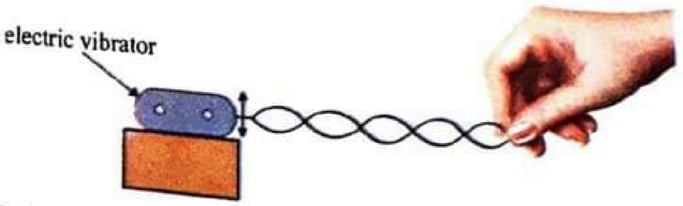


Figure 6.1 Illustration of production of stationary wave

Principle of Superposition

Waves have linearity property. When two (or) more waves pass the same point, the resultant wave at that point is the sum of the individual waves. This is called principle of superposition. If the resultant wave has a larger amplitude than that of individual waves, this is said to be constructive interference. If the resultant wave has a smaller amplitude than that of individual waves, this is said to be destructive interference. If the resultant wave has a zero amplitude, it is said to be completely destructive interference in Figure 6.2.

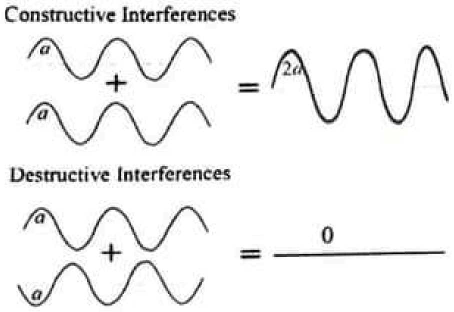


Figure 6.2 Superposition of two waves

Nodes and Antinodes

One characteristic of every stationary wave pattern is that there are points along the medium, which are called nodes and antinodes. The points marked N in Figure 6.3 are always stationary. They are called nodes.

The points between nodes are vibrating with different amplitudes. The mid-points between successive nodes have the largest amplitudes and are called antinodes which are marked A in Figure 6.3.

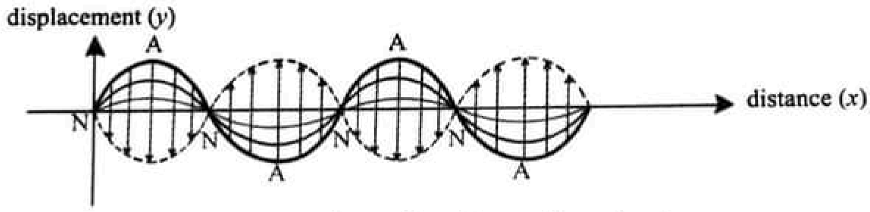


Figure 6.3 Illustration of nodes and antinodes

Nodes and antinodes always alternate and are equally spaced. The distance between two successive nodes (or) antinodes is equal to $\frac{\lambda}{2}$ where λ is the wavelength. The distance from a node to the nearest antinodes is equal to $\frac{\lambda}{4}$.

Example 6.1 If the distance between two consecutive nodes of a stationary wave in a stretched string is 0.5 m, (i) find the distance between two successive antinodes, (ii) find the distance between a node and the nearest antinode.

distance between two consecutive nodes $= \frac{\lambda}{2} = 0.5$ m, $\lambda = 1$ m

(i) distance between two successive antinodes $= \frac{\lambda}{2} = 0.5$ m

(ii) distance between a node and the nearest antinode $= \frac{\lambda}{4} = 0.25$ m

Reviewed Exercise

1. Describe how stationary waves can be produced.
2. How are antinodes and nodes created in a stationary wave?

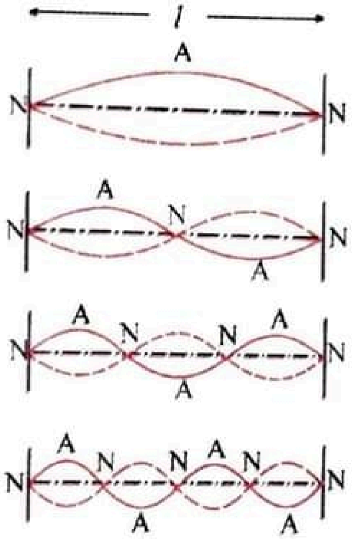
   (Hints: Nodes/Antinodes are produced at the locations where destructive /constructive interference occurs.)

Key Words: progressive wave, stationary wave, superposition, nodes, antinodes

6.2 VIBRATING STRINGS

In most of the musical instruments (for example, violin, mandolin, etc.) the stretched strings act as a source of sound. When the stretched strings are plucked, the stationary waves are produced. They have certain specific frequencies. To understand why only certain frequencies can occur, consider a string of length l rigidly fixed at both ends.

When the string is plucked the stationary waves with nodes at the fixed ends are formed. The waves that are formed on the string are called harmonics. The first four harmonics of the vibrating string are shown in Figure 6.4.



$l = \frac{\lambda_1}{2}, \quad \lambda_1 = 2l$ First harmonic (fundamental)

$l = \frac{2\lambda_2}{2}, \quad \lambda_2 = \frac{2l}{2}$ Second harmonic (first overtone)

$l = \frac{3\lambda_3}{2}, \quad \lambda_3 = \frac{2l}{3}$ Third harmonic (second overtone)

$l = \frac{4\lambda_4}{2}, \quad \lambda_4 = \frac{2l}{4}$ Fourth harmonic (third overtone)

Figure 6.4 Harmonics of vibrating string

The wavelength in Figure 6.4 can be labeled with a subscript n, where n is a positive integer and is called harmonic number.

For the n^{th} harmonic $\lambda_n = \frac{2l}{n} \quad (n = 1, 2, 3, ...)$ (6.1)

The corresponding frequencies are calculated from $v = f_n\lambda_n$, where v is the velocity of wave in a string. The frequencies of vibrating string of length l are,

$$f_n = \frac{nv}{2l} \quad (n = 1, 2, 3, ...) \qquad (6.2)$$

> Vibration of a string in one single segment is called the fundamental (or) first harmonic.
> A musical tone which is part of the harmonic series above a fundamental note is called an overtone.

The velocity of a wave in a vibrating string depends on the tension of the string and mass per unit length of the string as follows.

$$v = \sqrt{\frac{T}{\mu}} \qquad (6.3)$$

where T = tension of the string

μ = mass per unit length of the string

The frequencies of the vibrating string can also be written in terms of T and μ.

We obtain $$f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}} \qquad (n = 1, 2, 3, \ldots) \qquad (6.4)$$

First harmonic, $n = 1$, $$f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Second harmonic, $n = 2$, $$f_2 = \frac{2}{2l}\sqrt{\frac{T}{\mu}} = 2f_1$$

Third harmonic, $n = 3$, $$f_3 = \frac{3}{2l}\sqrt{\frac{T}{\mu}} = 3f_1$$

Mass per unit length of the string is the ratio of mass to length of the string.

$$\mu = \frac{m}{l}, \qquad \text{however, } m = \rho V = \rho A l$$

$$\mu = \frac{\rho A l}{l} = \rho A \qquad (6.5)$$

where A = uniform cross-sectional area of the string
ρ = density of material of the string
V = volume of the string
m = mass of the string

Example 6.2 Find the frequencies of the first three harmonics of the longest string in a grand piano. The length of the string is 1.98 m and the velocity of the wave in the string is 130 m s^{-1}.

The frequencies in a vibrating string,

$$f_n = \frac{nv}{2l} \qquad (n = 1, 2, 3, \ldots)$$

For the first harmonic, $n = 1$

$$f_1 = \frac{v}{2l} = \frac{130}{2\times 1.98} = 32.8 \text{ Hz}$$

For the second harmonic, $n = 2$

$$f_2 = \frac{2v}{2l} = 2f_1 = 2\times 32.8 = 65.6 \text{ Hz}$$

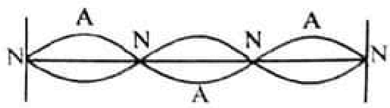
For the third harmonic, $n = 3$

$$f_3 = \frac{3v}{2l} = 3f_1 = 3\times 32.8 = 98.4 \text{ Hz}$$

Example 6.3 The wave velocity in the highest frequency violin string is 435 m s^{-1} and its length l is 0.33 m. If a violin player lightly touches the string at a point which is at a distance $l/3$ from one end, a node is formed at that point. What is the lowest frequency that can now be produced by the string?

$v = 435$ m s^{-1}, $l = 0.33$ m

The frequencies in a vibrating string,



$$f_n = \frac{nv}{2l} \qquad (n = 1, 2, 3, \ldots)$$

Since the string is now vibrating with third harmonic $n = 3$,

$$f_3 = \frac{3v}{2l} = \frac{3 \times 435}{2 \times 0.33} = 1977 \text{ Hz}$$

Example 6.4 The highest and lowest frequency strings of a piano are tuned to fundamentals of $f_H = 4186$ Hz and $f_L = 32.8$ Hz. Their lengths are 0.051 m and 1.98 m respectively. If the tension in these two strings is the same, compare the masses per unit length of the two strings.

The frequencies of a vibrating string,

$$f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}, \qquad (n = 1, 2, 3 \ldots)$$

For fundamental frequency, $n = 1$, $f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$$\mu = \frac{T}{(2l f_1)^2}$$

Thus the ratio of μ_L for the low frequency string to μ_H for the high frequency string of piano is,

$$\frac{\mu_L}{\mu_H} = \frac{T/(2l_L f_L)^2}{T/(2l_H f_H)^2} = \frac{(l_H f_H)^2}{(l_L f_L)^2}$$

$$\frac{\mu_L}{\mu_H} = \frac{(0.051 \times 4186)^2}{(1.98 \times 32.8)^2} = 10.8$$

Reviewed Exercise

- How does the velocity of a stationary wave formed in a string, with both ends firmly fixed, depend on the tension and mass per unit length of the string?

Key Words: harmonic, tension, mass per unit length, overtone

6.3 RESONANCE COLUMN AND ORGAN PIPES

Resonance and Resonant Frequency

Several pendulums of different lengths are suspended from a flexible beam as shown in Figure 6.5.

If one of them such as A, is set up into oscillation, the others will begin to oscillate because they are coupled by vibrations in flexible beam.

Pendulum C, whose length is the same as that of A, will oscillate with the greatest amplitude since its natural frequency matches that of pendulum A which provides the driving force.

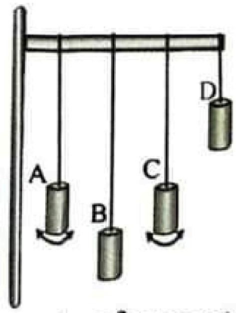


Figure. 6.5 Example of resonance

The amplitude of the motion reaches a maximum when the frequency of driving force equals the natural frequency of the system f_0. Under this condition, the system is said to be in resonance. f_0 is called the resonant frequency of the system.

Natural frequency is the frequency at which a system tends to oscillate in the absence of any driving force.

Resonance Column

If a vibrating tuning fork is placed over the open end of a glass tube partly filled with water as shown in the Figure 6.6, the sound of the tuning fork can be greatly amplified under certain conditions. The length of air column in the tube can be varied by raising (or) lowering the water level. At a certain length of air column, the loud resonant sound will be heard from the tube.

The resonant sound will be heard at certain different length of air column. The wave is sent down the air column in the tube and it is reflected upwards when it hits the water surface. Once again it is reflected downwards when it reached the source. If the air column is just the proper length the reflected wave will be reinforced by the vibrating source as it travels down the tube a second time.

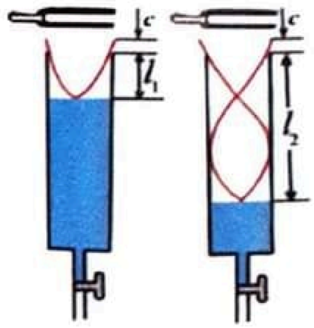
In this way the wave is reinforced for a number of times and resonance is obtained from these multiple reinforcements.

Figure 6.6 Adjustable resonance tube

The tube shown in Figure 6.7 will have an antinode near the open end and a node at the water surface. The water at that end will not allow the air molecules to move downward. So they cannot move at the closed end and a node will be formed. At the open end the air molecules can move out freely and an antinode will be formed at the open end.

Resonance can only be produced under the situation where a node is formed at water surface (closed end) and an antinode is formed at the open end.

The velocity of sound can be found using resonance phenomena. Figure 6.7 shows the resonance phenomena at the length of air column l_1 and l_2. Since the antinodes lie just beyond the end of tube, correction c is added to the length of the air column.



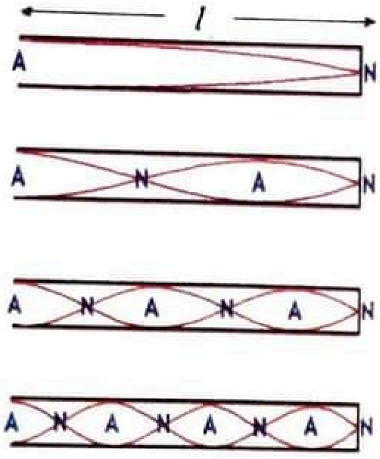
Therefore, $l_1 + c = \frac{\lambda}{4}$ and $l_2 + c = \frac{3\lambda}{4}$.

Figure 6.7 First and second resonance

Using these equations, the wavelength of vibrating air column is $\lambda = 2(l_2 - l_1)$. At resonance, the frequency f of the tuning fork is equal to the frequency of vibrating air column. Thus, the velocity of sound is $v = f\lambda$.

Organ Pipe

The resonance phenomenon also occurs in an organ pipe. The organ pipes produce sound from the vibrations of air column in a pipe. Organ pipes are two types, closed organ pipe and open organ pipe. In closed organ pipe, one end is opened and another end is closed. For example whistle is a closed organ pipe. In closed organ pipes, an antinode exists near the open end (blowing end) while a node is formed at the closed end. The resonant frequencies for a closed organ pipe are as shown in Figure 6.8.



$l = \frac{\lambda_1}{4}$, $\lambda_1 = 4l$ First Harmonic (fundamental)

$l = \frac{3\lambda_3}{4}$, $\lambda_3 = \frac{4l}{3}$ Third Harmonic (first overtone)

$l = \frac{5\lambda_5}{4}$, $\lambda_5 = \frac{4l}{5}$ Fifth Harmonic (second overtone)

$l = \frac{7\lambda_7}{4}$, $\lambda_7 = \frac{4l}{7}$ Seventh Harmonic (third overtone)

Figure 6.8 Resonance phenomena in closed organ pipe

The wavelength of the n^{th} harmonic for vibrating air column in closed organ pipe is

$$\lambda_n = \frac{4l}{n} \qquad (n = 1, 3, 5 \ldots) \qquad (6.6)$$

We can now easily find the corresponding resonant frequency.

Frequency of closed organ pipe is

$$f_n = \frac{nv}{4l} \qquad (n = 1, 3, 5 \ldots) \qquad (6.7)$$

where v = velocity of sound

For the first harmonic, $n = 1$

$$f_1 = \frac{v}{4l}$$

For the third harmonic, $n = 3$

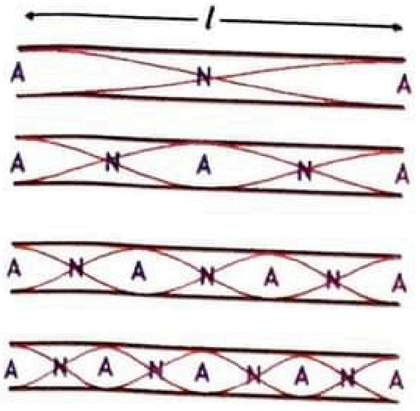
$$f_3 = \frac{3v}{4l} = 3f_1$$

For the fifth harmonic, $n = 5$

$$f_5 = \frac{5v}{4l} = 5f_1$$

Closed organ pipe produces only odd harmonics. Therefore, third harmonic and fifth harmonic are called first overtone and second overtone respectively.

The resonance phenomena in open organ pipe are shown in Figure 6.9. The stationary wave with a single node in the open organ pipe corresponds to fundamental frequency (first harmonic). Thus, the stationary wave with two nodes is second harmonic (or) first overtone. A flute is an open organ pipe.



$l = \frac{\lambda_1}{2}$,	$\lambda_1 = 2l$	First harmonic (fundamental)
$l = \frac{2\lambda_2}{2}$,	$\lambda_2 = \frac{2l}{2}$	Second harmonic (first overtone)
$l = \frac{3\lambda_3}{2}$,	$\lambda_3 = \frac{2l}{3}$	Third harmonic (second overtone)
$l = \frac{4\lambda_4}{2}$,	$\lambda_4 = \frac{2l}{4}$	Fourth harmonic (third overtone)

Figure 6.9 Resonance phenomena in open organ pipe

The wavelength for the n^{th} harmonic $\lambda_n = \frac{2l}{n}$ $(n = 1, 2, 3 ...)$ (6.8)

The frequencies of vibrating air column in open organ pipe are,

$$f_n = \frac{nv}{2l} \qquad (n = 1, 2, 3 ...) \qquad (6.9)$$

A flute can be modeled as a pipe opens at both ends, while clarinet can be modeled as a pipe closed at one end.

Beats

Beats are the periodic fluctuation heard in the intensity of sound when two sound waves of slightly different frequencies interfere with one another as shown in Figure 6.10.

The number of beats per second (or) beat frequency is the difference in frequency between the two sources.

$f_b = f_2 \sim f_1$ where f_b = beat frequency, f_1, f_2 = frequencies of the two sources

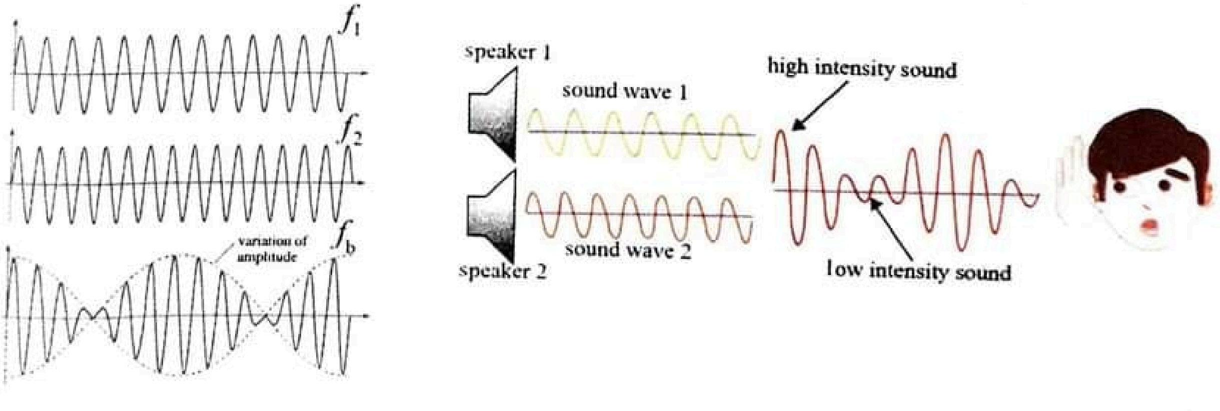


Figure 6.10 Beat frequency

Example 6.5 If two tuning forks with frequencies of 512 Hz and 516 Hz vibrate simultaneously, find the beat frequency.

$f_b = f_2 - f_1 = 516 - 512 = 4$ Hz

That is there would be four pulsating sounds will be heard.

Example 6.6 Find the harmonics which will be formed in a closed organ pipe of length 0.4 m. Velocity of sound in air is 340 m s^{-1}.

$l = 0.4$ m, $v = 340$ m s^{-1}

The harmonics formed in a closed organ pipe,

$$f_n = \frac{nv}{4l} \qquad (n = 1, 3, 5, \ldots)$$

For the first harmonic, $n = 1$

$$f_1 = \frac{v}{4l} = \frac{340}{4 \times 0.4} = 212.5 \text{ Hz}$$

For the third harmonic, $n = 3$

$$f_3 = \frac{3v}{4l} = 3f_1 = 3 \times 212.5 = 637.5 \text{ Hz}$$

For the fifth harmonic, $n = 5$

$$f_5 = \frac{5v}{4l} = 5f_1 = 5 \times 212.5 = 1062.5 \text{ Hz}$$

Example 6.7 A tuning fork is struck and placed over the open end of a resonance tube with adjustable air column. If resonances occur when the air column is 17.9 cm and 56.7 cm long, find the velocity of sound from these values. Frequency of tuning fork is 440 Hz.

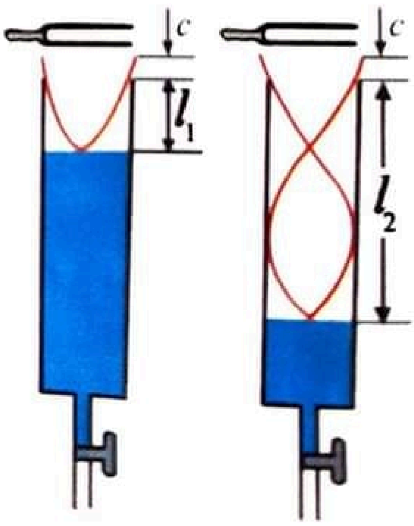
$l_1 = 17.9\,\text{cm}$, $l_2 = 56.7\,\text{cm}$, $f = 440$ Hz

For first resonance,

$$\frac{\lambda}{4} = l_1 + c$$

For second resonance,

$$\frac{3\lambda}{4} = l_2 + c$$



Therefore $\lambda = 2\,(l_2 - l_1)$

$= 2\,(56.7 - 17.9)$

$= 77.6$ cm

$v = f\lambda = 440 \times 77.6$

$= 34\,144$ cm s^{-1}

$= 341.44$ m s^{-1}

Reviewed Exercise

- A closed organ pipe has a fundamental frequency of 256 Hz. What is the length of the pipe? (Velocity of sound = 340 m s^{-1})

Key Words: resonance, resonant frequency, natural frequency, beat

6.4 INTENSITY OF WAVES

The intensity of a wave is the power (or) energy per unit time transported per unit cross-sectional area.

$$I = \frac{P}{A} \tag{6.10}$$

where I = intensity of a wave

P = power

A = cross-sectional area

In SI units, the unit of intensity of a wave is watt per metre squared (W m^{-2}).

If the air around the source is perfectly uniform, the sound power propagated in all directions is the same. In this situation, the propagated sound wave is represented by a spherical wave as shown in Figure 6.11.

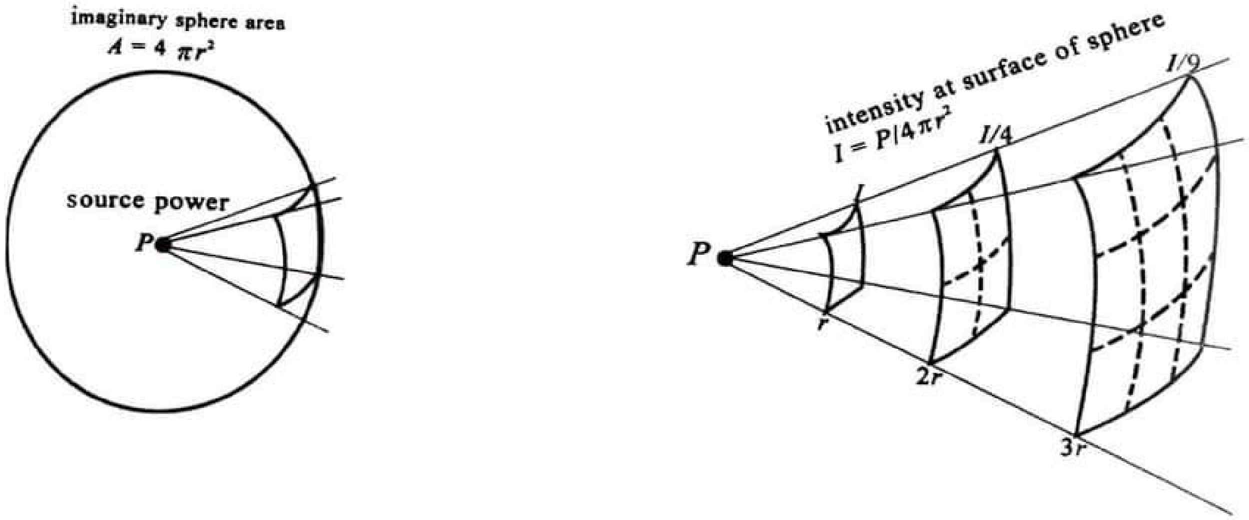


Figure 6.11 Variation of wave intensity with distance from the source

By considering source of sound as a point source, sound wave can be distributed uniformly over spherical wave front of area $4\pi r^2$. Hence, the wave intensity at the distance r from the source is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

For a particular source, intensity varies inversely with the square of the distance from the source as follow.

$$I \propto \frac{1}{r^2}$$

Intensity of a wave obeys inverse square law.

The intensity of a wave is directly proportional to the square of the amplitude of the wave.

Noise Exposure Limit

Noise is a sound especially that is loud (or) unpleasant that causes disturbance in hearing. Loudness of sound can be measured by decibel meter. The unit of loudness (sound level) is decibel (dB).

Sound with very high intensities can be dangerous. Above the threshold of pain (120 dB), sound is painfully loud to ear. Brief exposure to levels of 140 to 150 dB can rupture eardrums and cause permanent hearing loss.

Longer exposure to lower sound (noise) levels can also damage hearing. For example there may be a hearing loss for a certain frequency range. Table 6.1 is expressed the permissible noise exposure limits for maximum duration per day.

Table 6.1 Permissible Noise Exposure Limits

Maximum duration per day (hours)	Sound level (dB)
8	90
6	92
4	95
3	97
2	100
$1\frac{1}{2}$	102
1	105
$\frac{1}{2}$	110
$\frac{1}{4}$	115

Example 6.8 Find the sound intensity for a person sitting 2 metre from the 25 watt sound box.

$P = 25$ W, $r = 2$ m

Intensity of a sound wave,

$$I = \frac{P}{4\pi r^2} = \frac{25}{4\times 3.142\times (2)^2} = 0.50\ \text{W m}^{-2}$$

Reviewed Exercise

- If the distance from a point source of sound is increased by three times, by what factor does sound intensity decrease?

Key Words: intensity, amplitude, power, noise

SUMMARY

A **stationary wave** is the resultant wave by the superposition of two waves of the same type having equal amplitudes and velocities traveling in opposite directions.

Progressive waves spread out from the region in which they are produced. They carry energy through the medium.

The **intensity of a wave** is the power (or) energy per unit time transported per unit cross-sectional area.

EXERCISES

1. There are always points that do not move in stationary waves. (i) What are those points called? (ii) How is the distance between two such successive points related to the wavelength?
2. The distance between two successive nodes of stationary waves produced in a stretched string is 0.4 m. Find the wavelength of that stationary wave. If the frequency is 105 Hz, what is the velocity of the wave in the string?
3. Draw a graph which correctly describes the relation $f - \sqrt{T}$ for the stretched string (f = frequency of the string, T = tension in the string).
4. If the mass of a string of 1 m length is 0.3 g and its tension is 48 N, find the velocity of wave and find the fundamental (the lowest) frequency of the string.
5. What is the tension required for a violin string to vibrate at fundamental frequency of 440 Hz? The length of the violin string is 0.33 m, its diameter is 0.05 cm and the density of the material of which the string is made is 3.5×10^3 kg m^{-3}.
6. Find the fundamental frequency of an open tube of length 4.5 m and diameter 2.5 cm. (velocity of sound = 340 m s^{-1})
7. What is the beat frequency of two tones with the frequencies 256 Hz and 260 Hz?
8. A violinist with a perfectly tuned a string (f = 440 Hz) plays an A note with another violinist, and a beat frequency of 2 Hz is heard. What is the frequency of the tone from the other violin? Is there only one possibility?
9. How is fundamental frequency of vibration of an organ pipe altered as the velocity of sound increases?
10. A student is enjoying a picnic across the valley from a cliff. She is listening her radio and notice a faint echo from the cliff. She claps her hands and the echo takes 1.5 s to return. (i) Given that the speed of sound in air is 343 m s^{-1} on that day, how far away is the cliff? (ii) If the intensity of the music 1.0 m from the radio is 1.0×10^{-5} W m^{-2}, what is the intensity of the radio music arriving the cliff?

CONCEPT MAP

VIBRATION OF STRINGS, RESONANCE AND VIBRATION OF AIR COLUMNS

- **Two Types of Wave**
  - **Progressive Wave**
    e.g.- sound wave
    - water wave
    - Intensity of waves
      $I = \frac{P}{A}$
      $I \propto \frac{1}{r^2}$
    - Beat
      $f_b = f_1 - f_2$
    - Noise exposure limit
      120 dB
  - **Stationary Wave**
    e.g.- wave in hollow tube
    - wave in string musical instrument
    - Wave formed in vibrating
      $\lambda_n = \frac{2l}{n}$
      $f_n = \frac{nv}{2l}$
      $n = 1,2,3,.....$
      $v = \sqrt{\frac{T}{\mu}}$
    - Wave produced in vibrating air column
      - closed organ pipe
        $\lambda_n = \frac{4l}{n}$
        $f_n = \frac{nv}{4l}$
        $n = 1,3,5,.....$
      - open organ pipe
        $\lambda_n = \frac{2l}{n}$ $f_n = \frac{nv}{2l}$
        $n = 1,2,3,.....$

CHAPTER 7
REFRACTION OF LIGHT

Light is a form of energy which stimulates our sense of vision. It is essential for life on earth. Almost all of the natural light comes to us from sun.

Learning Outcomes

It is expected that students will

- summarize the nature of light.
- investigate the velocity of light.
- examine and explain the refraction of light, refractive index and the laws of refraction.
- examine total internal reflection.

In this chapter we will study the nature of light, the speed of light and refraction of light.

7.1 THE NATURE OF LIGHT

By the middle of the seventeenth century, two theories about the nature of light were introduced. They are corpuscular theory and wave theory of light.

Newton suggested that light was made up of a stream of tiny particles known as corpuscles. These corpuscles are given off (or) emitted by light sources such as the sun and the candle flame. They travel outward from light sources in straight lines. They can pass through transparent materials and are bounced back (or) reflected from surfaces of opaque materials through which they cannot pass. When they enter the eye, the sense of sight is stimulated. By using this corpuscular theory, Newton could explain the phenomena of reflection and refraction of light.

Huygens, contemporary of Newton, suggested that like water waves and sound waves, light also have wave nature. However, the majority of scientists did not accept the wave theory of light immediately. Water waves and sound waves can bend around obstacles on their path. Therefore, light, if it is to be considered as a wave motion, must also bend around obstacles and it should be possible to see objects hidden by an obstacle. However, since such objects cannot be observed, the majority did not accept the wave theory of light. Light, in fact, can bend round an obstacle. But the wavelengths of light are so short compared to those of water waves and sound waves that the bending of light cannot be ordinarily observed.

The phenomena of interference, diffraction and polarization could be well explained only if light was considered as a wave motion. In addition, it was discovered by the end of the nineteenth century that light consists of electromagnetic waves. However, it could not be said that Newton's corpuscular theory of light was completely wrong, because it was found at the beginning of the twentieth century that in addition to the wave nature, light has corpuscular or particle nature as well.

Light behaves like particles in some phenomena and acts as waves in others. Hence, light can exhibit both wave and particle characters. Today, it is generally accepted that light has wave-particle duality.

Therefore, the corpuscular theory and wave theory of light are not contradictory but are complementary to each other.

Example 7.1 Write down the names of the two theories concerning the nature of light that were introduced by the middle of seventeenth century. What is the main difference between these two theories?

Newton's corpuscular theory and Huygens' wave theory.

Newton suggested that light was made up of a stream of tiny particles known as corpuscles. Huygens suggested that light has wave nature.

Reviewed Exercise

1. What are the optical phenomena that cannot be explained by Newton's corpuscular theory?
2. Why can the bending of light not be seen although the bending of water waves can be seen?

Key Words: corpuscles, wave-particle duality, bending of light

7.2 VELOCITY OF LIGHT

The velocity of light is usually denoted by the symbol c and it appears in many fundamental formulae in advanced physics.

For example, Einstein has shown that the energy E released from an atom is given by $E = m\,c^2$. Here m is mass. Thus, c is an important physical constant. The velocity of light in free space c is one of the fundamental constants of nature. The value of this constant is taken as 3×10^8 m s^{-1}.

Some methods of measuring the velocity of light are described below.

Galileo's method of measuring the velocity of light was entirely correct in principle. His experiment failed since the method of measuring the extremely short time interval was not accurate enough.

Roemer's method was the first to successfully measure the velocity of light. According to the method, c = 130 000 miles per second (or) 2.1×10^8 m s^{-1}.

Fizeau's method was the first to successfully determine the velocity of light from purely terrestrial measurement. The velocity of light was found 3.1×10^8 m s^{-1} in this method.

The best value of velocity of light is 2.99793×10^8 m s^{-1}.The most precise measurement of light was made by Michelson. The value of the velocity of light obtained by him is c = 186 000 mi s^{-1} = 3×10^8 m s^{-1}. The velocity of light can be measured with the use of electronic devices in the laboratory.

Example 7.2 How long does the light take to reach the earth from the sun if the distance between the sun and the earth is 1.5×10^{11} m?

velocity of light $c = 3\times10^8$ m s^{-1}, $s = 1.5 \times10^{11}$ m

$$v = \frac{s}{t}$$

$$t = \frac{s}{v} = \frac{1.5\times10^{11}}{3\times10^8} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

Reviewed Exercise

1. Can an object move with a velocity greater than the velocity of light?
2. Choose the correct answer from the following.

   If c_1 is the velocity of light coming from the sun and c_2 is that coming from the candle flame, then A. $c_1 > c_2$ B. $c_1 < c_2$ C. $c_1 = c_2$
3. Why did Galileo not succeed in measuring the velocity of light?

Key Words: velocity of light

7.3 REFRACTION OF LIGHT

We shall study how light passes through transparent materials or transparent media. Transparent media such as air, water, oil and glass have different optical densities.

When light passes through two media of different optical densities, the direction of light changes in passing from the first to the second medium. This phenomenon is called refraction of light.

When light passes from one medium to another, the velocity of light changes in both magnitude and direction. That is why the refraction of light takes place.

The velocity of light in a medium depends upon the optical density of the medium it passes through. The more optically dense a medium is, the smaller is the velocity of light in that medium.

Examples for refraction of light are; the part of a stick under the water appears to be bent. a swimming pool looks less deep than it really is. This is due to the refraction of light coming from the immersed part of the stick as shown in Figure 7.1.

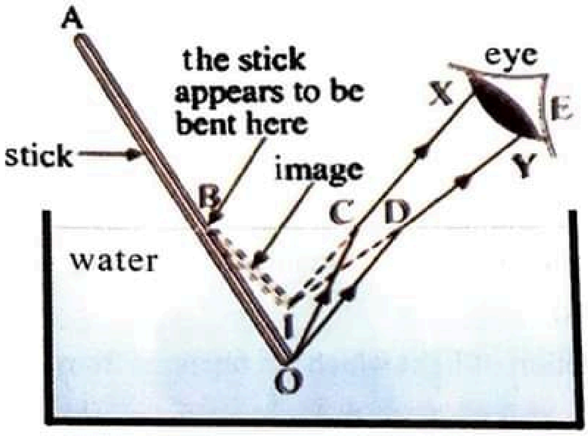
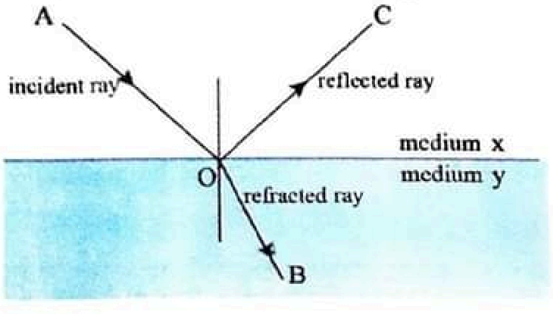


Figure 7.1 Illustration for refraction of light [CREDIT: Source from the Internet]

Example 7.3 Can refraction and reflection of light occur together? If so, illustrate with a diagram.
Yes. Refraction and reflection can occur together.



Reviewed Exercise

1. Draw a ray diagram to show why a stick bends when part of it is under water.
2. Can refraction of light take place in a single medium? Explain.

Key Words: refraction, optical densities

7.4 LAWS OF REFRACTION

In studying refraction through media, we shall assume that the boundary between two media is a plane surface. In Figure.7.2, x and y are media of different optical densities and PQ is the boundary between them.

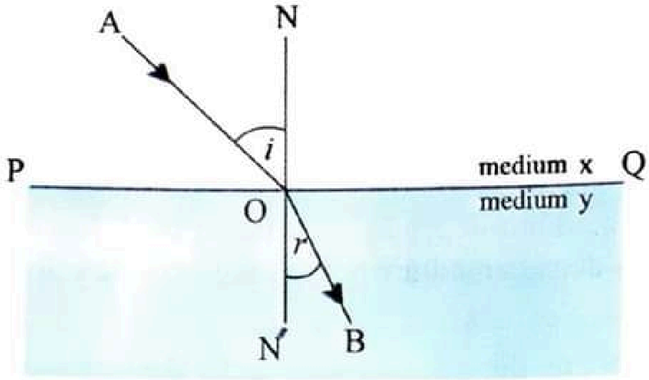


Figure 7.2 Refraction of light at a plane surface

Suppose that light travels from a less dense medium x to a more dense medium y.

The direction of incident ray AO changes when it passes from x to y. This phenomenon is called refraction in medium y. It travels along a new direction OB in medium y. OB is called the refracted ray. Since y is optically more dense medium, OB bends towards the normal.

The angle between the incident ray AO and the normal NON′ is called the angle of incidence and is represented by i. The angle between the refracted ray OB and the normal NON′ is called the angle of refraction and is represented by r. In this case r is smaller than i.

When the medium y is optically less dense medium, refracted ray OB bends away from the normal. In this case r is greater than i.

The laws governing the refraction of light which are obtained from experiments are stated below.

Laws of Refraction

(1) The incident ray, the refracted ray and the normal all lie in the same plane.

(2) For a particular wavelength of light and for a given pair of media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.
(The second law is called Snell's law since it was so named in honour of its discoverer - Dutch Scientist Willebrord Snell in 1621.)

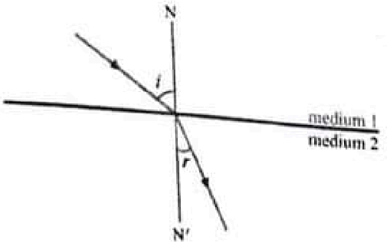
In symbols
$$\frac{\sin i}{\sin r} = \text{constant} \qquad (7.1)$$

where i = angle of incidence
r = angle of refraction

In refraction of light, the light rays also obey the principle of reversibility of light.

Example 7.4 A ray of light traveling in medium 1 strikes the interface to another transparent medium 2. If the speed of light is less in medium 2 than in medium 1, will the ray refract toward the normal? Draw ray diagram.

Yes. The ray will refract toward the normal because the more opitically dense a medium is, the smaller is the speed of light.



Reviewed Exercise

1. Do the light rays in refraction of light obey the principle of reversibility of light? Explain it with suitable ray diagram.
2. When the light ray passes from optically more dense medium to less dense medium, does the refracted ray bend away from the normal?

Key Words: optically more dense medium, optically less dense medium, refraction, angle of incidence, angle of refraction

7.5 REFRACTIVE INDEX

The ratio of the velocity of light in air c to the velocity of light in a particular medium v is called the refractive index n, of that medium. The refractive index is given by

$$n = \frac{c}{v} \qquad (7.2)$$

The refractive index of a medium can also be defined by using Snell's law as follows:
The ratio of the sine of the angle of incidence to the sine of the angle of refraction is always constant, That constant is the refractive index of the medium through which the refracted ray passes.
In Figure 7.2, the refractive index can be expressed as

$$_{x}n_{y} = \frac{\sin i}{\sin r} \qquad (7.3)$$

$_{x}n_{y}$ is called the refractive index of medium y with respect to medium x (or) refractive index for the refraction of light from medium x to medium y.

According to the principle of reversibility of light, if BO is an incident ray in medium y, it will be refracted away from the normal along OA in medium x. In this case, r is the angle of incidence, i is the angle of refraction and the refractive index is

$$_{y}n_{x} = \frac{\sin r}{\sin i} \qquad (7.4)$$

Multiplying equation (7.3) and (7.4), we obtain

$$_{x}n_{y} \times {}_{y}n_{x} = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1$$

$$_{x}n_{y} = \frac{1}{_{y}n_{x}} \qquad (7.5)$$

Therefore, the refractive index of the medium y with respect to x is equal to the reciprocal of the refractive index of medium x with respect to y.

Absolute Refractive Index

The above stated media x and y may be any two media. Specifically, if x is a vacuum the refractive index is denoted by ${}_{v}n_{y}$. The refractive index of the medium y for light travelling from vacuum into y is called the absolute refractive index of medium y.

Refractive Index of a Medium

The refractive index of a medium with respect to air is easier to determine than the refractive index of the medium with respect to vacuum. This refractive index of a medium with respect to air is normally used to compare the refractive indices of different media. For example, the refractive index of water with respect to air is written as n_w and the refractive index of glass with respect to air is written as n_g. Refractive indices of some media are given in Table 7.1.

Table 7.1 Refractive indices of some media

Substance	Refractive Index
Ice	1.31
Water	1.33
Ethyl Alcohol	1.36
Oleic Acid	1.46
Glycerine	1.47
Quartz	1.54
Glass	1.4 – 1.9
Diamond	2.42

Relation between Refractive Index of Media and Wavelength

Since light has wave nature, the wavelength of light wave also changes when light passes from one medium to another as shown in Figure 7.3. v_x and v_y are the velocities of light in media x and y respectively. n_x and n_y are the refractive indices of media x and y respectively.

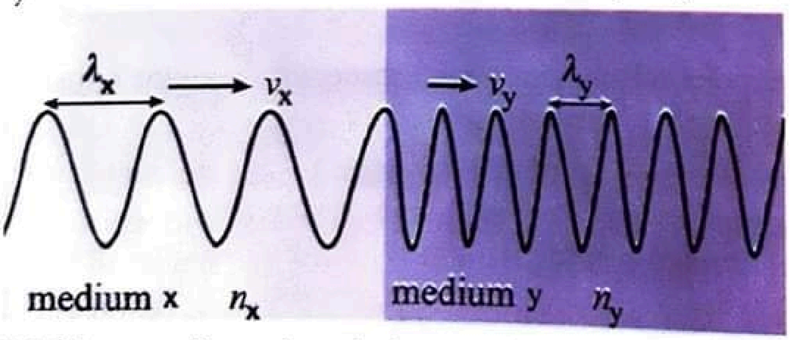


Figure 7.3 Change of wavelength due to refractive indices of media

$$n_x = \frac{c}{v_x} \text{ and } n_y = \frac{c}{v_y}$$

$$n_x v_x = c \text{ and } n_y v_y = c$$

Therefore, $$n_x v_x = n_y v_y \tag{7.6}$$

The velocity of light v, the frequency of light f and the wavelength of light λ are related as $v = f\lambda$. When light passes from medium x to medium y, the frequency remains the same. λ_x and λ_y are the wavelengths of light in medium x and y respectively, we get $v_x = f\lambda_x$ and $v_y = f\lambda_y$.

$$n_x f \lambda_x = n_y f \lambda_y$$

Therefore, $$n_x \lambda_x = n_y \lambda_y \qquad (7.7)$$

Example 7.5 The wavelength of a ray of light in air is 5×10^{-7} m. With what velocity will that ray pass through diamond whose refractive index is 2.42? Find the wavelength of that ray in diamond.

wavelength in air $\lambda_a = 5\times10^{-7}$ m

refractive index of air $n_a = 1$, refractive index of diamond $n_d = 2.42$

Let velocity of light in diamond be v_d

$$c = 3\times10^8 \text{ m s}^{-1}$$

$$n_d = \frac{c}{v_d}$$

$$v_d = \frac{3\times10^8}{2.42} = 1.24\times10^8 \text{ m s}^{-1}$$

$$n_a \lambda_a = n_d \lambda_d$$

$$1\times5\times10^{-7} = 2.42\, \lambda_d$$

$$\lambda_d = 2.07\times10^{-7} \text{m}$$

Wavelength in diamond is 2.07×10^{-7} m.

Example 7.6 The angle of incidence of a ray of light passing from air to a transparent medium x is 30° and the angle of refraction is 19° 28′. If another ray is incident at 35° on that medium find the angle of refraction.

$i = 30°$, $r = 19° 28'$

By Snell's law,

$$n_x = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 30^\circ}{\sin 19^\circ 28'} = 1.5$$

For another ray, $i_1 = 35°$, angle of refraction is r_1.

$$n_x = \frac{\sin i_1}{\sin r_1}$$

$$\sin r_1 = \frac{\sin 35^\circ}{1.5}$$

Therefore, $$r_1 = 22^\circ 29'$$

The angle of refraction is 22° 29′ for the angle of incidence 35°.

Reviewed Exercise

1. Explain the statement: "The refractive index of glass is 1.5".
2. Which quantity can be determined by using Snell's law?

Relation between Angle of Incidence and Angle of Emergence for a Ray passing through a Glass Slab with Parallel Sides

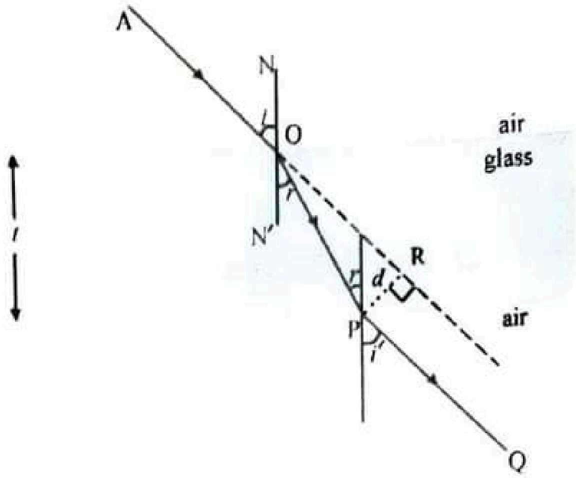


Figure 7.4 Refraction through a glass slab

In Figure7.4, an incident ray AO in air is refracted along OP in the glass slab and it emerges along PQ into the air, i is the angle of incidence and i' is the angle of emergence.

Since glass is a more optically dense medium, the refracted ray OP is bent towards the normal in the glass. Again, OP is refracted along PQ away from the normal into the air.

For the first refraction, light passes from air to glass $\quad {}_{a}n_{g} = \dfrac{\sin i}{\sin r}$

For the second refraction, light emerges to air from glass $\quad {}_{g}n_{a} = \dfrac{\sin r}{\sin i'}$

Since $\quad {}_{a}n_{g} \times {}_{g}n_{a} = 1$

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i'} = 1$$

$$\sin i = \sin i'$$
$$i = i'$$

Therefore, the angle of incidence i is equal to the angle of emergence i' for a ray passing through a glass slab with parallel sides. This holds true not only for glass and air but also for any two media having parallel boundary surfaces between them. In other words, the incident ray and the emergent ray are parallel for such a pair of media.

Lateral Displacement of a Ray passing through a Glass Slab with Parallel Sides

In Figure 7.4, the perpendicular distance PR between AO produced beyond O and the emergent ray PQ is a lateral displacement of AO, denoted by d. The thickness of the glass slab, that is, the distance between its parallel sides is denoted by t. We can easily derive lateral displacement d.

The lateral displacement $\quad d = \dfrac{t \sin(i-r)}{\cos r} \qquad (7.8)$

Refraction through three Parallel Media

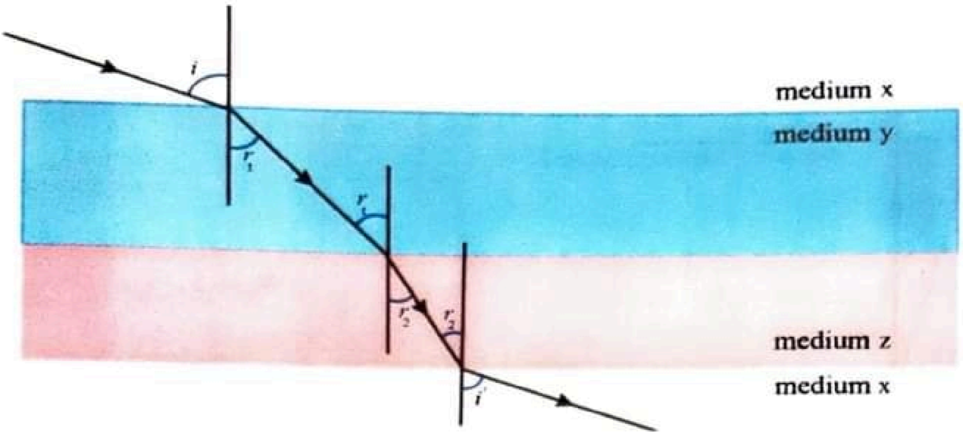


Figure 7.5 Refraction through three parallel media

In Figure 7.5 media x, y and z have different refractive indices. A ray in medium x is refracted through media y and z and emerges into medium x. The refraction occurs three times and the angle of incidence i is equal to the angle of emergence i'.

For the refraction from medium x to y, $\quad {}_xn_y = \dfrac{\sin i}{\sin r_1}$

For the refraction from medium y to z, $\quad {}_yn_z = \dfrac{\sin r_1}{\sin r_2}$

For the refraction from medium z to x, $\quad {}_zn_x = \dfrac{\sin r_2}{\sin i'}$

$$ {}_xn_y \times {}_yn_z \times {}_zn_x = \frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i'} = 1 $$

$$ {}_yn_z = \frac{1}{{}_xn_y \times {}_zn_x} $$

Since $\quad {}_xn_z = \dfrac{1}{{}_zn_x}, \quad {}_yn_z = \dfrac{{}_xn_z}{{}_xn_y}$

If medium x is air, we have $\quad {}_yn_z = \dfrac{n_z}{n_y} \qquad (7.9)$

Therefore, the refractive index of medium z with respect to medium y is the ratio of the refractive index of medium z with respect to air to the refractive index of medium y with respect to air.

Refractive Index Related to Real and Apparent Depths

Figures 7.6 and 7.7 show the positions of objects in one medium and their respective images when viewed from the neighbouring medium. Medium y has greater refractive index than medium x.

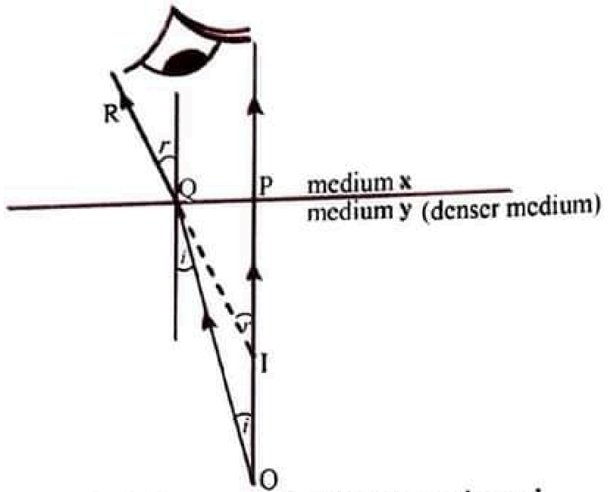


Figure 7.6 Apparent depth when viewed from a less dense medium

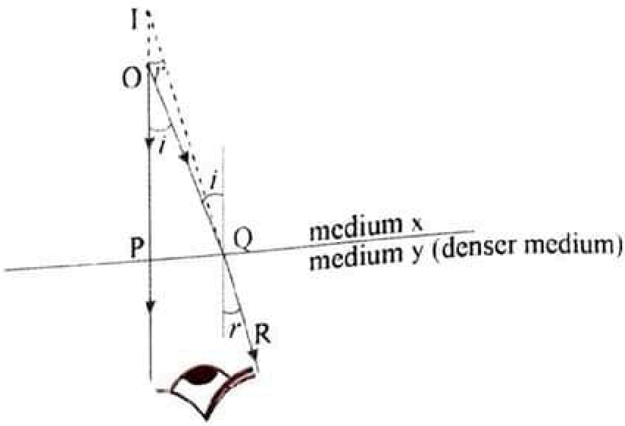


Figure 7.7 Apparent depth when viewed from a denser medium

In Figure 7.6 an object O is in medium y and the observer in medium x, looks at it directly from above. Due to the refraction of light the point I is the position of the image of O. Therefore, the observer in medium x, viewing O directly from above, sees it in the position I. In other words, the object appears nearer to the observer.

In Figure 7.7 an object O is in medium x and the observer in medium y. The point I is the image of O. Therefore, the observer in the medium y, viewing O, sees it in the position I. In other words, the object appears farther away from the observer.

Since the refracted ray QR enters the observer's eyes, Q is actually very close to P in practice.

The perpendicular distance from the object O to the x-y boundary surface is called the real depth and is represented by u. The perpendicular distance from the image I to the x-y boundary surface is called the apparent depth and is represented by v.

In Figure7.6 and 7.7 $PO = u$, $PI = v$ and we have

$$\sin i = \frac{PQ}{QO}, \qquad \sin r = \frac{PQ}{QI}$$

Then $$n = \frac{\sin i}{\sin r} = \frac{PQ/QO}{PQ/QI} = \frac{QI}{QO}$$

Since P is very close to Q, $QI = PI$ and $QO = PO$.

Therefore $$n = \frac{\sin i}{\sin r} = \frac{PI}{PO} = \frac{v}{u} \qquad (7.10)$$

By the Snell's law

For refraction in Figure 7.6, when viewed from less dense medium,

$${}_{y}n_{x} = \frac{\sin i}{\sin r} = \frac{v}{u} = \frac{\text{apparent depth}}{\text{real depth}}$$

However, ${}_{x}n_{y} = \frac{1}{{}_{y}n_{x}}$ Therefore, ${}_{x}n_{y} = \frac{\text{real depth}}{\text{apparent depth}}$

For refraction in Figure 7.7, when viewed from denser medium,

$${}_{x}n_{y} = \frac{\sin i}{\sin r} = \frac{v}{u} = \frac{\text{apparent depth}}{\text{real depth}}$$

Example 7.7 The refractive index of a liquid is 1.32 and that of glass is 1.5. If a ray of angle of incidence 30° enters from liquid to glass find the angle of refraction.

$n_l = 1.32$, $n_g = 1.5$

Refractive index of glass with respect to liquid

$$ {}_l n_g = \frac{n_g}{n_l} = \frac{1.5}{1.32} = 1.136 $$

$i = 30°$, angle of refraction in glass is r.

By the Snell's law,

$$ {}_l n_g = \frac{\sin i}{\sin r} $$

$$ \sin r = \frac{\sin i}{{}_l n_g} = \frac{\sin 30°}{1.136} = 0.4386 $$

That is $r = 26° 6'$

Example 7.8 When a drop of ink at the bottom of a glass slab 6 cm thick is viewed from above, it is seen at a spot 2 cm above the bottom. Find the refractive index of glass.

real depth = 6 cm,

Since the drop of ink is at the bottom of the glass slab,

apparent depth = 6 − 2 = 4 cm

The refractive index of glass $n_g = \frac{\text{real depth}}{\text{apparent depth}} = \frac{6}{4} = 1.5$

Reviewed Exercise

- The object O is in medium y and the observer is in air. Show that n_y = real depth / apparent depth.

Key Words: refractive index, real depth, apparent depth

7.6 CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

When light passes from a medium to a more optically dense medium both reflection and refraction will occur for all angles of incidence. But when light passes from a medium to a less optically dense medium refraction will occur only for some angles of incidence. However, reflection will occur for all angles of incidence.

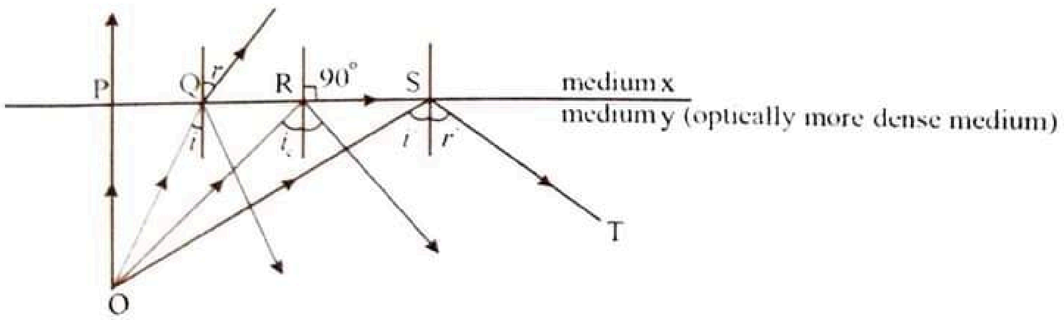


Figure 7.8 Illustration of total internal reflection

In Figure 7.8, an object O is in medium y which has greater refractive index than medium x. The angle of refraction r is greater than the angle of incidence i. As the angle of incidence increases, the angle of refraction also increases. At a certain angle of incidence, the angle of refraction becomes 90°. This means that the refracted ray lies in the boundary plane between the two media. The angle of incidence corresponding to the angle of refraction 90° is called the critical angle and is denoted by i_c.

In Figure 7.8, the incident ray OR from O is refracted along the x-y boundary and the angle of incidence i_c is the critical angle. When the angle of incidence is greater than i_c light does not enter medium x at all, but is reflected back into medium y.

The light in one medium does not enter the optically less dense medium and is reflected back into the first medium for all angles of incidence greater than the critical angle. This phenomenon is called total internal reflection.

Relation between Critical Angle and Refractive Index

In the case of refraction in Figure 7.8,

By using Snell's law

$$_y n_x = \frac{\sin i}{\sin r}$$

When $i = i_c$, $r = 90°$

Therefore $$_y n_x = \frac{\sin i_c}{\sin 90°} = \sin i_c$$

Since $$_x n_y = \frac{1}{_y n_x}$$

$$_x n_y = \frac{1}{\sin i_c}$$

If medium x is air, $$n_y = \frac{1}{\sin i_c} \qquad (7.11)$$

The refractive index of the medium in which the object is situated is equal to the reciprocal of the sine of the critical angle.

Example 7.9 (i) Find the critical angle of a liquid of refractive index 1.32. (ii) Find the refractive index of diamond of critical angle 24° 27′.

(i) refractive index of liquid, $n_l = 1.32$

$$\sin i_c = \frac{1}{n_l} = \frac{1}{1.32}$$

Therefore the critical angle of a liquid is $i_c = 49° 15'$

(ii) $i_c = 24° 27'$

$$n_d = \frac{1}{\sin i_c} = \frac{1}{\sin 24° 27'} = 2.42$$

The refractive index of diamond is 2.42

Reviewed Exercise

1. A diver shines a flash light upward from beneath the water at a 42.5° angle to the vertical. Will the light leave the water? (refractive index of water = $\frac{4}{3}$)
2. Under what conditions can the total internal reflection occur?

Refraction through a Prism

A prism is a transparent object usually made of glass. It has two plane surfaces, inclined to each other (Figure 7.9).

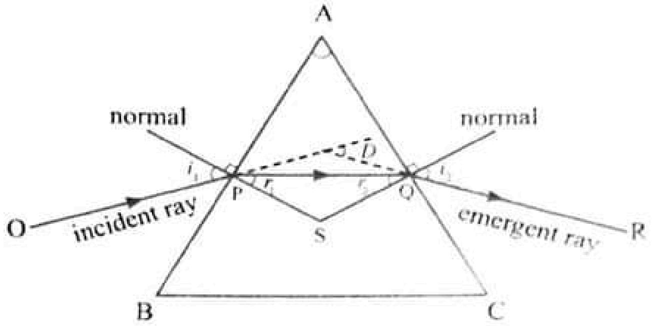


Figure 7.9 Refraction by a prism

Figure 7.9 shows refraction by a prism. The angle A is known as the angle of prism and BC is its base. An incident ray OP is refracted along PQ in the prism and emerges from the surface AC into the air along QR.

AB is called the incident surface and AC the emergent surface. The emergent ray QR is directed towards the base and OPQR is the path of light travelling through the prism. If a ray is incident at the surface AC along RQ it will travel along RQPO.

The incident ray OP after refraction through the prism emerges along QR and the direction of OP is changed or deviated by it. This is called the deviation of light by the prism. The angle D, between the direction of incident ray OP and that of emergent ray QR is known as the angle of deviation.

When the angle of incidence i is varied the angle of deviation D also varies. When i is increased gradually, D decreases gradually to a minimum value and then increases with the further increase of i. If the experimental values of i are plotted against those of D the appearance of an i-D graph obtained is shown in Figure 7.10.

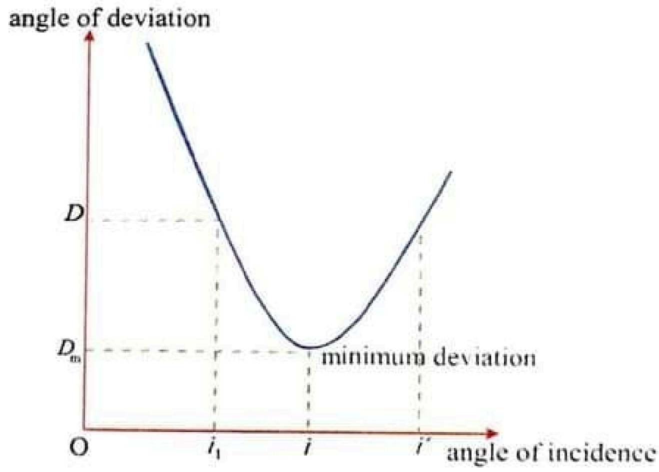


Figure 7.10 Relation between angle of incidence and angle of deviation (i-D graph)

The angle of minimum deviation is denoted by D_m. It is found that the angle of incidence is equal to the angle of emergence when the angle of deviation is minimum.

In the minimum deviation case the formula for the refractive index of a prism can be obtained as follows.

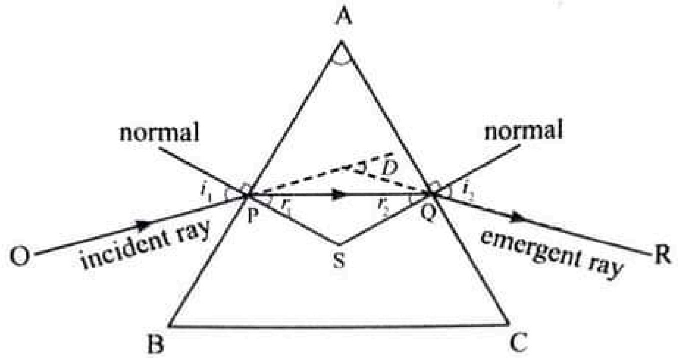


Figure 7. 11 Deviation by a prism

From Figure 7.11, $A + \angle PSQ = 180°$ $(A = \angle BAC)$

From the triangle PQS, $r_1 + r_2 + \angle PSQ = 180°$

$$A = r_1 + r_2$$

Angle of deviation $D = (i_1 - r_1) + (i_2 - r_2)$

When $D = D_m,\ i_1 = i_2,\ r_1 = r_2,\ A = 2r_1$

$$D_m = 2i_1 - A \text{ and } i_1 = \frac{(A + D_m)}{2}$$

By Snell's law the refractive index of the prism material is $n = \dfrac{\sin i_1}{\sin r_1}$

Substituting the values of i_1 and r_1, we get

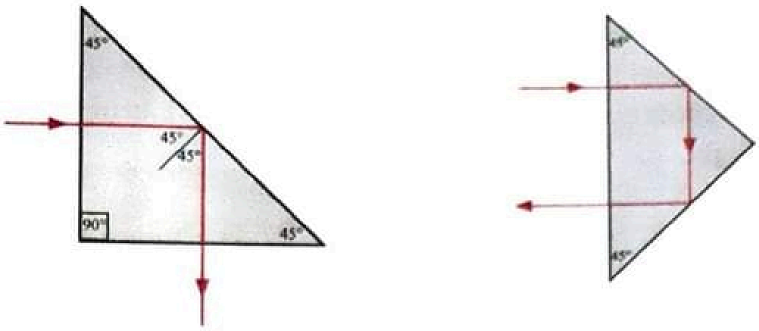
$$n = \frac{\sin \dfrac{(A+D_m)}{2}}{\sin \dfrac{A}{2}} \tag{7.12}$$

The refractive index of the prism material can be calculated from this formula if the angle of prism A and angle of minimum deviation D_m are provided.

Some Applications of Total Internal Reflection

If the refractive index of glass is 1.5, the critical angle of glass will be 42°. Thus, the total internal reflection can occur in a glass prism of angles 90°- 45° - 45°. A prism having such angles can be used as a total reflecting prism.

In a total reflecting prism 100 percent of the light is reflected while other reflecting surfaces reflect only some of the light incident on them.

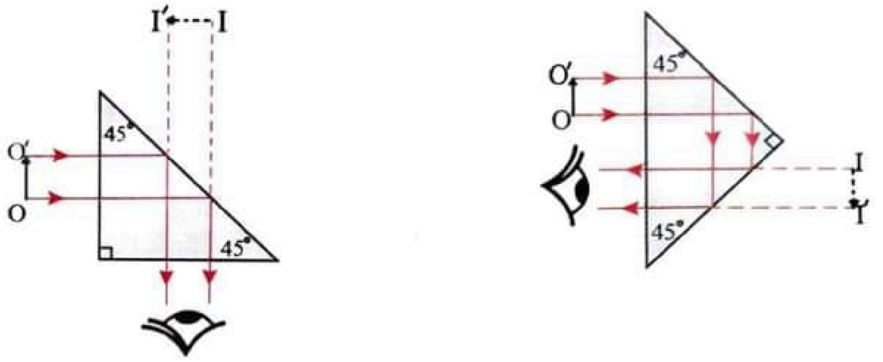


(a) (b)

Figure7.12 Reflection at hypotenuse surface and side surfaces of total reflecting prism

A ray in the air is incident normally on a side surface of the prism as shown in Figure 7.12 (a). In this case the deviation is 90°. In Figure7.12 (b), a ray is incident normally on the hypotenuse surface. The deviation in this case is 180°.

The images seen in the 90°- 45°- 45° prisms are shown in Figure 7.13.



(a) (b)

Figure 7.13 Formation of image by a total reflecting prism

Total reflecting prisms are used in periscopes and binoculars.

The concept of total internal reflection is also used in cutting the faces of diamond for its brightness. If the rays entering the diamond are totally reflected from its base and emerge from the surfaces, the diamond becomes brighter. In order to enhance the brilliance, the facets of diamond must be cut systematically.

Suppose that a ray enters one end of a glass rod or a transparent plastic rod. If the successive total internal reflections occur in the rod the ray will emerge from the other end (Figure7.14). Such a transparent rod is called a light pipe.

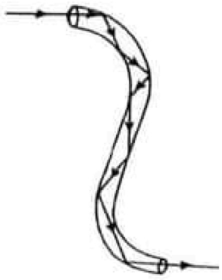
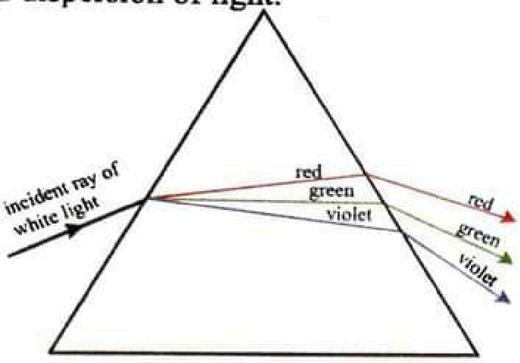


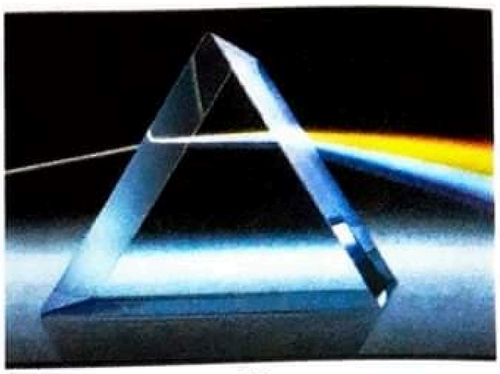
Fig. 7.14 Total internal reflection in a light pipe

Dispersion and Formation of Spectrum by a Prism

When a narrow pencil of white light passes through a prism, it is split into bands of different colours. Such a band of different colours is called a spectrum. Splitting of white light into different colour-bands is called dispersion of light.



(a)



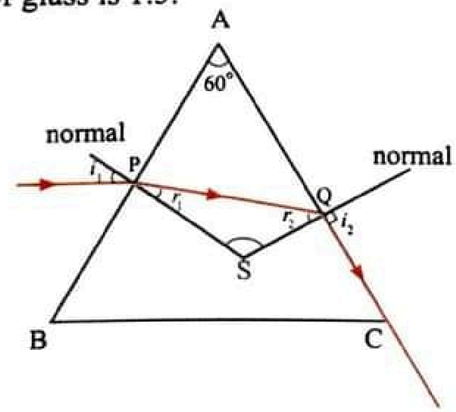
(b)

Figure 7.15 (a) Dispersion by a prism (b) formation of spectrum [CREDIT: Source from the Internet]

The violet colour is deviated the most from the original path of white light. The red colour is deviated the least .

The refractive index of the prism material for violet colour has the largest value. The refractive index of the prism material for red colour has the smallest value.

Example 7.10 A ray of light in air enters a prism (having an angle 60°) from one surface and emerges into the air from the other surface. If the emergent ray lies in the surface of the prism, find the angle of incidence. The refractive index of glass is 1.5.



$A = 60°$, refractive index of glass, $n_g = 1.5$

If the emergent ray lies in the surface of the prism, $i_2 = 90°$, $r_2 = i_c$

At surface AC

$$\sin i_c = \frac{1}{n_g} = \frac{1}{1.5} = 0.667$$

$$i_c = 41° 48'$$

$$\therefore r_2 = 41° 48'$$

Since, $A = r_1 + r_2$, $r_1 = 60° - 41°48' = 18°12'$

At surface AB

By Snell's law,

$$n_g = \frac{\sin i_1}{\sin r_1}$$

$$1.5 = \frac{\sin i_1}{\sin 18^\circ 12'}$$

$$\sin i_1 = 1.5 \times 0.3123$$

$$= 0.4685$$

$$i_1 = 27^\circ 56'$$

The angle of incidence is $27^\circ 56'$.

Reviewed Exercise

- Draw a ray diagram showing (i) 90° deviation case and (ii) 180° deviation case by a total reflecting prism. How much is the percent of reflection by total reflecting prism? Give some examples of their usages.

Key Words: critical angle, total internal reflection, light pipe, dispersion of light, light spectrum

SUMMARY

The ratio of the velocity of light in air c to the velocity of light in a particular medium v is called the **refractive index** n, of that medium.

The angle of incidence corresponding to the angle of refraction 90° is called the **critical angle**.

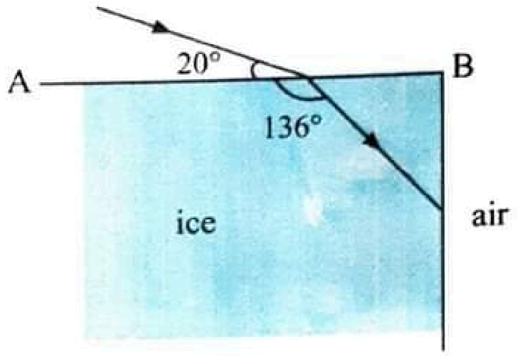
When a narrow pencil of white light passes through a prism, it is split into bands of different colours. Such a band of different colours is called a spectrum. Splitting of white light into different colour-bands is called **dispersion** of light.

EXERCISES

1. Choose the correct answer from the following.
   A. Light has only particle nature.
   B. Light has only wave nature.
   C. Light has both particle and wave nature.
2. Choose the correct answer from the following.
   A. All optical phenomena can be explained by Huygens' wave theory.
   B. All optical phenomena can be explained by Newton's corpuscular theory.
   C. The statement gives in A. and B. are both wrong.
3. If the velocity of light in a medium is 2.3×10^8 m s^{-1}, find the refractive index of the medium.
4. A ray of light in water has a wavelength of 4.42×10^{-7} m. What is the wavelength of that ray while passing through ice? ($n_w = 1.33$, $n_{ice} = 1.31$)

5. When a ray of light is incident on the surface of a glass slab, both reflection and refraction of light take place. If the angle of incidence of the ray is 30° and the refractive index of glass is 1.5, find the angle between the reflected ray and the refracted ray.

6. The path of a ray of light through one corner of a block of ice is shown below.



Find (i) the angle of incidence on the face AB, (ii) the angle of refraction at this face, (iii) the refractive index of ice, (iv) the critical angle for ice and (v) determine whether the ray will emerge from the block of ice.

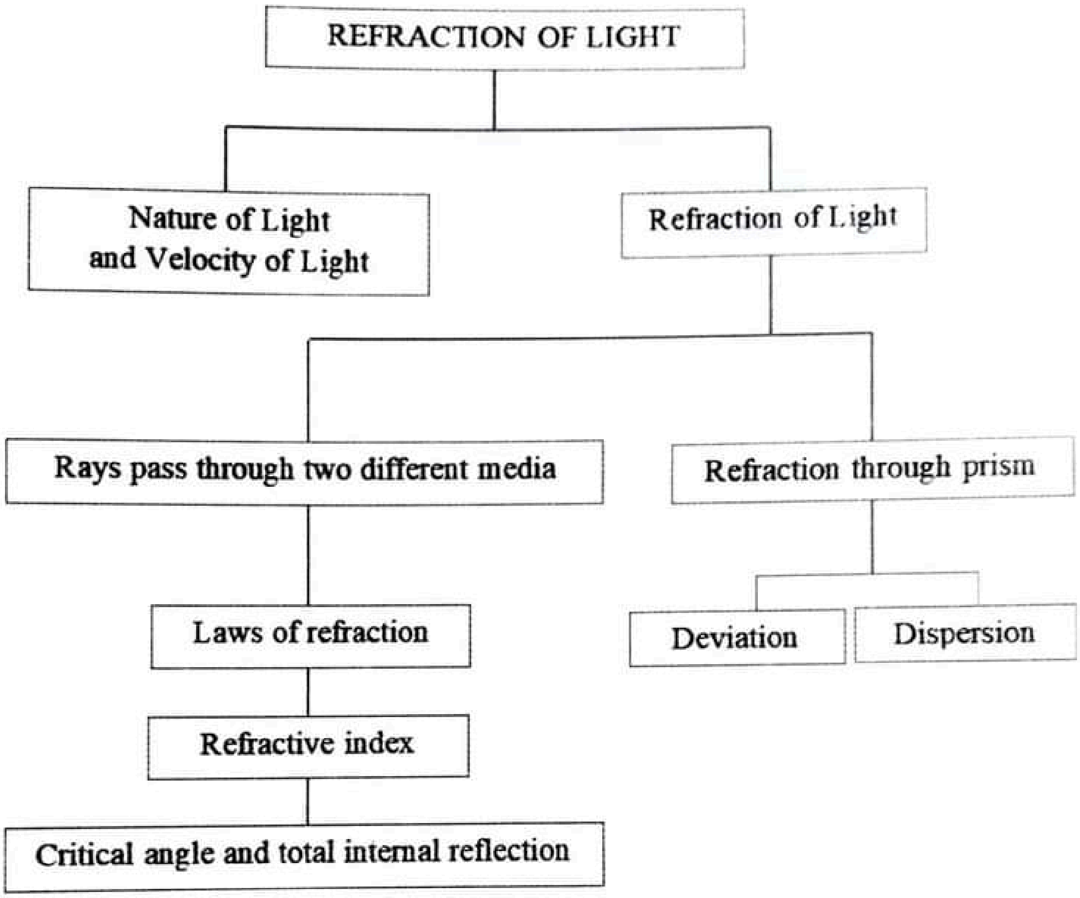
7. A ray of light in air is incident on the surface of a glass slab 4 cm thick at an angle of 60°. It emerges from the slab and travels into the air from the other side of the glass slab. If the refractive index of glass is 1.5, find the lateral displacement between the incident ray and the emergent ray.

8. A cube of ice of refractive index 1.31 is placed on a glass slab of refractive index 1.6. If a ray of light passing from the glass slab to the ice has an angle of incidence of 35°, will the ray enter the ice?

9. (i) The angle of a glass prism is 60° and the angle of minimum deviation is 39°. Find the refractive index of glass. (ii) If the refractive index of glass is 1.66 and the angle of prism is 60° find the angle of minimum deviation.

10. What is the critical angle at Lucite-glass interface? The refractive index of Lucite is 1.49 and that of glass is 1.66. To be totally reflected, which medium the light must start from?

CONCEPT MAP


REFRACTION OF LIGHT
Nature of Light and Velocity of Light
Refraction of Light
Rays pass through two different media
Refraction through prism
Laws of refraction
Deviation
Dispersion
Refractive index
Critical angle and total internal reflection

CHAPTER 8
LENSES

Lenses produce images similar to images formed by curved mirrors, but they do so by refracting light rather than reflecting it.

Learning Outcomes

It is expected that students will

- describe the refraction of light at a curved surface.
- examine the refraction of light through lenses and apply the lens equation.
- examine the power and the magnification of lenses.

As a preliminary study for refraction through lenses, we first look at refraction at a single spherical surface.

8.1 REFRACTION AT A CURVED SURFACE

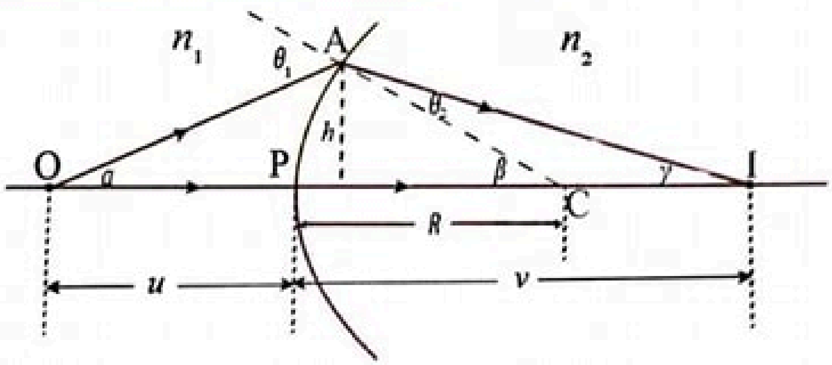


Figure 8.1 Refraction at a spherical surface

Consider a spherical surface of radius R separating the two media 1 and 2 as shown in Figure (8.1). The two rays from point object O are incident on the curved surface. The ray incident at A will be refracted at the surface and meet the ray propagating along the axis at point I. The light ray along the axis is incident on the surface normally and hence is not bent. The point object O thus has its image at I. If the rays are paraxial, then the angles α, β, γ, θ_1 and θ_2 are very small.

From Snell's law,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

If an angle is small and expressed in radian,

$$\sin\theta_1 = \theta_1 \quad \text{and} \quad \sin\theta_2 = \theta_2$$

$$n_1\theta_1 = n_2\theta_2$$

In triangle OAC, $\theta_1 = \alpha + \beta$ and in triangle IAC, $\beta = \theta_2 + \gamma$.

But $\beta = h/R$, $\alpha = h/u$ and $\gamma = h/v$. Thus,

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R} \qquad (8.1)$$

If the first medium is air, $n_1 = 1$, $n_2 = n$ $$\frac{1}{u} + \frac{n}{v} = \frac{n-1}{R} \quad (8.2)$$

Sign Convention for Radius of Curvature of a Spherical Surface
For refraction at spherical surface, the radius R is positive if the surface is convex toward the object, whereas R is negative if the surface is concave toward the object.

Example 8.1 An air bubble is at the centre of a glass sphere of radius 6 cm and refractive index 1.5. Find the distance of the image when viewed from outside.

$n_1 = 1.5$, $n_2 = 1$, $R = -6$ cm, $u = 6$ cm

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\frac{1.5}{6} + \frac{1}{v} = \frac{1-1.5}{-6}$$

$$v = -6 \text{ cm}$$

The distance of virtual image is 6 cm.

Reviewed Exercise

- State the sign convention for radius of curvature for a refraction of light at spherical surface. Illustrate your answer with a diagram.

Key Words: spherical surface, convex, concave

8.2 THE LENS EQUATION

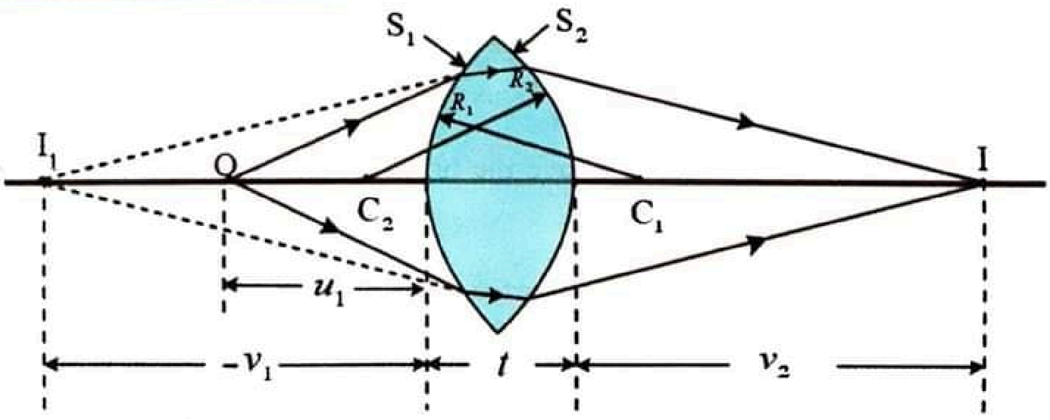


Figure 8.2 Path of light rays from a point object at O through a lens to the image at I

For the first refraction at surface S_1, $$\frac{n_1}{u_1} + \frac{n_2}{v_1} = \frac{n_2 - n_1}{R_1} \quad \text{(i)}$$

For the second refraction at surface S_2, $$\frac{n_2}{u_2} + \frac{n_1}{v_2} = \frac{n_2 - n_1}{R_2} \quad \text{(ii)}$$

The image I_1 formed at the first surface acts as the object for the second surface of the lens. Hence, $u_2 = -v_1 + t$, where t is the thickness of the lens. Negative sign is inserted because I_1 is the virtual

image for surface S_1 and real object for surface S_2.
Since the thickness of the lens t is small compared to object and image distances, $u_2 = -v_1$.
Using $u_2 = -v_1$ in Eq (ii) and with Eq (i), we obtain

$$\frac{n_1}{u_1} + \frac{n_1}{v_2} = (n_2 - n_1)(\frac{1}{R_1} - \frac{1}{R_2}) \qquad \text{(iii)}$$

Considering the lens as a single entity, the object distance for the lens is $u = u_1$ and the image distance is $v = v_2$. If medium 1 is air, $n_1 = 1$ and $n_2 = n$, Eq. (iii) becomes

$$\frac{1}{u} + \frac{1}{v} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2}) \qquad (8.3)$$

We define focal length, f of a lens as the image when the object is at infinity. Hence, subsisting $u = \infty$ and $v = f$ in Eq. (8.3) gives the following lens-makers' equation.

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2}) \qquad (8.4)$$

Combining Eq. (8.3) and Eq. (8.4), we obtain the lens equation.

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad (8.5)$$

This is general formula for the thin lens. The object distance u and the image distance v are measured from the centre of the lens P. The position and nature of image formed by the lens can be calculated by using lens formula.

The Sign Convention for the Lens

The images formed by the lenses may either be real (or) virtual, and erect (or) inverted. Hence the following sign conventions are required in applying lens formulae to solve the problems.

(i) Distances of real object, real image and real focus are positive. Distances of virtual object, virtual image and virtual focus are negative.
(ii) The perpendicular distance measured above the principal axis is positive and that below the principal axis is negative.

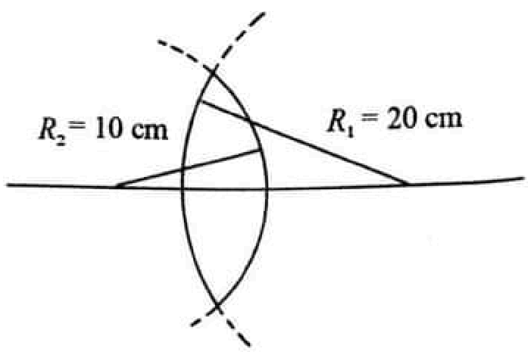
Example 8.2 The radii of curvature of two curve surfaces are 20 cm and 10 cm, in given figure. Find the focal length of a lens whose refractive index is 1.5.

$n = 1.5$, $R_1 = 20$ cm, $R_2 = -10$ cm

By using lens-makers' equation, $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$\frac{1}{f} = (1.5-1)\left[\frac{1}{20} - \frac{1}{-10}\right]$$

$$f = 13.33 \text{ cm}$$



Reviewed Exercise

- Write the lens-makers' equation for a lens which is immersed in water.

Key Words: radius of curvature, focal length, lens-makers' equation

8.3 REFRACTION THROUGH LENSES

A transparent material which can diverge or converge rays of light is called a lens. A lens has at least one curved surface. Generally, lenses are made up of glass with two spherical surfaces.

There are two main types of lenses, known as convex (or) converging lens and concave (or) diverging lens.

Lenses have different shapes and are very useful objects. They are used in spectacles, cameras, projectors, telescopes and microscopes. The convex lens (or) converging lens is used as a magnifying glass. The lenses in spectacles used by a short-sighted person are concave lenses. The lenses in spectacles used by a long-sighted person are convex lenses.

Convex Lens

A lens which can converge the parallel rays is called a convex lens. It is thicker in the middle than at the edges.

Three types of convex lens are bi-convex, plano-convex and converging meniscus (Figure 8.3).

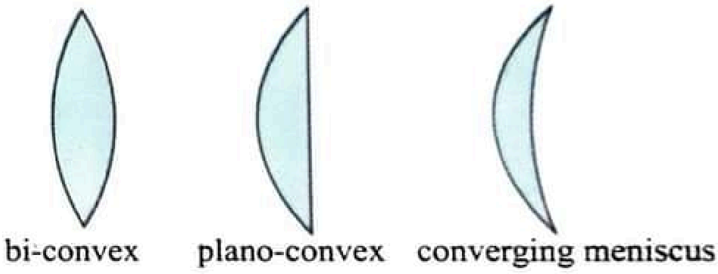


Figure 8.3 Convex (converging) lenses

Concave Lenses

A lens which can diverge the parallel rays is called concave lens. It is thinner in the middle than at the edges.

Three types of concave lens are bi-concave lens, plano-concave lens and diverging meniscus (Figure 8.4).

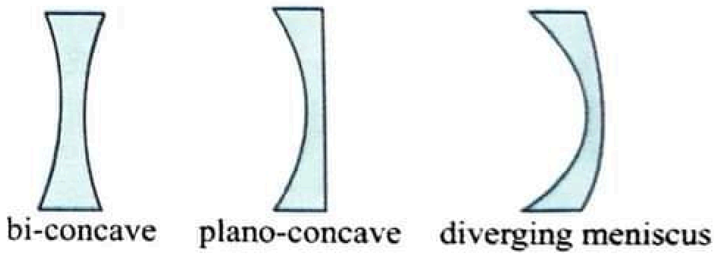


Figure 8.4 Concave (diverging) lenses

Bi-convex and bi-concave lens are widely used. For simplicity, a bi-convex lens will be called a convex lens and a bi-concave will be a concave lens.

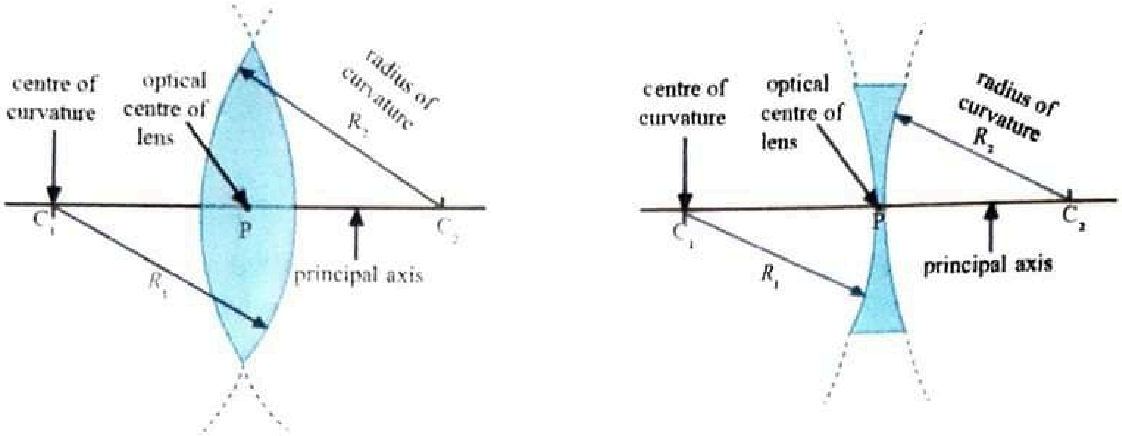


Figure 8.5 Terms in connection with lenses

Each surface of a lens has a centre of curvature. Since a lens has two surfaces it has two centres of curvature.

Principal Axis of a Lens

The line joining the centres of curvature of two surfaces is the principal axis of a lens. It passes through the middle of the lens.

Symmetric Lens

If the radii of curvature of the two surfaces of a lens are equal, the lens is said to be symmetric. (i.e. $R_1 = R_2$)

Optical Centre of a Lens

A point in the middle of a symmetric lens on the principle axis is the centre of a lens (or) the optical centre of a lens.

Principal Focus and Focal Length

The rays parallel to the principal axis converge at a point on the principal axis after refraction through a convex lens. This point is called the principal focus of the convex lens and is denoted by F. Since these rays actually pass through the focus, the focus of the convex lens is real (Figure 8.6).

The distance between centre of a convex lens and its focus is called focal length *f* of the convex lens.

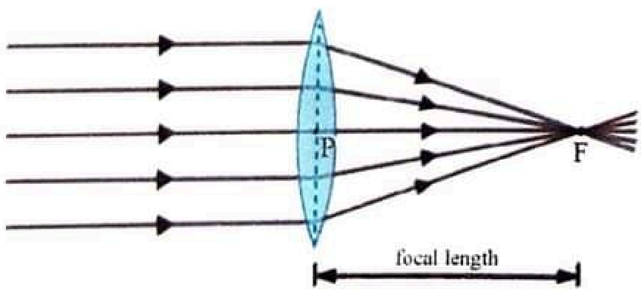


Figure 8.6 Principal focus and focal length of a convex lens

The rays parallel to the principal axis are divergent after refraction through a concave lens. Those divergent rays appear to come from a point on the principal axis. This point is called the principal focus of the concave lens. Since the divergent rays do not actually pass through that point, the focus of the concave lens is virtual as shown in Figure 8.7.

The distance between centre of a concave lens and its focus is called the focal length of the concave lens.

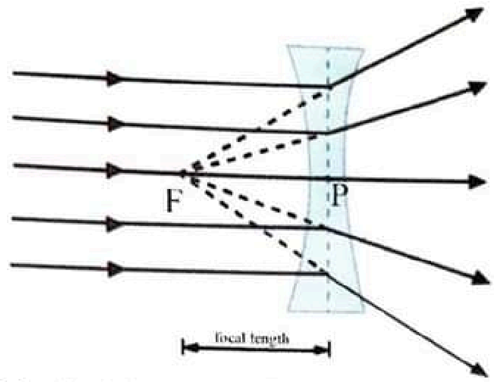


Figure 8.7 Principal focus and focal length of a concave lens

The rays parallel to the principal axis enter the convex lens from the left and pass through the focus on the right [Figure 8.8 (a)]. If the rays parallel to the principal axis enter the convex lens from the right, they will pass through the focus on the left [Figure 8.8 (b)]. Thus, a lens has two foci.

Like a convex lens, a concave lens has two focal points (foci). The rays parallel to the principal axis enter the concave lens from the left, the focus is also on the left [Figure 8.8(c)]. If the rays parallel to the principal axis enter the concave lens from the right, the focus is on the right [Figure 8.8(d)].

The points at a distance of twice the focal length from the centre of lens are represented by 2F. These points are very important for the lens.

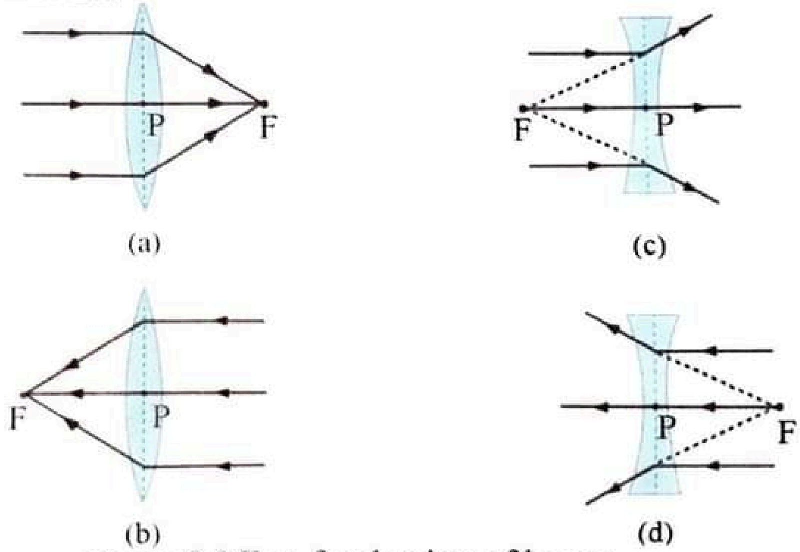


Figure 8.8 Two focal points of lenses

Principal Rays for Lenses

The images formed by lenses can be studied by means of ray diagram which can be drawn using the principal rays stated below.

(i) A ray parallel to the principal axis passes through the focus after refraction through a convex lens. A ray parallel to the principal axis is refracted through a concave lens and the refracted ray produced backward passes through the focus F [Figure 8.9 (a)].

(ii) A ray passing through the focus of a convex lens emerges parallel to the principal axis after refraction by the lens. A ray on one side of a concave lens directed towards the focus on the other side, emerges parallel to the principal axis after refraction through the lens [Figure 8.9 (b)].

(iii) A ray passing through the centre of lens emerges in the same direction [Figure 8.9 (c)].

Only two of the above rays are sufficient to locate the image of an object in various positions.

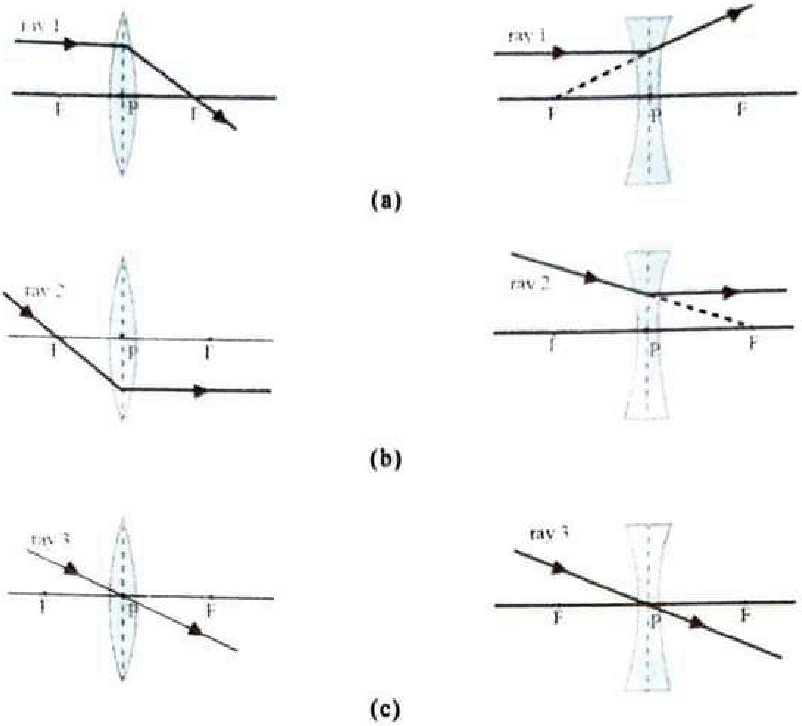


Figure 8.9 Principal rays

Formation of Images by Lenses

The formations of images in a convex lens by drawing ray diagrams are shown below. In these diagrams we will assume that an object OO′ is placed upright on the principal axis.

In Figure 8.10 (a) the object is at infinity and its image is

1. at F,
2. real,
3. inverted, and
4. smaller than the object.

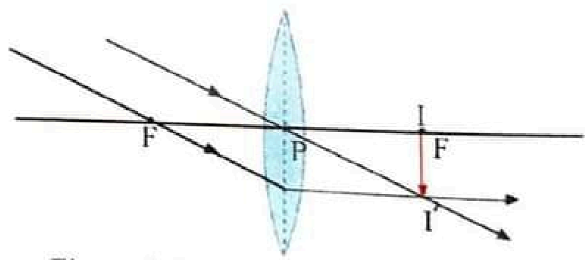


Figure 8.10 (a) The object is at infinity

Due to this property, convex lens can be used as object lens of a telescope.

In Figure 8.10(b) the object is beyond 2F and its image is

1. between F and 2F,
2. real,
3. inverted, and
4. smaller than the object.

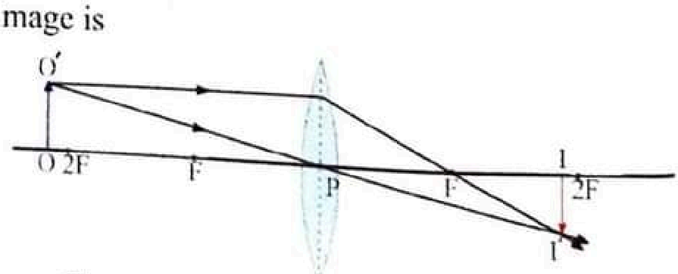


Figure 8.10 (b) The object is beyond 2F

Due to this property, convex lens can be used in a camera.

In Figure 8.10 (c) the object is at 2F and its image is
1. at 2F,
2. real,
3. inverted, and
4. of the same size as the object.

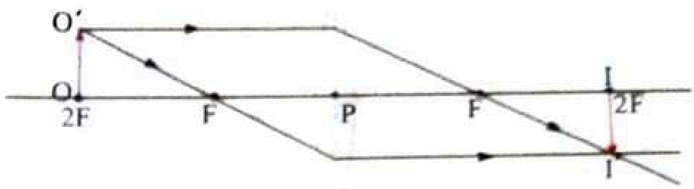


Figure 8.10 (c) The object is at 2F

Due to this property, convex lens can be used in a photocopier making equal-sized copy.

In Figure 8.10 (d) the object is between F and 2F and its image is
1. beyond 2F,
2. real,
3. inverted, and
4. larger than the object.

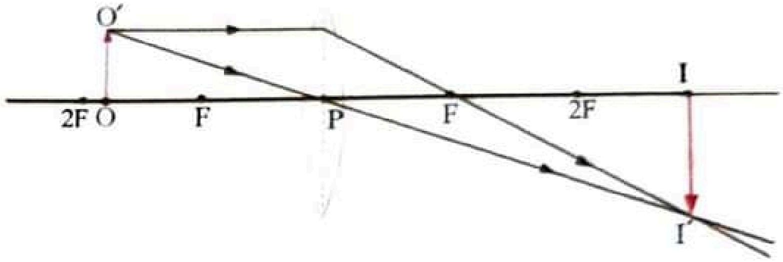


Figure 8.10 (d) The object is between F and 2F

Due to this property, convex lens can be used in a projector .

In Figure 8.10 (e) the object is at F and its image is at infinity.

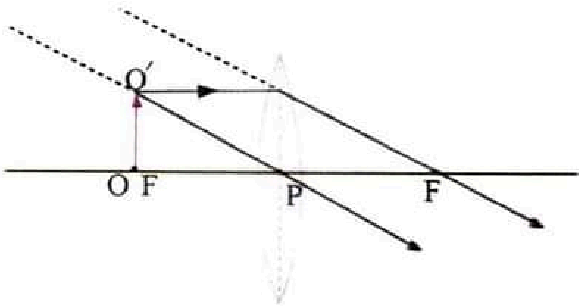


Figure 8.10 (e) The object is at F

Due to this property, convex lens can be used as in a spotlight to produce parallel beam.

In Figure 8.10(f) the object is between F and P and its image is
1. behind the object,
2. virtual,
3. erect (upright), and
4. larger than the object.

Due to this property a convex lens can be used as a magnifying glass.

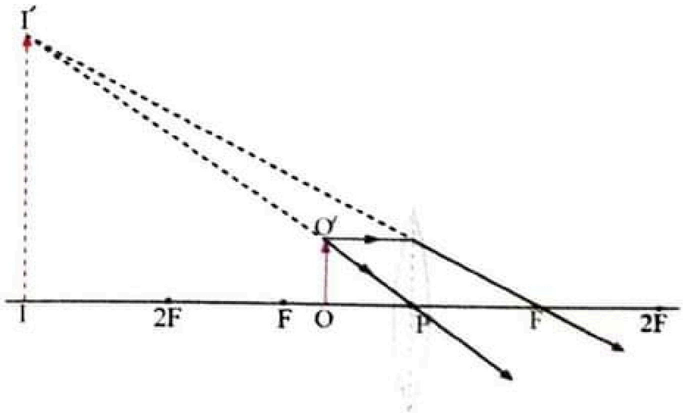


Figure 8.10 (f) The object is between F and P

The image formed by a concave lens is always virtual, erect (upright) and smaller than the object. It is formed between P and F on the same side of the lens as the object. Figure 8.11 shows the image formed by the concave lens.

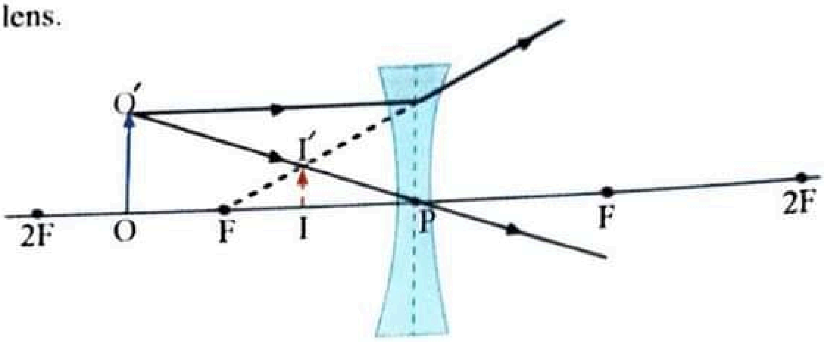


Figure 8.11 Image formed by a concave lens

Convex lens can produce both real and virtual images depending on the position of the object. However, a concave lens can produce only virtual image. The virtual image formed by the convex (or) concave lens is the same size as the object only when the object is in contact with the lens.

Magnification

The magnification is the ratio of the height of the image to the height of the object. It is usually denoted by m.

If OO' is the height or the size of the object and II' is the height or the size of the image, then

$$m = \frac{II'}{OO'}$$

The magnification can be expressed as,

$$m = \frac{II'}{OO'} = -\frac{v}{u} \qquad (8.6)$$

v = image distance

u = object distance

Example 8.3 (i) An object is placed 30 cm from a convex lens of focal length 10 cm. Find the position of its image and the magnification. (ii) An object is placed 30 cm from a concave lens of focal length 10 cm. Find the position of its image and the magnification.

(i) $f = +10$ cm, $u = +30$ cm

Using lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{+30} + \frac{1}{v} = \frac{1}{+10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30}$$

$$v = 15 \text{ cm}$$

Hence, the image is real and 15 cm from the lens.

$$m = -\frac{v}{u} = -\frac{15}{30} = -0.5$$

Since m has minus sign, the image is inverted.

(ii) $f = -10$ cm $\quad u = +30$ cm

Using lens formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{+30}+\frac{1}{v}=\frac{1}{-10}$$

$$\frac{1}{v}=-\frac{1}{10}-\frac{1}{30}$$

$$v = -7.5 \text{ cm}$$

The image, therefore, is virtual and 7.5 cm from the lens.

$$m=-\frac{v}{u}=-\frac{(-7.5)}{30} = 0.25$$

Since m has a plus sign, the image is erect.

Reviewed Exercise

1. What are the differences between real and virtual images formed by a convex lens?
2. Draw any two ray diagrams to show how real and virtual images can be formed by a convex lens.

Key Words: convex lens, concave lens, magnification, erect image, inverted image

8.4 POWER OF A LENS

The power of a lens is inversely proportional to the focal length of the lens. It is denoted by the letter P. The lens having greater power can make the rays parallel to the principal axis more convergent (or) divergent.

The shorter the focal length the greater is the power of the lens (Figure 8.12).

Since the focal length of a convex lens is positive in sign it has a positive power. The focal length of a concave lens is negative so that it has a negative power. The sign of the powers of the lenses used here are the same as those used by the lens makers.

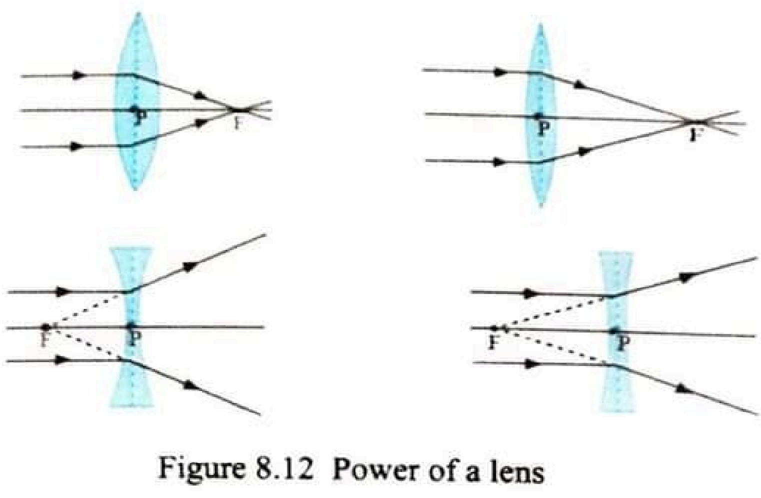


Figure 8.12 Power of a lens

Unit of Power of a Lens (dioptre)

If a lens has a focal length of 1 metre, it has one unit power (or) one dioptre. It is denoted by D.
If the focal length f is measured in metre,

$$P = \frac{1}{f(\text{m})} \tag{8.7}$$

If the focal length f is measured in centimetre,

$$P = \frac{100}{f(\text{cm})}$$

For example, if the power of a lens is + 2 D, it is a convex lens and its focal length is 0.5 m (or) 50 cm. If the power of a lens is − 4 D, it is a concave lens and its focal length is 0.25 m (or) 25 cm.

Example 8.4 An object is 30 cm from a lens and its image is formed 10 cm on the same side as the object from the lens. (i) Find the type of the lens and its focal length. (ii) Find the power of the lens.

(i) Since the image is formed on the same side as the object, it is a virtual image.
In addition, $u = 30$ cm and $v = 10$ cm so that the image is between the object and the lens. Thus the lens is a concave lens.
$u = +30$ cm, $v = -10$ cm (virtual image)

Using lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{+30} + \frac{1}{-10} = \frac{1}{f}$$

$$\frac{1}{f} = -\frac{1}{10} + \frac{1}{30}$$

$$f = -15 \text{ cm}$$

The focal length of the concave lens is 15 cm.

(ii) The power of lens

$$P = \frac{100}{f(\text{cm})}$$

$$P = \frac{100}{-15} = -6.67 \text{ D}$$

Example 8.5 An image, which is five times the size of an object, is to be produced by a convex lens of power + 2D on the same side as the object. How far should the object be placed from the lens?

$P = +2$ D, $II' = +5 \times OO'$ (or) $\frac{II'}{OO'} = +5$ (or) $m = +5$

The power of a lens,

$$P = \frac{100}{f(\text{cm})}$$

$$+2 = \frac{100}{f}$$

Therefore,

$$f = 50 \text{ cm}$$

The magnification

$$m = -\frac{v}{u}$$

$$+5 = -\frac{v}{u}$$

Therefore,

$$v = -5u$$

Using lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{u} + \frac{1}{-5u} = \frac{1}{50}$$

$$u = 40 \text{ cm}$$

The object is placed 40 cm from the lens.

Example 8.6 An image which is ten times the size of an object is formed on the wall by a convex lens of focal length 10 cm. (i) How far is the object from the lens? (ii) How far is the wall from the lens?

$f = +10$ cm

Since the image is real and inverted $II' = -10 \times OO'$ (or) $\frac{II'}{OO'} = -10$ (or) $m = -10$

(i) The magnification

$$m = -\frac{v}{u}$$

$$-10 = -\frac{v}{u}$$

$$v = 10u$$

Using lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{u} + \frac{1}{10u} = \frac{1}{+10}$$

$$u = 11 \text{ cm}$$

The object is placed 11 cm from the convex lens.

(ii) $v = 10u = 10 \times 11 = 110$ cm

The wall is 110 cm from the convex lens.

Reviewed Exercise

1. The human eye has a lens of focal length + 5 cm. Find the power of the eye.
2. An image is formed by a convex lens on a screen which is 60 cm from the lens. If the height of image is one-fourth the height of the object, what is the power of the lens?

Key Words: power of a lens, dioptre

SUMMARY

A transparent material which can converge (or) diverge rays of light is calld **a lens**. A lens has at least one curved surface.

The distance between centre of a convex lens and its focus is called the **focal length of the convex lens.**

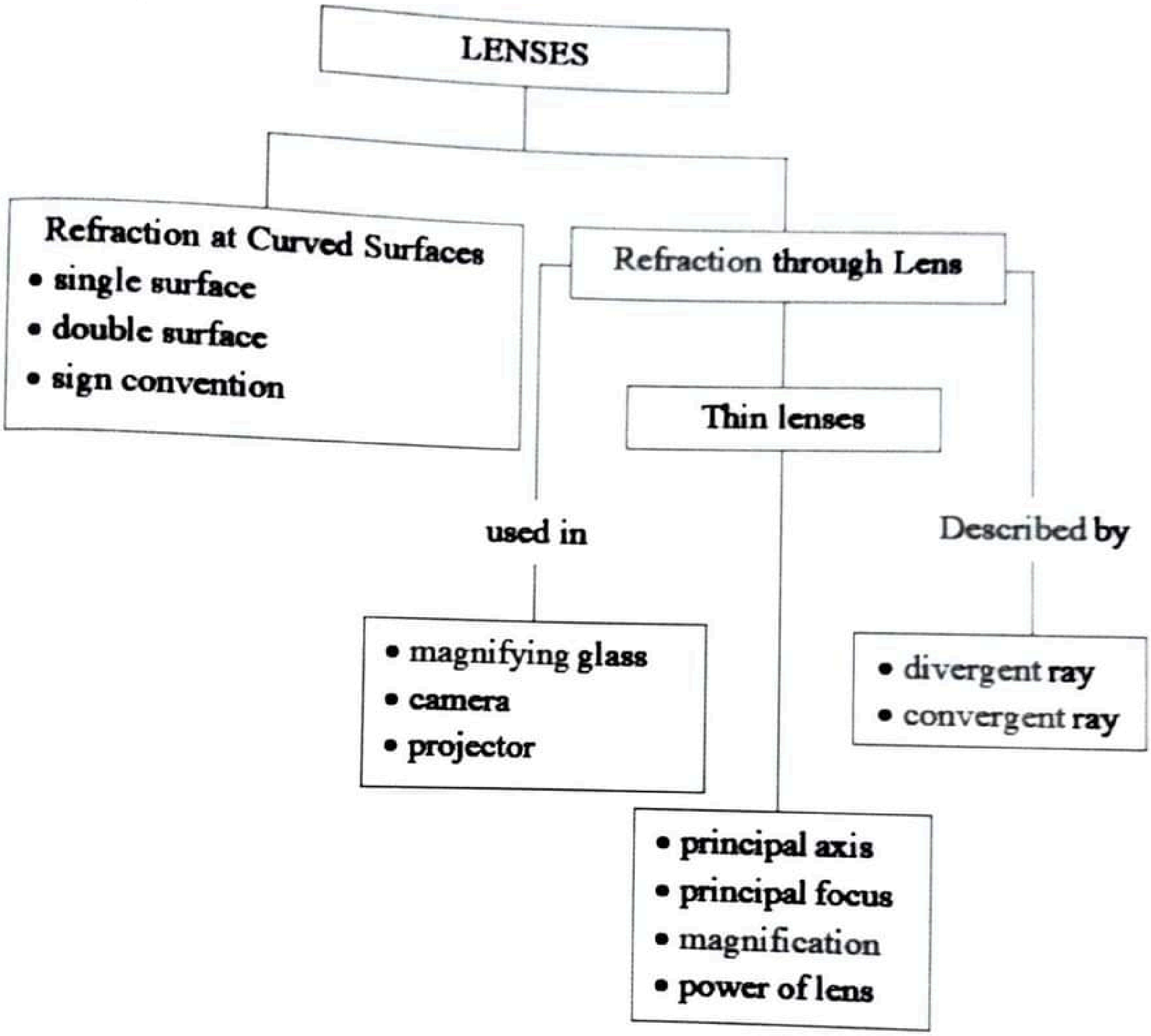
The distance between centre of a concave lens and its focus is called the **focal length of the concave lens.**

The power of lens is inversely proportional to the focal length of the lens .

EXERCISES

1. (a) What is a lens? State some of its uses.
   (b) What do you understand by focus of a convex lens and focus of a concave lens?
2. Choose the correct answer from the following.
   When a pencil 10 cm long is placed vertically 100 cm from a lens of focal length + 50 cm, the image is
   A. erect and 5 cm tall. B. inverted and 5 cm tall.
   C. erect and 10 cm tall. D. inverted and 10 cm tall.
3. Choose the correct answer from the following.
   An object is 10 cm from a lens. The image of the object is formed on the same side as the object. If the image is 10 cm from the object, the focal length of the lens is
   A. + 6.7 cm. B. − 6.7 cm.
   C. + 20 cm. D. − 20 cm.
4. State the sign convention for lenses.
5. (a) State the properties of an image formed by a concave lens.
   (b) How far must the object be placed from a concave lens of focal length 10 cm to obtain an image 4 cm from the lens? Draw a ray diagram to show formation of the image.
6. An object 3 cm tall is 30 cm from a convex lens of focal length 20 cm. (i) Find the size of the image and the image distance. (ii) If the object is moved 5 cm closer to the lens how far does the image move?
7. The virtual image of an object is formed 24 cm from a lens of focal length 8 cm. (i) Find the distance between the object and the lens. (ii) How far must the object be placed from the lens to obtain a real image of the same size as the virtual image obtained previously?
8. A magnifying glass of focal length 9 cm is used to produce an image which is three times the size of an object. How far must the magnifying glass be placed from the object?
9. When an object is placed 12 cm from a convex lens a real image formed is three times the size of the object. If a real image which is four times the size of the object is required, how far must the object be moved?
10. An object is placed 18 cm from a screen. Where must a lens of focal length 4 cm be placed between the screen and the object to produce an image on the screen?
11. An object is placed 60 cm in front of a screen. Is it possible to obtain a sharp image larger than the size of an object on the screen by placing a convex lens of focal length 15 cm somewhere between the screen and the object? Answer this by doing the necessary calculations. What changes can occur when the object and the screen are interchanged?
12. An object 1.05 cm tall is 80 cm away from the screen and the size of its image on the screen is 0.35 cm. Find the position and focal length of the lens.
13. Determine the nature of the images formed by the lenses for the magnifications given below:
   (i) magnification is between − 1 and 0,
   (ii) magnification is between 0 and +1 and
   (iii) magnification is greater than + 1.

CONCEPT MAP


LENSES
Refraction at Curved Surfaces
• single surface
• double surface
• sign convention
Refraction through Lens
Thin lenses
used in
Described by
• magnifying glass
• camera
• projector
• divergent ray
• convergent ray
• principal axis
• principal focus
• magnification
• power of lens

CHAPTER 9

ELECTRIC FIELD

Two electric charges, which are not in contact, exert electrical forces on each other. The concept of electric field is used to explain this phenomenon.

Learning Outcomes

It is expected that students will

- identify and apply Coulomb's law.
- investigate electric fields, electric field intensity, electric lines of force, electric potential and potential difference.
- examine electric potential of the earth, the potential between two parallel charged plates, electric charge distribution and equipotential surfaces.

Just as there is a gravitational force between two masses, there is an electric force between two charges. Electrical forces bind electrons and nuclei to form atoms. In addition, these forces hold atoms to form molecules, liquids and solids.

The gravitational forces are appreciable only when the masses of the bodies are very large. However, the electrical forces are so much greater in magnitude than gravitational forces.

9.1 COULOMB'S LAW

The French scientist Charles-Augustin de Coulomb, studied systematically the attractive and repulsive forces acting between pairs of charges and discovered a law, known as Coulomb's law.

Coulomb's law states that:

The electric force between two charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

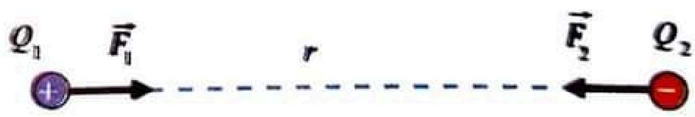


Figure 9.1 Two charges separated by a distance ($|\vec{F}_1| = |\vec{F}_2|$)

In Figure 9.1, Q_1 and Q_2 are electric charges and r is the distance between them. If F is the electric force between Q_1 and Q_2, Coulomb's law can be expressed as

$$F \propto \frac{Q_1 Q_2}{r^2}$$

In equation

$$F = K\frac{Q_1 Q_2}{r^2} \tag{9.1}$$

K is a constant. The value of K depends upon the units of F, Q_1, Q_2 and r and upon the medium in which the charges Q_1 and Q_2 are located.

In the SI system, charge Q is measured in coulomb, the distance r in metre and the force F in newton, then

$$K = \frac{1}{4\pi\varepsilon}$$

ε is a constant called the permittivity of the medium in which the charges are located.

When charges are located in vacuum, $K = \frac{1}{4\pi\varepsilon_0}$

ε_0 is the permittivity of vacuum, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

The value of K in vacuum is,

$$K = \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 8.987\ 42 \times 10^9\ N\ m^2\ C^{-2}$$

For convenience in calculation, the value of K in vacuum will be taken as $K = 9 \times 10^9\ N\ m^2\ C^{-2}$.
The value of K in air is approximately equal to that of K in vacuum.

Electric force is, of course, a vector quantity. The direction of electric force is always along the line joining the two charges.

If the charges are like charges, the force between them is repulsive and is directed outwards. If the charges are unlike charges, the electric force between them is attractive and directed inwards (Figure 9.2).

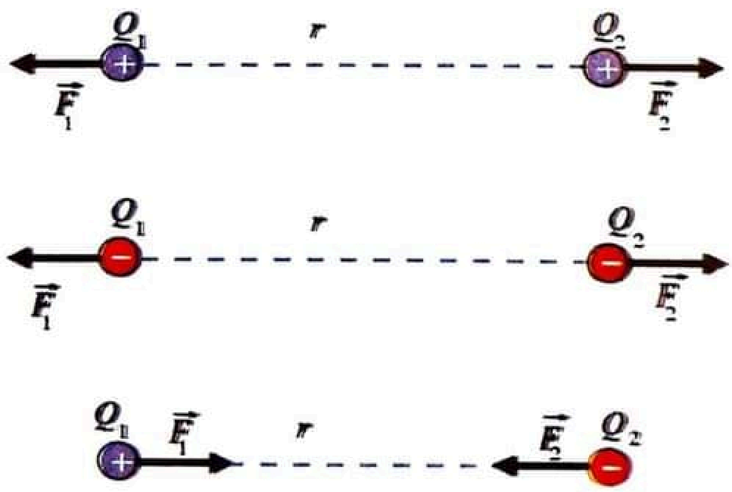


Figure 9.2 Direction of force between two charges

Coulomb's law equation in vector form is,

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}\,\hat{r} \quad (9.2)$$

$\hat{r}$ is the unit vector, its direction is always along the line joining between the two charges and outwards.

Since the electric force is inversely proportional to the square of the distance between the two charges $F \propto \frac{1}{r^2}$, Coulomb's law is also called an inverse square law.

A point charge is a charge without dimension (or) with dimension so much smaller than other dimensions appearing in the problem.

Example 9.1 Find the electric force between two charges of 1 C each that are 1 m apart.

$Q_1 = 1\ \text{C}, \quad Q_2 = 1\ \text{C}, \quad r = 1\ \text{m}$

By Coulomb's law,
$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$$
$$F = 9 \times 10^9\ \frac{1\times 1}{1} = 9 \times 10^9\ \text{N}$$

The charge 1 coulomb is too large in magnitude so microcoulomb is introduced in static electricity.

Example 9.2 (i) Calculate the values of two equal charges if they repel one another with a force of 0.1 N when situated 50 cm apart in vacuum. (ii) Calculate the values of two equal charges if they repel one another with a force of 0.1 N when situated 50 cm apart in a liquid whose permittivity is 10 times that of vacuum.

(i) $r = 50\ \text{cm} = 0.5\ \text{m}, \ F = 0.1\ \text{N}$

By Coulomb's law,
$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$$

Since they are equal charges, $Q_1 = Q_2 = Q$

Therefore,
$$0.1 = 9\times 10^9 \frac{Q^2}{(0.5)^2}$$
$$Q = 1.67 \times 10^{-6}\ \text{C} = 1.67\ \mu\text{C}$$

(ii) The permittivity of the liquid medium $\varepsilon = 10\ \varepsilon_0$
$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1Q_2}{r^2}$$
$$F = \frac{1}{4\pi(10\varepsilon_0)}\frac{Q^2}{r^2}$$
$$0.1 = \frac{9\times 10^9}{10}\frac{Q^2}{(0.5)^2}$$
$$Q = 5.27 \times 10^{-6}\ \text{C} = 5.27\ \mu\text{C}$$

Example 9.3 If the electric force acting on a charge Q, 6 cm from a charge of $+50\times10^{-8}$ C is 0.. find the magnitude of Q.

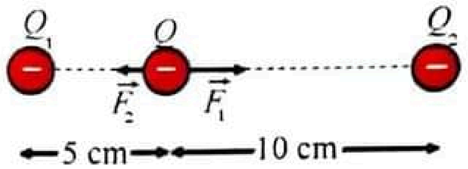
$Q_1 = +50\times10^{-8}$ C, $Q_2 = Q$, r = 6 cm = 0.06 m, F = 0.24 N

By Coulomb's law, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1Q_2}{r^2}$

$$0.24 = 9\times10^9\ \frac{50\times10^{-8}Q}{(0.06)^2}$$

$$Q = \frac{0.24\times(0.06)^2}{9\times10^9\times50\times10^{-8}} = 1.92\times10^{-7}\ \text{C}$$

Example 9.4 A test charge of -5×10^{-5} C is placed between two other charges so that it is 5 cm from a charge of -3×10^{-5} C and 10 cm from a charge of -6×10^{-5} C. If the three charges lie on a straight line, find the magnitude and, the direction of the electric force on the test charge.



$Q = -5\times10^{-5}$ C, $Q_1 = -3\times10^{-5}$ C, $Q_2 = -6\times10^{-5}$ C, $r_1 = 0.05$ m, $r_2 = 0.1$ m

By Coulomb's law,

The magnitude of force on Q exerted by Q_1

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q}{r_1^2}$$

$$F_1 = 9\times10^9\ \frac{(3\times10^{-5})(5\times10^{-5})}{(0.05)^2} = 5400\ \text{N}$$

$\vec{F_1}$ is directed towards Q_2.

The magnitude of force on Q exerted by Q_2,

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q}{r_2^2}$$

$$F_2 = 9\times10^9\ \frac{(6\times10^{-5})(5\times10^{-5})}{(0.1)^2} = 2700\ \text{N}$$

$\vec{F_2}$ is directed towards Q_1.

The resultant electric force acting on Q is, $\vec{F} = \vec{F_1} + \vec{F_2}$

Since $\vec{F_1}$ and $\vec{F_2}$ are in opposite direction, the magnitude of $\vec{F}$ is

$$F = F_1 - F_2 = 5400 - 2700 = 2700\ \text{N}$$

$\vec{F}$ is directed towards Q_2.

Reviewed Exercise

- If distance between two charges is doubled, how much does the electric force change?

Key Words: electric charge, electric force, repulsive force, attractive force, permittivity

9.2 ELECTRIC FIELD AND ELECTRIC FIELD INTENSITY

The concept of electric field can explain by the electric forces acting between charges.

An electric field can be defined as a region where electrical forces act. Any electric charge gives rise to an electric field in its vicinity.

When a small charge q is brought near to a charge Q, it is found that there is an electric force acting upon it. Hence, the electric force on q is due to the electric field of Q.

It is noticed that q also produces an electric field around in its vicinity.

> In order to test whether an electric field exists at a certain point, a test charge must be placed at that point. If an electric force is exerted on the test charge, then we can say that an electric field exists at that point.

The Electric Field Intensity

When an electric charge is placed in an electric field, an electric force is exerted on it. It is necessary to know the electric field intensity in order to specify an electric field. The electric field intensity is defined as follows.

The electric field intensity at a point in an electric field is the electric force acting upon a unit positive charge placed at that point.

The electric field intensity is a vector quantity.

If the electric force on q is $\vec{F}$, the force exerted on a unit positive charge is $\frac{\vec{F}}{q}$. The electric field intensity can be expressed as,

$$\vec{E} = \frac{\vec{F}}{q} \qquad (9.3)$$

In the SI system, the unit of electric field intensity is newton per coulomb ($N\ C^{-1}$).

> The direction of $\vec{E}$ is the same as that of $\vec{F}$ acting upon the positive charge but is opposite to that of $\vec{F}$ acting upon the negative charge.

Calculation of the Electric Field Intensity from Coulomb's Law

The electric field intensity at a point, a certain distance from the charge, can be calculated by using Coulomb's law.

Figure 9.3 Force on a charge in an electric field

Suppose that a small positive charge q is placed at point A in the electric field surrounding the charge Q as shown in Figure 9.3.

By Coulomb's law the electric force F on q due to Q is, $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$.

The force on a unit positive charge due to Q is the electric field intensity; $E = \frac{F}{q}$

Therefore,
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad (9.4)$$

Equation (9.4) gives the magnitude of the electric field intensity at point A.

The vector form of the electric field intensity is written as $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$.

The direction of the electric field intensity at the point A is along the line joining q and Q and away from Q.

If a negative charge Q is put in place of the positive charge Q, the magnitude of the electric field intensity at the point A will not change. But its direction will be along the line joining q and Q and towards Q.

If the electric field intensities at a point due to the charges Q_1, Q_2, Q_3, ... are $\vec{E}_1, \vec{E}_2, \vec{E}_3$,.. respectively then the resultant electric field intensity $\vec{E}$ at that point is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... \quad (9.5)$$

Example 9.5 The magnitude of electric field intensity at a point in an electric field is $2\times10^5\,\mathrm{N\,C^{-1}}$. If a charge of magnitude $5\times10^{-6}\,\mathrm{C}$ is placed at that point, find the magnitude of the electric force on that charge.

$E = 2\times10^5\,\mathrm{N\,C^{-1}}$, $q = 5\times10^{-6}\,\mathrm{C}$

The magnitude of the electric force, $F = qE$

$$F = 5\times10^{-6} \times 2\times10^5 = 1\ \mathrm{N}$$

Example 9.6 If the magnitude of the electric field intensity at a point 9 m from a charge $+Q$ is $2\times10^3\ \mathrm{N\,C^{-1}}$ (i) find the magnitude of $+Q$ (ii) find the magnitude of the electric field intensity at a point 18 m from $+Q$.

(i) $E_1 = 2 \times 10^3\ \mathrm{N\,C^{-1}}$, $r_1 = 9\ \mathrm{m}$

The electric field intensity at a point due to Q,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_1^2}$$

$$2\times10^{3} = 9\times10^{9}\ \frac{Q}{9^2}$$

$$Q = 18\times 10^{-6}\,C = 18\ \mu C$$

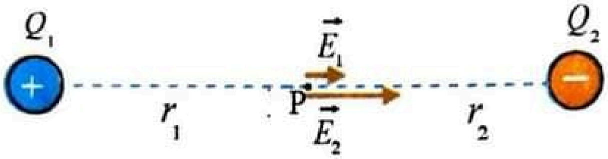
(ii) The magnitude of the electric field intensity at a point 18 m from $+Q$ is E_2, $r_2 = 18$ m

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_2^2}$$

$$= 9\times10^{9}\ \frac{18\times10^{-6}}{(18)^2}$$

$$= 500\ N\,C^{-1}$$

Example 9.7 Two charges of $+2\ \mu C$ and $-5\ \mu C$ are 6 m apart. Find the electric field intensity at the point midway between them.



$Q_1 = +2\ \mu C = 2\times10^{-6}\ C$, $Q_2 = -5\ \mu C = 5\times10^{-6}\ C$, $r_1 = r_2 = \frac{6}{2} = 3$ m

The magnitude of the electric field intensity at P due to Q_1

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2}$$

$$= 9\times10^{9}\ \frac{2\times10^{-6}}{3^2}$$

$$= 2\times10^{3}\ N\,C^{-1}$$

The direction of $\vec{E}_1$ is towards Q_2.

The magnitude of the electric field intensity of P due to Q_2

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2^2}$$

$$= 9\times10^{9}\ \frac{5\times10^{-6}}{3^2}$$

$$= 5\times10^{3}\ N\,C^{-1}$$

The direction of $\vec{E}_2$ is towards Q_2.

The resultant electric field intensity at P, $\vec{E} = \vec{E}_1 + \vec{E}_2$

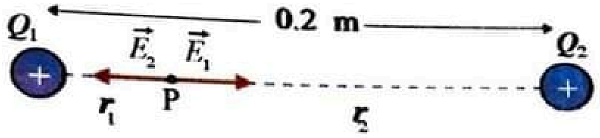
$\vec{E}_1$ and $\vec{E}_2$ are in the same direction.

Therefore, $E = E_1 + E_2 = 2\times10^{3} + 5\times10^{3}$

$E = 7\times10^{3}\ N\,C^{-1}$

The direction of $\vec{E}$ is towards Q_2.

Example 9.8 A charge + 1.5 × 10⁻⁶ C is 0.2 m away from another charge + 6 × 10⁻⁶ C. Where is the electric field in their vicinity equal to zero?



$Q_1 = +1.5 \times 10^{-6}$ C, $Q_2 = +6 \times 10^{-6}$ C, $r = 0.2$ m

Assume P is the point where electric field intensity is zero.

Let the distance of point P be x m from Q_1.

Distance of point P from Q_1, $r_1 = x$ m

Distance of point P from Q_2, $r_2 = (0.2 - x)$ m

The electric field intensity at P due to Q_1 is $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2}$

The electric field intensity at P due to Q_2 is $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2^2}$

Since the electric field intensity at P is zero,

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2^2}$$

$$\frac{1.5\times10^{-6}}{x^2} = \frac{6\times10^{-6}}{(0.2-x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(0.2-x)^2}$$

$$(0.2-x)^2 = 4x^2$$

$$0.2 - x = 2x$$

$$x = 0.067 \text{ m}$$

Reviewed Exercise

1. Why is the electric field intensity a vector?
2. The electric field intensity at 2 cm from a certain charge has a magnitude of 10^5 N C^{-1}. What is the value of the electric field intensity at 1 cm from the charge?

Key Words: electric force, electric field, electric field intensity

9.3 ELECTRIC LINES OF FORCE

The concept of lines of force was introduced by Michael Faraday as an aid in visualizing an electric field. Electric field can be represented by electric lines of force.

Electric lines of force do not really exist. They are only imaginary lines.
An electric line of force is a path along which a small positive charge will move in an electric field.

The electric lines of force in an electrostatic field are continuous lines which start from a positive charge and end on a negative charge.

In Figure 9.4 (a) the electric lines of force around a single positive charge are directed radially outward. They will terminate on negative charges situated at infinity.

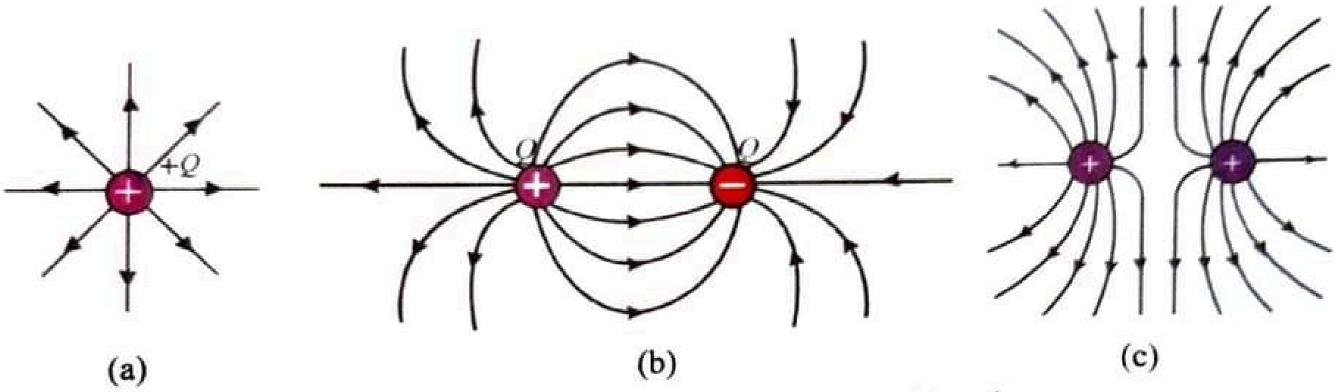


Figure 9.4 Electric lines of force around (a) a single positive charge
(b) two equal unlike charges (c) two equal positive charges

The electric lines of force around two equal charges, one positive and one negative, are shown in Figure 9.4(b). Figure 9.4(c) shows the electric lines of force around two equal positive charges.

The electric lines of force are close together when the electric field intensity is large and far apart when the electric field intensity is small.

In addition, the electric lines of force never intersect. Because the electric field intensity at any point can have only one direction, only one electric line of force can pass through that point.

Example 9.9 A body whose mass is 10^{-6} kg carries a charge $+10^{-6}$ C. If the body is suspended in equilibrium at a point above the ground by an electric field, find the magnitude of the electric field. If the magnitude of electric field is doubled, what is the direction of motion of the body? ($g = 9.8$ m s^{-2})

The gravitational force on the body $w = mg$

If the magnitude of the electric field is E, the electric force acting on the body is $F = qE$.

Since the body is in equilibrium

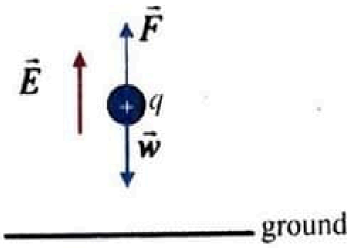
$$w = F$$

$$mg = qE$$

$$E = \frac{mg}{q}$$

$$E = \frac{10^{-6} \times 9.8}{10^{-6}}$$

$$E = 9.8\ \text{N C}^{-1}$$



If the magnitude of electric field is doubled, the electric force is doubled. Since the electric force is greater than weight of the body, the direction of motion of the body is upward.

Reviewed Exercise

1. Why don't the electric lines of force intersect one another?
2. Draw the electric lines of force around a single negative charge.

Key Words: electric lines of force, electric field, electric force

9.4 ELECTRIC CHARGE DISTRIBUTION

When a charge is given to a conducting object of any shape, the charge is found to be spread out over the outer surface of the object. But the charge is not uniformly distributed. The more highly curved parts of the objects have greater concentration of charge than the less curved parts [Figure 9.5 (a),(b),(c)].

Therefore, we can say that charges are highly concentrated at the pointed portion of the object. For a charged pointed rod shown in Figure 9.5 (d) the charge concentration at the pointed end is so large that some of the charges leak off into the air. This makes a pointed rod very useful as a lightning conductor.

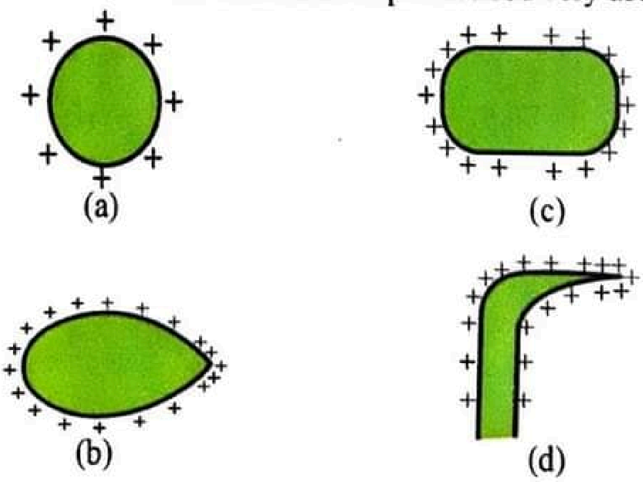


Figure 9.5 Charge distribution on a conductor

The Electric Field inside a Charged Conducting Object

It has been stated that there is an electric field surrounding a charged conducting object. However, the electric fields due to the individual charges on the surface of the charged conducting object all cancel out inside the object. Therefore, the electric field is zero everywhere inside a charged conducting object of any shape.

> The motion of charges or the current is not observed in a charged conducting object, because there is no electric field inside it.

Non-Uniform Electric Field and Uniform Electric Field

If the electric field intensity varies from point to point in an electric field, such an electric field is called a non-uniform electric field.

For example, the electric field around a point charge is a non-uniform electric field. The electric lines of force which represent a non-uniform electric field are not parallel but are continuous curves.

If the electric field intensity at every point is the same in magnitude and direction, such an electric field is called a uniform electric field. For example, the electric field between two oppositely charged parallel plates is a uniform electric field.

As shown in Figure 9.6 a uniform electric field is represented by electric lines of force which are uniformly spaced parallel lines of the same length. The arrows indicate the direction of the electric field.

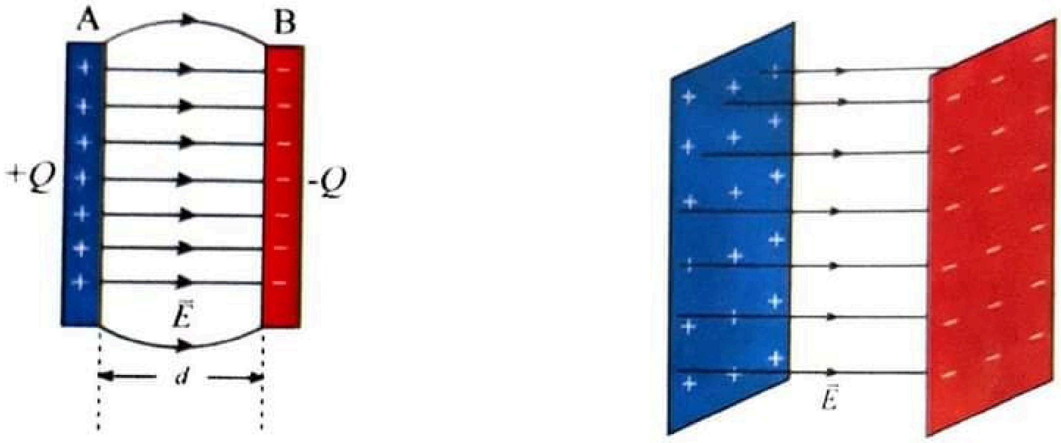


Figure 9.6 Uniform electric field between two parallel charged plates

Example 9.10 An electron of charge 1.6×10^{-19} C is situated in a uniform electric field of intensity 1.2×10^{5} N C^{-1}. (i) Find the force on the electron. (ii) Find the acceleration of the electron. (iii) How long does the electron take to travel a distance 20 mm from rest? (Mass of electron = 9.1×10^{-31} kg)

$q = 1.6 \times 10^{-19}$ C, $E = 1.2 \times 10^{5}$ N C^{-1}

(i) The electric force on the electron $F = qE$

$$F = 1.6 \times 10^{-19} \times 1.2 \times 10^{5} = 1.92 \times 10^{-14} \text{ N}$$

The direction of electric force on the electron is opposite to that of electric field.

(ii) If a is the acceleration of the electron, $F = ma$

$$a = \frac{F}{m}$$

$$a = \frac{1.92 \times 10^{-14}}{9.1 \times 10^{-31}} = 2.11 \times 10^{16} \text{ m s}^{-2}$$

The direction of acceleration of the electron is same as that of electric force.

(iii) $s = 20$ mm $= 0.02$ m

Since the electron starts from rest, $v_0 = 0$.

Therefore,

$$s = v_0 t + \frac{1}{2}at^2$$

$$s = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}}$$

$$t = \sqrt{\frac{2 \times 0.02}{2.11 \times 10^{16}}} = 1.37 \times 10^{-9}\ \text{s}$$

Reviewed Exercise

1. When one million electrons are placed on a solid copper sphere, how are the electrons distributed ?
2. What is the difference between the electric lines of force which represent a non-uniform electric field and those which represent a uniform electric field?
3. Explain why the electric field intensity is zero everywhere inside a charged conductor.

Key Words: uniform electric field, non-uniform electric field, electric lines of force

9.5 ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

Electric Potential Energy and Electric Potential

Work must be done to separate two bodies having opposite charges since they attract each other. Likewise, work must be done to bring closer two bodies having the same kind of charge since they repel each other. In both cases the work done is stored up in the charged objects as electric potential energy.

Suppose that a charge q is initially at infinity. The work done in bringing the charge q against the electric force from infinity to a point in an electric field is the electric potential energy of charge q at that point.

The electric potential at a point in an electric field is the work done in bringing a unit positive charge against the electric force from infinity to that point.

Let W be the work done in bringing the small positive charge q from infinity to a point in the electric field. Then the electric potential at that point can be expressed as,

$$V = \frac{W}{q} \qquad (9.6)$$

Since the electric potential is actually the amount of work done, it is a scalar quantity. The electric potential is the electric potential energy per unit positive charge.

The electric potential at infinity is taken as zero by convention. The electric potential at a point in the electric field is expressed relative to the electric potential at infinity.

The Unit of Electric Potential

The practical unit of electric potential is the volt (V). If the work done in bringing +1 coulomb from infinity to a point in an electric field is 1 joule, the electric potential at that point is 1 joule per coulomb ($1\ \text{J C}^{-1}$) or 1 V.

The Electric Potential Difference

The electric potential difference between two points in an electric field is the work done in bringing a unit positive charge from one point to another against electric force. Let V_A be the electric potential at A and V_B be the electric potential at B.

$$V_B - V_A = \frac{W_{A \to B}}{q} \qquad (9.7)$$

The SI unit of electric potential difference is volt. The practical unit of electric potential difference is also volt.

A positive charge will move from a point of higher electric potential to a point of lower electric potential. A negative charge will move from a point of lower electric potential to a point of higher electric potential.

The Electric Potential due to a Point Charge

Figure 9.7 Electric potential due to a point charge

The electric potential at a distance r from a point charge $+Q$ can be expressed as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \qquad (9.8)$$

Therefore, the electric potential V at a point is directly proportional to the charge Q and inversely proportional to the distance r from the charge.

Suppose that the total electric potential at a point due to several point charges is to be determined. First, the electric potentials at that point due to the individual charges must be calculated. In doing so the signs of the individual charges must be taken into account. That is to say the individual electric potentials must be added algebraically. If the electric potentials due to the charges Q_1, Q_2, Q_3, ..., are V_1, V_2, V_3, ..., respectively, the total electric potential V is

$$V = V_1 + V_2 + V_3 + \ldots\ldots \qquad (9.9)$$

Example 9.11 (i) Find the electric potential at a point 3 m from a point charge of $+6.0 \times 10^{-9}$ C.
(ii) Find the electric potential at a point 6 m from a point charge -3.0×10^{-9} C.

(i) $Q = +6.0 \times 10^{-9}$ C, $r = 3$ m

The electric potential due to Q

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$= 9 \times 10^9 \frac{(+6.0 \times 10^{-9})}{3} = +18 \text{ V}$$

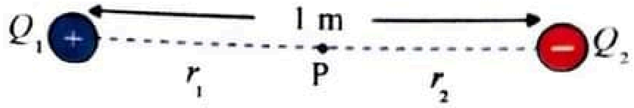
(ii) $Q = -3.0 \times 10^{-9}$ C , $r = 6$ m

The electric potential due to Q

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$= 9 \times 10^9 \frac{(-3.0 \times 10^{-9})}{6} = -4.5 \text{ V}$$

Example 9.12 Two point charges of $+4.0 \times 10^{-8}$ C and -3.0×10^{-8} C are 1 m apart. (i) Find the electric potential at P midway between the two charges. (ii) Find the work done in bringing a charge $+3.0 \times 10^{-9}$ C from infinity to P.



(i) $Q_1 = +4.0 \times 10^{-8}$ C, $Q_2 = -3.0 \times 10^{-8}$ C , $r_1 = r_2 = \frac{1}{2} = 0.5$ m

The electric potential at P due to Q_1

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1}$$

$$= 9 \times 10^9 \frac{(+4.0 \times 10^{-8})}{0.5}$$

$$= +720 \text{ V}$$

The electric potential at P due to Q_2,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2}$$

$$= 9 \times 10^9 \frac{(-3.0 \times 10^{-8})}{0.5}$$

$$= -540 \text{ V}$$

Total electric potential at P,

$$V = V_1 + V_2 = 720 + (-540) = 180 \text{ V}$$

(ii) $q = +3.0 \times 10^{-9}$ C

If W is the work done in bringing the charge q from infinity to P

$$W = Vq$$

$$= 180 \times 3.0 \times 10^{-9}$$

$$= 0.54 \times 10^{-6} \text{ J}$$

The Path of the Charge and the Work Done

In Figure 9.8 the points A and B are situated in an electric field due to the charge $+Q$. A and B are at distances of r_a and r_b from $+Q$ respectively. A unit positive charge may be taken from A to B along the path 1 (or) 2 (or) any other path.

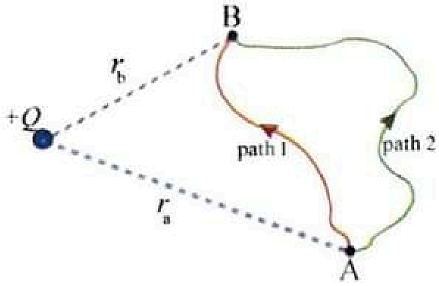


Figure 9.8 Work done is independent of the path taken

Electric potential at A, $V_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_a}$

Electric potential at B, $V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_b}$

The work done is independent of the path taken by the charge, it only depends on the electric potential difference between two points $[\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_b} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_a}]$. In other words, the same amount of work must be done whenever the unit positive charge is taken along any path from A to B.

Example 9.13 If the points A and B are at distances of 0.5 m and 1 m respectively from the charge $+5.0 \times 10^{-9}$ C, (i) find the electric potential difference between them (ii) how much work is done when the charge $+2.0 \times 10^{-9}$ C is brought from B to A.

(i) $Q = +5.0 \times 10^{-9}$ C, $r_a = 0.5$ m, $r_b = 1$ m

The electric potential at A, $V_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A} = 9\times10^9\frac{(+5.0\times10^{-9})}{0.5} = +90$ V

The electric potential at B, $V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B} = 9\times10^9\frac{(+5.0\times10^{-9})}{1} = +45$ V

The electric potential difference between A and B,

$$V_{AB} = V_A - V_B = 90 - 45 = 45 \text{ V}$$

(ii) $q = +2.0 \times 10^{-9}$ C

If W is the work done in bringing the charge q from B to A,

$$W_{\text{B to A}} = (V_A - V_B)\,q$$

$$= 45 \times 2.0 \times 10^{-9} = 9.0 \times 10^{-8} \text{ J}$$

Reviewed Exercise

1. How does the electric potential relate to the electric potential energy?
2. Why is electric potential a scalar quantity?
3. Can electrons by themselves move from a point of lower electric potential to a point of higher electric potential? Why?
4. If a small positive charge moves from a point of higher electric potential to a point of lower electric potential, does it gain (or) lose electric potential energy? Why?

Key Words: electric potential energy, electric potential, electric potential difference

9.6 EQUIPOTENTIAL SURFACES

A surface drawn through the points at the same potential is called an equipotential surface.
The equipotential surfaces around a charge Q are shown in Figure 9.9. They are the spherical surfaces centred about the charge Q. The electric lines of force are radially outward and perpendicular to the equipotential surfaces.

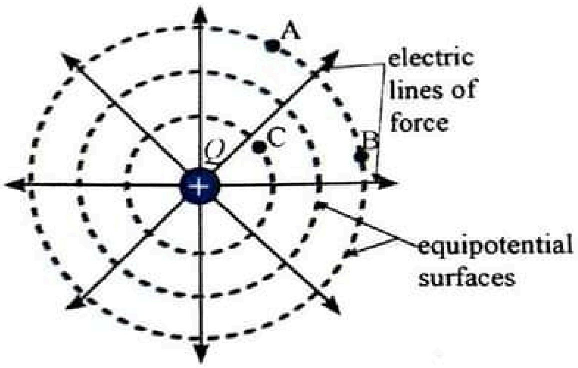


Figure 9.9 Electric lines of force are perpendicular to equipotential surfaces

The surface of a charged conducting sphere is an equipotential surface. This is because the charges, distributed uniformly on its surface, are stationary. If the surface is not an equipotential surface, there will be the potential difference and the charges would move from point to point.

The charged conductors may have any shape but their surfaces are all equipotential surfaces as shown in Figure 9.10. In addition, the electric lines of force are perpendicular to the surface of the charged conductor.

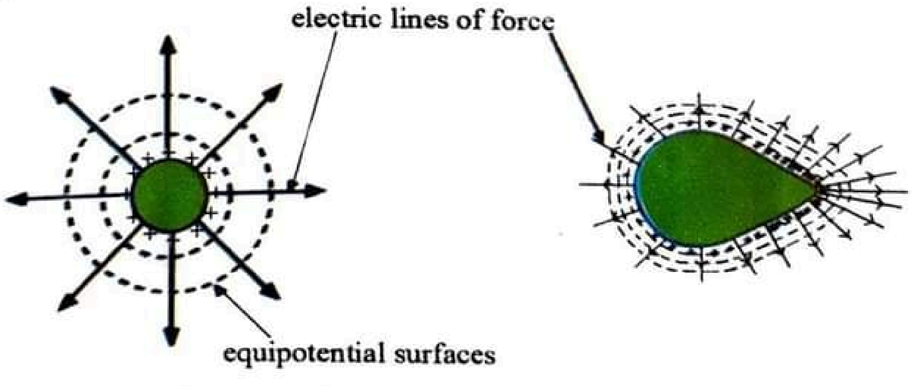


Figure 9.10 Equipotential surfaces

Example 9.14 What is the radius of an equipotential surface of 30 V surrounding a point charge of $+1.5 \times 10^{-6}$ C ?

$V = 30$ V , $Q = +1.5 \times 10^{-6}$ C

The electric potential of a equipotential surface is, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$$30 = 9\times10^9 \frac{(+1.5\times10^{-6})}{r}$$

$$r = 450 \text{ m}$$

The radius of an equipotential surface is 450 m

Reviewed Exercise

1. How much work is done in moving a charge of $+1.6 \times 10^{-19}$ C from one point to another on an equipotential surface of 200 V?
2. Explain why the electric lines of force are perpendicular to equipotential surfaces for a point charge.

Key Words: equipotential surface, electric line of force

The work done is zero in bringing a charge from one point to another point on the equipotential surface.

9.7 ELECTRIC POTENTIAL OF THE EARTH

The electric potentials of charged conductors are expressed relative to the electric potential of the surface of the earth. That is the electric potential of the earth is taken as zero.

The earth is a conductor. Moreover, since it is very large compared to other conductors it can receive as well as give out quite a number of electrons. When compared to the size of the earth the number of electrons gained or lost by it is very small so that the net charge of the earth does not change. The electric potential of a conductor becomes zero when it is connected to the earth.

Suppose a positively charged body is connected to the earth as shown in Figure 9.11 (a), electrons flow from the earth to the sphere at higher potential. When a negatively charged body is connected to the earth as shown in Figure 9.11 (b), electrons flow from the sphere at the lower potential to the earth. In both cases, the electron flow takes place until the sphere has no net charge.

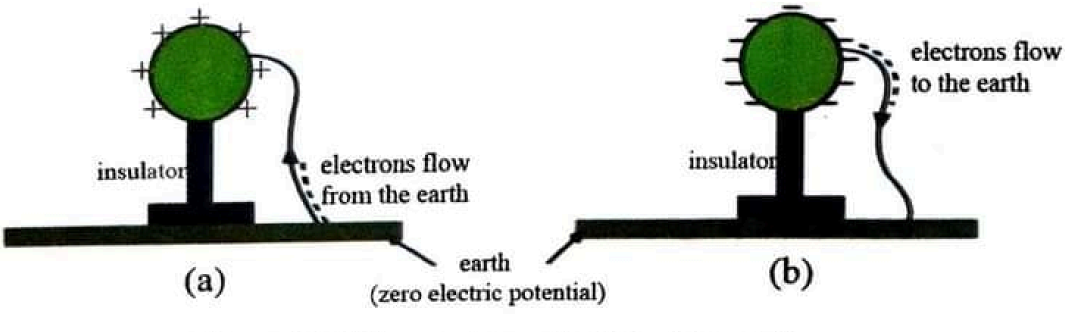


Figure 9.11 The electric potential of the earth

Reviewed Exercise

1. **Why can the earth be regarded as a body having zero electric potential?**
2. **What will happen to electric potential of a charged conductor if it is connected to the earth?**

Key Words: electric potential of the earth, conductor, electron

9.8 POTENTIAL DIFFERENCE BETWEEN TWO PARALLEL CHARGED PLATES

Figure 9.12, A and B are two parallel charged plates. The distance between A and B is d. The charge on A is $+Q$ and that on B is $-Q$. The electric field between the plates is uniform.

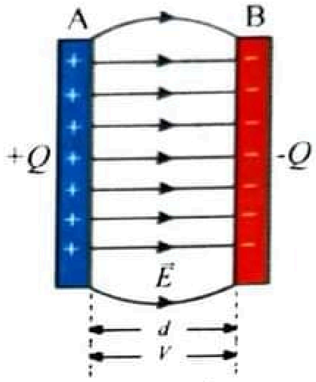


Figure 9.12 The electric potential difference between two parallel charged plates

Suppose that the electric field intensity between the parallel plates is $\vec{E}$. Consider a unit positive charge between the two plates. Therefore, the force acting upon a unit positive charge is $\vec{E}$. The work done in bringing the charge from B to A against that force is $W = Ed$.

Since this work done is the electric potential difference V between the two plates, $V = W$.

Therefore, the relation between potential difference and the electric field intensity between two plates is,

$$V = E\,d \qquad (9.10)$$

The unit of electric field intensity is also expressed in volt per metre ($V\ m^{-1}$) according to Eq (9.10).

> Parallel charged plates are two parallel metal plates of same size and same material, carrying charges of equal magnitude and opposite signs separated by a distance.

Example 9.15 A 6 V battery is connected to two parallel metal plates. (i) If the distance between the two plates is 5 cm, find the electric field intensity between them. (ii) The plate with lower electric potential is taken to be zero volt. What is the electric potential at mid point between the two plates?

(i) $V = 6$ V, $d = 5$ cm $= 0.05$ m

$$V = E\,d$$

$$E = \frac{V}{d} = \frac{6}{0.05} = 120\ N\ C^{-1}\ (\text{or})\ V\ m^{-1}$$

(ii) Since the electric field between the plates is uniform, E = constant.

$$d = 2.5\ cm = 0.025\ m$$

$$V = E\,d = 120 \times 0.025 = 3\ V$$

Example 9.16 A 6 V battery is connected to two parallel metal plates. The electric field intensity between the plates is 300 V m^{-1}. If an electron is placed on the negatively charged plate what is the velocity of the electron when it strikes the positively charged plate?

Suppose that v is the velocity of electron when it strikes the plate.

Kinetic energy of the electron when it strikes the positively charged plate,

$$KE = \frac{1}{2}mv^2$$

The work done in carrying an electron from one plate to another

$$W = Vq = 6 \times 1.6 \times 10^{-19} = 9.6 \times 10^{-19}\ J$$

KE of the electron = the work done in carrying an electron from one plate to another

Therefore, $$\frac{1}{2}mv^2 = W$$

$$\frac{1}{2} \times (9.1 \times 10^{-31})\, v^2 = 9.6 \times 10^{-19}$$

$$v^2 = \frac{2 \times 9.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$v = 1.45 \times 10^6\ m\ s^{-1}$$

Reviewed Exercise

1. Draw the equipotential surfaces between two parallel plates having charges of equal magnitude and opposite sign.
2. If an electron is placed between two parallel metal plates having charges of equal magnitude and opposite signs, find the direction of motion of the electron? Can the work be done on the electron by the electric force in this case?
3. Show that the units V m^{-1} and N C^{-1} for the electric field intensity are indeed equivalent.

Key Words: parallel plates, work, electric field

SUMMARY

An **electric field** can be defined as a region where electrical forces act.

The **electric field intensity** at a point in an electric field is the electric force acting upon a unit positive charge placed at that point.

The work done in bringing the charge q against the electric force from infinity to a point in an electric field is the **electric potential energy** of charge q at that point.

The **electric potential** at a point in an electric field is the work done in bringing a unit positive charge against the electric force from infinity to that point.

The **electric potential difference** between two points in an electric field is the work done in bringing a unit positive charge from one point to another against electric force.

Physical quantities of electric field

Physical quantity	Formula	Unit
Electric force	$F = K\frac{Q_1 Q_2}{r^2}$	newton (N)
Electric field intensity	$\vec{E} = \frac{\vec{F}}{q}$ $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$	newton per coulomb ($N\ C^{-1}$)
Electric potential	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$	volt (V)
Electric potential energy	$W = Vq$	joule (J)

EXERCISES

1. A positive charge of 4.0×10^{-6} C exerts a force of repulsion of 7.2 N on a second charge 0.25 m away. What is the sign and magnitude of the second charge?
2. Two charges of unknown magnitude and sign are observed to repel one another with a force of 0.1 N when they are 5 cm apart. Find the repulsive force between them when they are (i) 10 cm apart (ii) 50 cm apart (iii) 1 cm apart.
3. How far apart are two electrons if the force each exerts on the other is equal to the weight of an electron? ($g = 10\ m\ s^{-2}$)
4. Two charges, $+1 \times 10^{-4}$C and -1×10^{-4}C, are 40 cm apart. A particle carrying a charge of $+6 \times 10^{-5}$C is located halfway between them. If all charges lie on the same straight line, find the force acting on the charge located halfway between them.
5. Two metal spheres of the same size, one with a charge of $+2 \times 10^{-5}$ C and the other with a charge of -1×10^{-5} C are 10 cm apart. (i) What is the force between them? (ii) The two spheres are brought into contact, and then separated again to 10 cm. What is the force between them now?
6. Two charges of $+4 \times 10^{-6}$ C and $+8 \times 10^{-6}$ C are 2 m apart. What is the electric field intensity midway between them?
7. An electron is accelerated to $10^{8}\ m\ s^{-2}$ by an electric field. What is the direction and magnitude of the field?
8. Two charges, -20×10^{-6} C and $+5 \times 10^{-6}$ C, are 2 m apart. Where is the electric field intensity in their vicinity equal to zero?

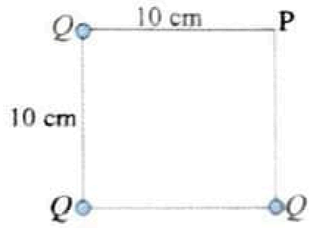
9. Explain how work is done in carrying a unit positive charge from a point of higher electric potential to a point of lower electric potential and how work is done in carrying a unit positive charge from a point of lower electric potential to a point of higher electric potential.

10. If the electric field intensity at a point in an electric field is zero, is the electric potential at that point necessarily zero?

11. A uranium nucleus has a charge of 92e. (i) Find the direction and the magnitude of the electric field intensity due to the nucleus at a point 10^{-10} m from the nucleus. (ii) Find the direction and magnitude of the force on an electron placed at that point. (iii) Find the electric potential at that point due to the nucleus.

12. The electric potential and the magnitude of the electric field intensity at a point at some distance from a point charge are 300 V and 100 N C^{-1} respectively. (i) How far is the point from the charge? (ii) What is the magnitude of the charge?

13. Find the total electric potential at the point P in the diagram given below. The value of Q is $+5.0 \times 10^{-9}$ C.



14. The electric potential difference between two parallel metal plates which are 0.5 cm apart is 0.5×10^{3} V. Find the force on an electron located between the plates.

15. An electron is accelerated by a uniform electric field from rest to a velocity of 10^{6} m s^{-1}. If the accelerating region is 0.2 m long, find the magnitude of the electric field.

CONCEPT MAP

- **ELECTRIC FIELD**
  - **Coulomb's law**
    - **Electric Force**
      - **Electric field intensity**
        $E = \frac{F}{q}, E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
        - Electric lines of force
        - Electric charge distribution
      - **Electric potential**
        - electric potential energy
        - electric potential difference
        $V = \frac{W}{q}, V_B - V_A = \frac{W_{AtoB}}{q}, V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
        - Electric potential of the earth
        - Equipotential surfaces
        - Electric potential between two parallel plates $V = E\,d$

CHAPTER 10
ELECTRIC CURRENT AND MAGNETIC EFFECT OF ELECTRIC CURRENT

Most applications of electricity and magnetism involve moving charges (or) electric currents in conductors. The stationary electric charge and the magnetic field do not affect each other. However, a moving electric charge (or) an electric current and the magnetic field have mutual effects between them. This means that the electric and magnetic phenomena are related. When an electric current flows through substances, it can produce three main effects. We shall discuss magnetic effect of electric current in this chapter.

Learning Outcomes

It is expected that students will

- examine current and the effects of current.
- examine electrical resistance, resistors in series and in parallel, and the application of Ohm's law.
- examine the magnetic field produced as a result of an electric current.
- examine the creation of electromagnets and identify some of the uses of electromagnets and permanent magnets.
- demonstrate the function and use of an ammeter and a voltmeter.

10.1 CURRENT AND EFFECTS OF CURRENT

The flow of electrons from a place of lower potential to a place of higher potential is called an electric current. In general, an electric current is a flow of electric charge from one place to another.

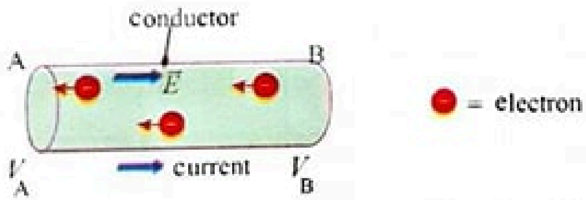


Figure 10.1 Moving charges constitute the electric current

Conductors contain a large number of free electrons. If the potential difference is established between the two ends of a conductor, electrons will flow from the end of lower potential to that of higher potential. In Figure 10.1 the potential of the end A is assumed to be higher than that of the end B. Thus the electrons will flow from B to A because the electric field in the conductor exerts a force ($F = qE$) upon them. This means that an electric current flows through that conductor. An electric current flowing through a conductor is defined as follows.

The amount of charge passing through a cross-sectional area of a conductor in one second is called an electric current.

(or)

The rate of flow of electric charge through a cross-sectional area of a conductor is called an electric current.

Current is a scalar quantity by the above definition. Suppose that the amount of charge Q passes through a cross-sectional area of a conductor in time t, the current I flowing through the conductor is

$$I = \frac{Q}{t} \tag{10.1}$$

If the number of electrons passing through a cross-sectional area of a conductor is n, the amount of charge Q is,

$$Q = n\,e$$

Therefore,

$$I = \frac{ne}{t} \tag{10.2}$$

Unit of Current

In the SI system, the unit of electric current is ampere (A) in honour of the French physicist, Andre Ampere. The unit ampere is defined as follows.

If the amount of charge 1 coulomb passes through a cross-sectional area of a conductor in 1 sesond, the current is 1 ampere. Therefore, 1 A is equivalent to 1 C s^{-1}.

In measuring the very small current the following sub-multiple units are also used.

1 milliampere (mA) = 10^{-3} A
1 microampere (μA) = 10^{-6} A

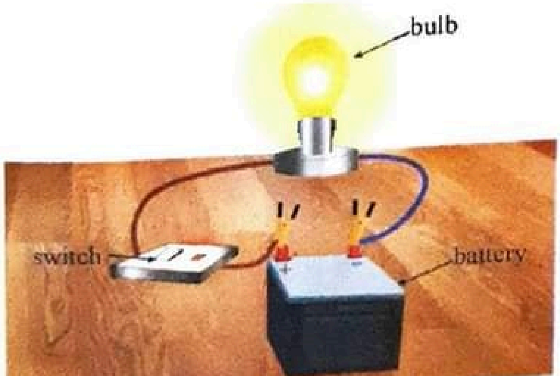
The direction of current is conventionally defined as the direction of the flow of positive charge from a higher potential to a lower potential. Hence, the direction of current is opposite to that of the flow of electrons.

Effects of Electric Current

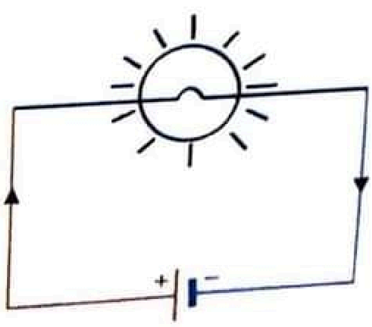
When an electric current is passed through substances, it can produce three main effects. They are (1) heating effect (2) chemical effect and (3) magnetic effect.

(1) Heating Effect

A small bulb glows when a battery is connected to it as shown in Figure 10.2 (a) and (b). As an electric current flows through, the tungsten wire in the bulb becomes hot and emits light. Thus a metal conductor produces heat energy when a current passes through it. Practical application of the heating effect of current is utilized in electrical appliances such as electric stove, electric iron and immersion heater.



(a)



(b)

Figure 10.2 Heating effect of current [CREDIT: Source from the Internet]

(2) Chemical Effect

When a current is passed through copper sulphate solution with copper plates A (anode) and C (cathode) dipping into it, some copper is deposited on the plate C after sometime as shown in Figure 10.3 (a) and (b). Thus, electric current produces chemical effect. The chemical effect of current is used in charging batteries, purifying metals and electro-plating.

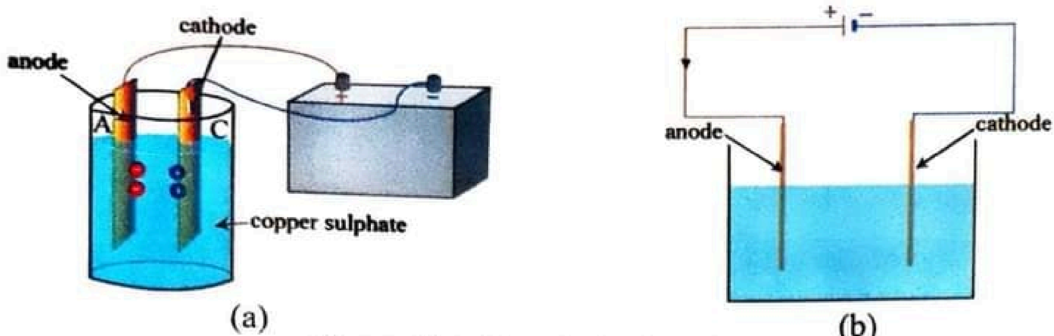


Figure 10.3 Chemical effect of current

(3) Magnetic Effect

When a current flows through a coil of insulated wire which is wound round a bar of soft iron, the bar becomes a magnet and attracts steel pins as shown in Figure 10.4 (a) and (b). Thus, electric current produces magnetic effect. The magnetic effect of current is used in making electromagnets. Electromagnets are used in electrical devices such as electric bell, magnetic relay and electric motor.

Figure 10.4 Magnetic effect of current

Example 10.1 A charge of 6 C passes through a cross-sectional area of a conductor in 2 s. (i) Find the current flowing through the conductor. (ii) How many electrons pass through that area in 1 s?

(i) $Q = 6\ \text{C},\ t = 2\ \text{s}$

The current flowing through the conductor,

$$I = \frac{Q}{t} = \frac{6}{2} = 3\ \text{A}$$

(ii) The magnitude of the charge of an electron, $e = 1.6 \times 10^{-19}$ C. If n is the number of electrons passing through the cross-sectional area in 1 s,

$$I = \frac{ne}{t}$$

$$3 = \frac{n \times 1.6 \times 10^{-19}}{1}$$

$$n = 1.88 \times 10^{19}\ \text{electrons}$$

Reviewed Exercise

1. In which direction do electrons flow in a conductor?
2. Name some electrical appliances which apply heating effect of current.

Key Words: electric current, electric charge, electron, effect of current, electromagnet

10.2 OHM'S LAW AND ELECTRICAL RESISTANCE

When there is a potential difference between the two ends of a conductor, a current flows through it. In 1827 the German physicist George Simon Ohm carried out experiments how the current through the conductor depended on the potential difference applied across its ends. He discovered a law. That law is called Ohm's law and states as follows.

If a conductor is kept at a constant temperature, the current flowing through it is directly proportional to the potential difference between its ends.

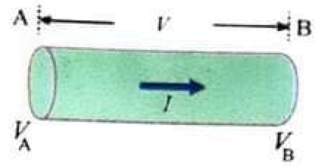


Figure 10.5 Electric current passing through a conductor

In Figure10.5 since the potential at A is assumed to be higher than the potential at B, a current will flow from A to B. If the potential difference between A and B is V and the current flowing through the conductor is I, by Ohm's law,

$$I \propto V$$

$$I = \frac{1}{R}V$$

$$V = IR \qquad (10.3)$$

Here R is a constant which is called the resistance of the conductor. The SI unit of the resistance R is ohm (Ω).

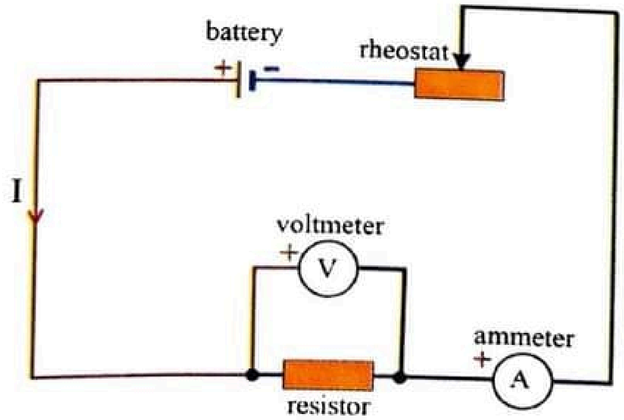


Figure 10.6 Electric circuit for measurement of current and potential difference

Figure 10.6 shows the electric circuit for the measurement of current and potential difference across a resistor. Ammeter is a device_to measure the current flowing through resistor and voltmeter is a device to measure the potential difference across a resistor. The potential difference across the resistor is varied by using rheostat (variable resistor). Figure 10.7 shows that the current is directly proportional to the potential difference (p.d) by using the data of idealize experiment.

p.d (V)	Current (A)
0	0.0
1	0.2
2	0.4
3	0.6
4	0.8
5	1.0

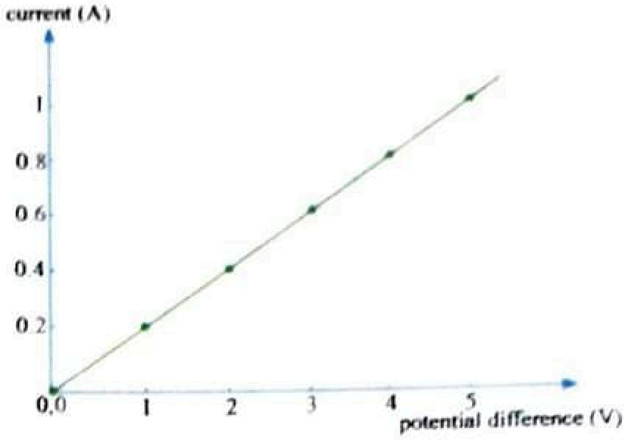


Figure 10.7 Relation between the potential difference and the current

Resistance of a Conductor

At a given temperature, the resistance of a conductor is the ratio of the potential difference between two ends of a conductor to the current flowing through it.

$$R = \frac{V}{I} \tag{10.4}$$

where R is the resistance of a conductor, V is the potential difference and I is the current.

At a given temperature,the resistance of a conductor does not depend on the potential difference between its ends and the current flowing through it.

Resistivity of a Conductor

At a given temperature the resistance of a conductor is directly proportional to its length and inversely proportional to its cross-sectional area, where the constant of proportionality is known as resistivity.

In Figure 10.8 the conductor shown has a cross-sectional area of A and length of l. If its resistance is R, then

$$R \propto \frac{l}{A}$$

$$R = \frac{\rho l}{A} \tag{10.5}$$

where ρ is a constant called the resistivity of the conductor.

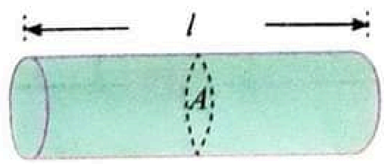


Figure 10.8 Dependence of resistance on length and cross-sectional area

Resistivity of conductor is defined as the resistance of a conductor having one unit cross-sectional area and one unit length. The unit of resistivity is ohm metre (Ω m).

If the length or the cross-sectional area of a substance changes, its resistance will also change. But its resistivity remains the same. This means that a particular substance has only a single value of resistivity. Although the resistivity varies slightly with temperature it can be taken as a constant.

Temperature Coefficient of Resistance

The resistance of a conductor increases with increasing temperature. However, the resistances of carbon (a non-metal), semiconductors such as silicon and germanium, and electrolytes decreases with increasing temperature.

Suppose that R_0 is the resistance of a conductor at the temperature of 0 °C and R_t is its resistance at t °C. R_t is related to R_0 as follows:

$$R_t = R_0\,(1 + \alpha t) \qquad (10.6)$$

where α is a constant called the temperature coefficient of resistance.

The unit of α is per °C (°C^{-1}). The resistivities and the values of the temperature coefficients of some substances are given in Table 10.1.

Table 10.1 Resistivities and the temperature coefficients of some substances

Substance	Resistivity ρ at 20 °C (Ω m)	Temperature coefficient of resistance α (°C^{-1})
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Copper	1.72×10^{-8}	4.3×10^{-3}
Iron	9.80×10^{-8}	5.6×10^{-3}
Silver	1.62×10^{-8}	3.9×10^{-3}
Tungsten	5.50×10^{-8}	5.8×10^{-3}
Mercury	95.77×10^{-8}	0.9×10^{-3}
Carbon (graphite)	3×10^{-5} to 60×10^{-5}	$(-0.6$ to $-0.1) \times 10^{-3}$

Example 10.2 A current of 2 A flows through a conductor when the potential difference between its ends is 12 V. If the potential difference is reduced to 3 V how much does the value of current change?

$V = 12$ V, $I = 2$ A

By Ohm's law $\qquad V = IR$

$$R = \frac{V}{I} = \frac{12}{2} = 6\ \Omega$$

Assuming the resistance of the conductor R remains constant,

$V_1 = 3\text{V}, R = 6\ \Omega$

$$R = \frac{V_1}{I_1}$$

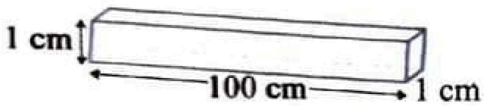
$$6 = \frac{3}{I_1}$$

$$I_1 = 0.5\text{ A}$$

The current change $I - I_1 = 2 - 0.5 = 1.5\text{ A}$

The current decreases by 1.5 A.

Example 10.3 A rectangular silver slab has dimensions 1 cm × 1 cm × 100 cm. What is the resistance between its two square surfaces? The resistivity of silver is $1.62 \times 10^{-8}\ \Omega$ m.



Cross-sectional area of the slab, A = 1 cm × 1 cm= 10^{-2} m × 10^{-2} m = 10^{-4} m^2

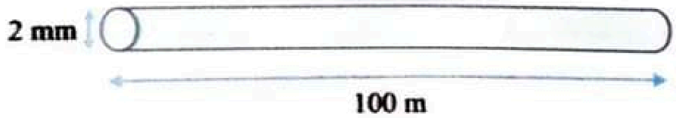
Length of the slab, l = 100 cm = 1m

$\rho = 1.62 \times 10^{-8}\ \Omega$ m

The resistance between two square surfaces of the slab,

$$R = \frac{\rho l}{A} = \frac{1.62 \times 10^{-8} \times 1}{10^{-4}} = 1.62 \times 10^{-4}\ \Omega$$

Example 10.4 A tungsten wire has a length of 100 m, a diameter of 2 mm and a resistivity of $4.8 \times 10^{-8}\ \Omega$ m. Find its resistance.



l = 100 m, diameter of the wire d = 2 mm = 2×10^{-3}m, $\rho = 4.8 \times 10^{-8}\ \Omega$ m

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi \frac{d^2}{4}} = \frac{4\rho l}{\pi d^2}$$

$$= \frac{4 \times 4.8 \times 10^{-8} \times 100}{3.142 \times \left(2 \times 10^{-3}\right)^2} = 1.53\ \Omega$$

The resistance of a tungsten wire is 1.53 Ω.

Example 10.5 When a platinum resistance thermometer is placed in a mixture of ice and water at 0 °C its resistance is 10 Ω. When it is placed in a furnace of unknown temperature its resistance is 100 Ω. If the temperature coefficient of platinum is 0.0036 °C^{-1}, find the temperature of the furnace.

Resistance of platinum at 0 °C, R_0 = 10 Ω

Resistance of platinum at t °C, $R_t = 100\ \Omega$,

$\alpha = 0.0036\ °C^{-1}$

Temperature of the furnace, $t = ?$

$$R_t = R_0 (1 + \alpha t)$$

$$100 = 10 (1 + 0.0036\, t)$$

$$t = 2500\ °C$$

Temperature of furnace is 2500 °C.

Reviewed Exercise

1. How can the electric current be kept flowing inside a conductor?
2. Two wires of equal lengths, one of copper and the other of manganin (alloy of copper, manganese, and nickel) have the same resistance. Which wire will be thicker? (Hint: the resistivity of manganin is greater than that of copper).
3. If a wire is stretched to double its original length, how will the resistivity and resistance of the wire be affected?

Key Words: potential difference, current, resistance, temperature, resistivity

10.3 RESISTORS

A resistor is an electrical device which is used to control the electric current flowing in a circuit. It is made of a substance having resistance.

Radio and television receivers contain a large number of resistors. Resistors have resistances from a few ohms to millions of ohms.

There are two types of resistors: fixed resistors and variable resistors. Figure 10.9 (a) shows a fixed resistor and its circuit symbol in Figure 10.9 (b). A rheostat (variable resistor) and its symbol are shown in Figure 10.10 (a) and (b) respectively.

(a) (b)

Figure 10.9 (a) A fixed resistor (b) Circuit symbol for a fixed resistor

(a) (b)

Figure 10.10 (a) Rheostat and potentiometer (b) Circuit symbol for a variable resistor

Example 10.6 When the resistance value of a variable resistor is increased from 100 Ω to 1000 Ω, what will happen to the electric current passing through it? The potential difference across the variable resistor is kept constant.

By Ohm's law, $V = IR$ (or) $I = \frac{V}{R}$

Since V is constant, $I \propto \frac{1}{R}$

When resistance value (R) is increased from 100 Ω to 1000 Ω, the electric current (I) passing through it will decrease by a factor of one-tenth.

Reviewed Exercise

1. Why are the resistors used in an electric circuit?
2. Name two types of resistor.

Key Words: resistor, fixed resistor, variable resistor, resistance

10.4 RESISTORS IN SERIES AND RESISTORS IN PARALLEL

Resistors in Series

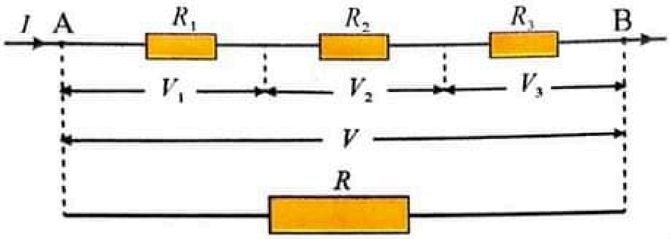


Figure 10.11 Resistors in series

The resistors are said to be connected in series if they are connected in such a way that the same current flows through each resistor as shown in Figure 10.11. The resistors R_1, R_2 and R_3 are connected in series. The point A is assumed to have a higher potential than the point B. Then a current will flow from A to B through the resistors.

If the individual potential differences across the resistors are V_1, V_2 and V_3 respectively, and the total potential difference across the combination is V, then

$$V = V_1 + V_2 + V_3$$

By Ohm's law, $$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3$$

When the above equations are substituted, $V = I\,(R_1 + R_2 + R_3)$

If the equivalent resistance of the combination of the resistors is R the potential difference across the combination is $V = IR$.

From the above equations,

$$R = R_1 + R_2 + R_3 \qquad (10.7)$$

If n resistors of resistances R_1, R_2, R_3, ..., R_n are connected in series and the equivalent resistance is R, then

$$R = R_1 + R_2 + R_3 + \text{........} + R_n \qquad (10.8)$$

> The equivalent resistance of the resistors in series is equal to the sum of the resistances of the individual resistors.

Resistors in Parallel

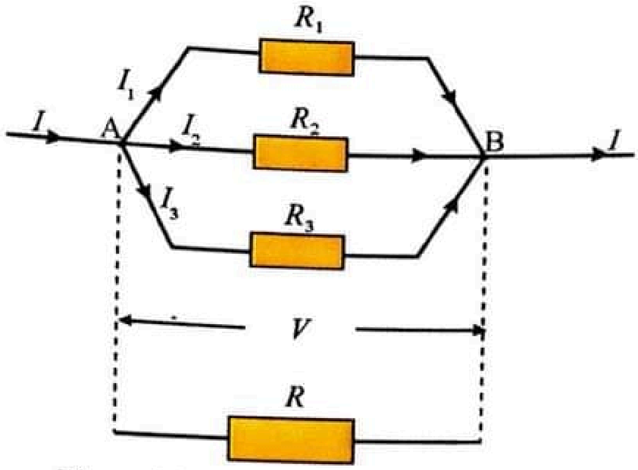


Figure 10.12 Resistors in parallel

Resistors are said to be connected in parallel if they are connected in such a way that the same potential appears across each and every resistor as shown in Figure 10.12. The resistors R_1, R_2 and R_3 are connected in parallel. One end of each resistor is joined at the point A and the remaining ends are joined at the point B.

The point A is assumed to have a higher potential than the point B. The current I divides into three components of current at the point A and flow through the resistors. These three currents recombine at the point B. The current leaving the point B is also I.

Let I_1, I_2 and I_3 are currents flowing through the resistors R_1, R_2 and R_3 respectively.

$$I = I_1 + I_2 + I_3$$

The potential difference across each resistor is the potential difference V between A and B.

By Ohm's law, potential difference across the resistor R_1, $V = I_1 R_1$
potential difference across the resistor R_2, $V = I_2 R_2$
potential difference across the resistor R_3, $V = I_3 R_3$

Substituting the above equations,

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

If the equivalent resistance of the combination of the resistors is R and the potential difference between point A and B is $V = IR$.

$$I = \frac{V}{R}$$

From the above equations

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (10.9)$$

If n resistors of resistances R_1, R_2, R_3, ...,R_n are connected in parallel and the equivalent resistance is R, then

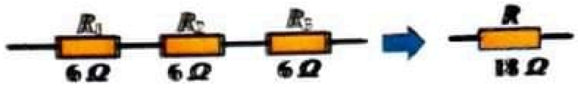
$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots\ldots\ldots+\frac{1}{R_n} \qquad (10.10)$$

The reciprocal of the equivalent resistance of resistors connected in parallel is equal to the sum of the reciprocal of the individual resistances.

Example 10.7 Find the equivalent resistance when three 6 Ω resistors are connected (i) in series and (ii) in parallel. (iii) Find the equivalent resistance when two resistors in parallel are connected to the remaining resistor in series.

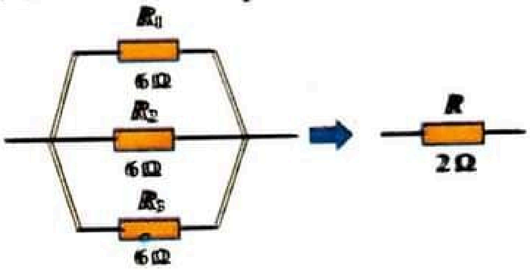
(i) If the equivalent resistance of three 6 Ω resistors connected in series is R, then

$$R = R_1 + R_2 + R_3$$
$$= 6 + 6 + 6 = 18\ \Omega$$



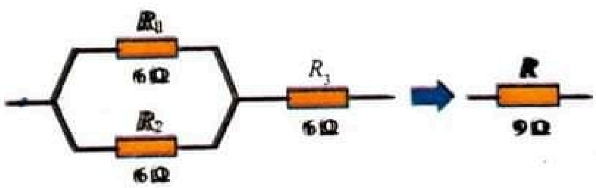
(ii) If the equivalent resistance of three 6 Ω resistors connected in parallel is R, then

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$
$$\frac{1}{R}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$$
$$R = 2\ \Omega$$



(iii) If the equivalent resistance of two 6 Ω resistors connected in parallel is R_p, then

$$\frac{1}{R_p}=\frac{1}{R_1}+\frac{1}{R_2}$$
$$\frac{1}{R_p}=\frac{1}{6.}+\frac{1}{6}=\frac{2}{6}$$
$$R_p = 3\ \Omega$$



If the equivalent resistance of R_p and the 6 Ω resistor connected in series is R, then

$$R = R_p + 6$$
$$R = 3 + 6 = 9\ \Omega$$

Reviewed Exercise

1. When the parallel combination of two resistors having different resistances is connected to a battery, which resistor will draw a greater current?
2. Draw diagrams to show that resistances of 20 Ω and 12.5 Ω can be obtained by using one 10 Ω resistor and two 5 Ω resistors.

Key Words: resistors, current, potential difference

10.5 MAGNETIC FIELD DUE TO AN ELECTRIC CURRENT

The magnetic effect of an electric current will be studied now. That effect was discovered by Oersted in 1820. Compass needle is shown in Figure 10.13 (a). When a straight wire carrying a current was placed above a compass needle as shown in Figure 10.13 (b), the needle was deflected. When the wire was placed below the needle as shown in Figure 10.13 (c), it was deflected in the opposite direction. This experiment was first done by Oersted.

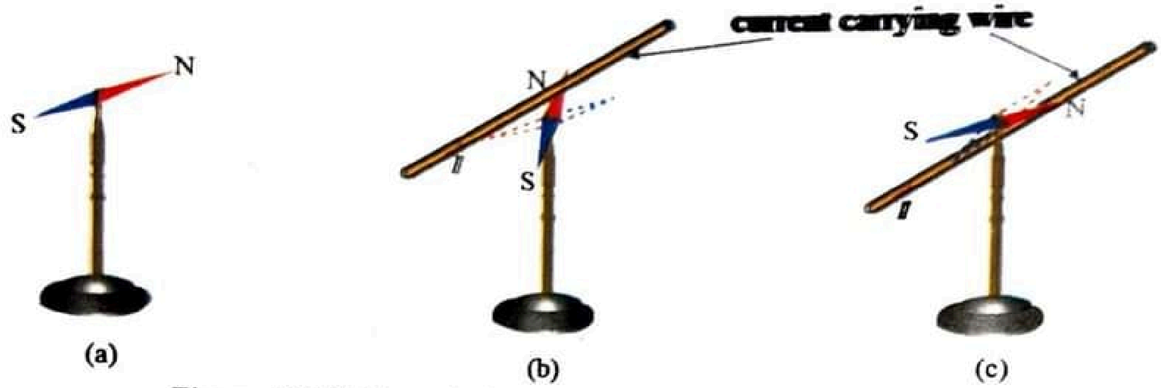


Figure 10.13 Magnetic field due to a current carrying wire

The deflection of the needle is due to a magnetic force acting on it. In other words, there is magnetic field in the neighbourhood of the wire. This magnetic field is produced by the current flowing in the wire.

The direction of the magnetic field due to the current flowing through the wire can be found by using the right-hand rule in Figure 10.14 (a). Imagine the wire to be grasped in the right hand with the thumb pointing along the wire in the direction of the current. The direction of the fingers will give the direction of the magnetic field. The north pole of a compass needle indicates the direction of the magnetic field in Figure 10.14 (c).

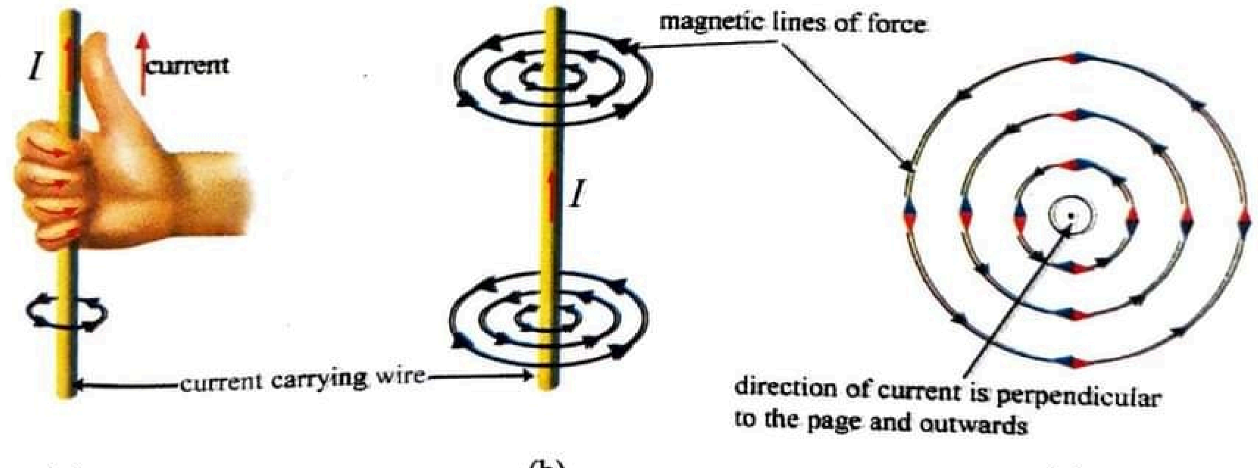


Figure 10.14 Application of the right-hand rule

As an electric field is represented by electric lines of force a magnetic field can also be represented by magnetic lines of force. The magnetic lines of force around the wire carrying a current *I* are shown in

Figure 10.14 (b). Figure 10.14 (c) shows the cross section of the wire as seen from the top. The dot in the cross section indicates that the current is flowing out of the page. The magnetic lines of force are closed circular loops around the wire and they are in the plane perpendicular to the wire. The orientation of the north pole of a compass needle along the magnetic line of force is shown in Figure 10.14 (c).

If the current flowing through the wire is reversed, the direction of the magnetic field will also be reversed. However, the magnetic lines of force will still be closed circular loops.

Magnetic Field of a Solenoid

A solenoid is a cylindrical coil of wire, which has a magnetic field inside and in its vicinity when a current flows through it. The magnetic field of a solenoid is identical with that of a bar magnet. Thus a solenoid can be considered as a bar magnet. One end of a solenoid acts like a north pole and the other like a south pole. It can be used in a electrical device such as electric bell, electric motor and magnetic relay that converts electrical energy into mechanical energy.

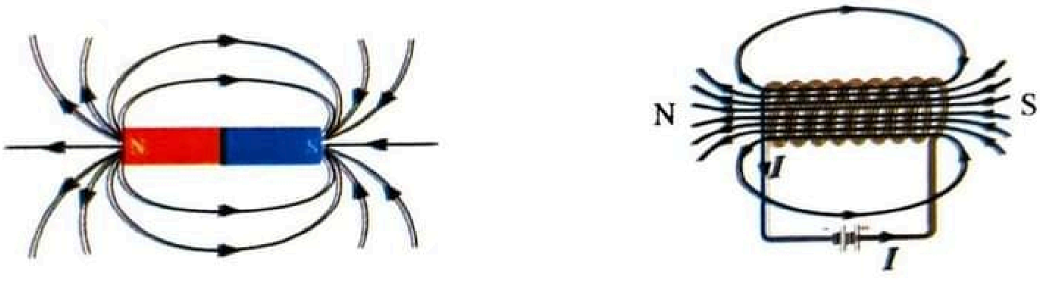


Figure 10.15 Magnetic field of a solenoid

The magnetic poles of a solenoid carrying a current can be found as follows. When viewing one end of the solenoid, that end will be a south pole if the current is seen flowing in a clockwise direction and a north pole if the current is seen flowing in an anticlockwise direction. The right end of the solenoid shown in Figure10.15 is the south pole and the left end is the north pole.

Fleming's Left-hand Rule

When a current-carrying conductor is placed in a magnetic field, the conductor experiences a force as shown in Figure 10.16 (a). The direction of a force on a current-carrying conductor in magnetic field can be found by the use of Fleming's left-hand rule. Place the fore finger, the second finger and the thumb of the left hand mutually at right angles to one another. If the fore finger points in the direction of the magnetic field and the second finger in the direction of the electric current, then the thumb will point in the direction of the motion along which the force acts [Fig 10.16 (b)].

(a) (b)

Figure10.16 Fleming's left-hand rule

Force on a Charged Particle Moving in a Magnetic Field

When a charged particle moves across a magnetic field, it experiences a force. In Figure 10.17, a charged particle $+q$ is moving perpendicular to the magnetic field B with a velocity v. The direction of the force F acting on that particle is perpendicular to those of B and v. The direction of F can be found by applying Fleming's left-hand rule. The second finger must point in the direction of velocity v in this case.

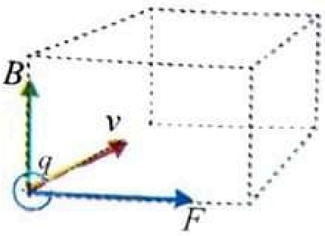


Figure 10.17 Force on a moving charge

If the particle in Figure 10.17 is a negatively charged one, the force acting on that particle will be in the opposite direction.

Reviewed Exercise

1. In which type of devices use solenoid?
2. Why can a current carrying solenoid be considered as a bar magnet?

Key Words: solenoid, magnetic force, magnetic field

10.6 ELECTROMAGNETS

The best method of making a magnet is to use the magnetic effect of an electric current. In Figure 10.18, a steel bar is placed inside a solenoid of insulated wire. When a large current flows through the solenoid the steel bar becomes magnetized permanently. Such a magnet is called a permanent magnet.

If the solenoid consists of many turns and a very large current flows through it a powerful magnetic field is obtained.

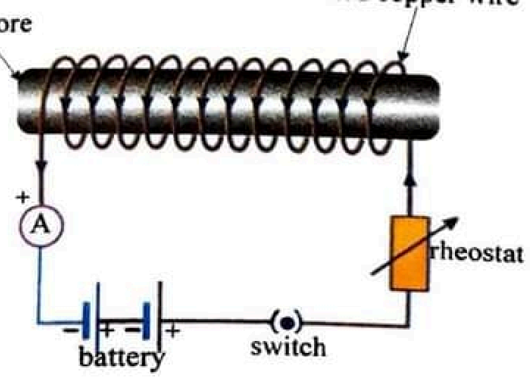


Figure 10.18 Magnetization by electric current

If a soft iron bar is placed inside the solenoid of insulated wire and a current flows through it, the bar becomes magnetized. It is demagnetized when the current stops. A soft iron core surrounded by a coil of wire, which acts as a magnet when a current flows through the coil is called a temporary magnet (or) an electromagnet.

Increasing the current will create a stronger magnetic field strength. Reversing the direction of current will resverse the direction of magnetic field.

Some Applications of Electromagnets

1. **Electric Bell**

An electric bell is an example of the use of the magnetic effect of current. The construction of an electric bell is shown in Figure 10.19.

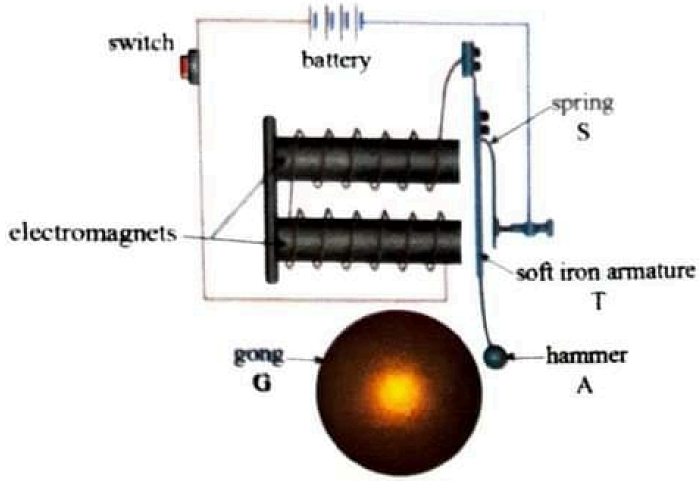


Figure 10.19 Electric bell

The soft iron armature (a piece of soft iron) T is mounted on a spring S. A small metal plate which is attached to the armature acts as a contact. When the switch is pressed the current flows through the circuit and the soft iron bars become magnetized. As they attract the armature T, the hammer A attached to it strikes the gong G. At that moment the metal plate and the end of the screw are separated so that the current stops. When this happens, the magnetism in the bars disappear and the armature is returned by the spring to its original position. Contact is now remade and the action is repeated. The armature vibrates and the hammer attached to it strikes the gong G repeatedly.

2. Circuit Breaker

A circuit breaker is also another example of the use of the magnetic effect of current. Figure 10.20 shows a circuit breaker designed to switch off the current in a circuit. When an excessive current flows through circuit breaker, it comes over rating.

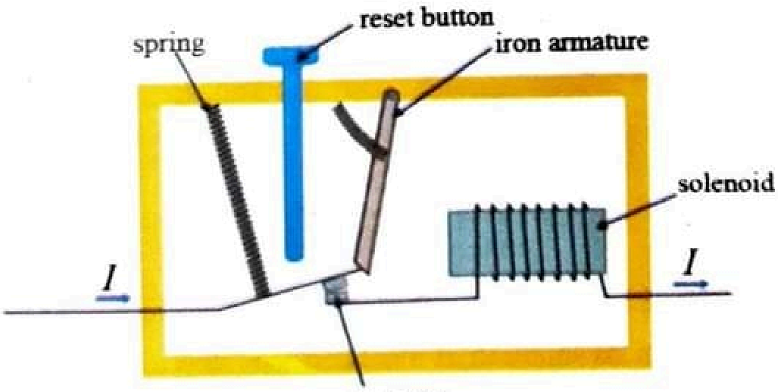


Figure 10.20 Circuit breaker

The current flows along the copper strip, through the iron armature and solenoid. The electromagnet will attract the iron armature if the current is large, thus breaking the circuit.

Reviewed Exercise

1. Name three devices which use the electromagnet.
2. Draw a diagram for a device consisting of an electromagnet.

Key Words: electromagnet, temporary magnet, soft iron core

10.7 AMMETER AND VOLTMETER

Ammeter and voltmeter are devices for measuring current and potential difference (voltage), respectively. They are constructed based on the principle of a moving coil galvanometer.

Moving Coil Galvanometer

A moving coil galvanometer is an instrument which is used to measure electric currents. It is a sensitive electromagnetic device which can measure low currents even of the order of a few microamperes. The construction and principle of function of moving coil galvanometer are expressed as follows.

The moving coil galvanometer is made up of a rectangular coil that has many turns of fine copper wire wounded on a frame. The coil is suspended in a uniform radial magnetic field and is free to rotate about a fixed axis. A cylindrical soft iron core is symmetrically positioned inside the coil to improve the strength of the magnetic field and to make the field radial. The coil is attached to a spring as shown in Figure 10.21. The spring is used to produce a counter torque which balances the magnetic torque producing a steady angular deflection.

The principle of function of a moving coil galvanometer is that the current-carrying coil when placed in an external magnetic field experiences magnetic torque. The angle through which the coil is deflected due to the effect of the magnetic torque is proportional to the magnitude of current in the coil.

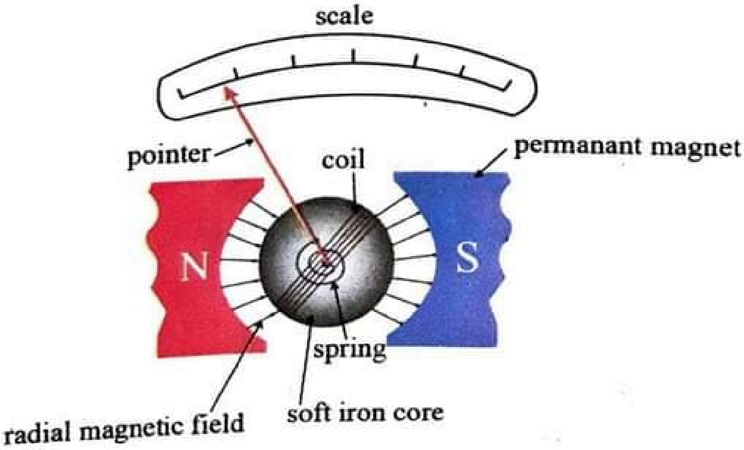


Figure 10.21 Moving coil galvanometer

Modification of Galvanometer to Ammeter

A galvanometer is converted into an ammeter by connecting it in parallel with a wire of low resistance called shunt resistance. Suitable shunt resistance is chosen depending on the range of the ammeter.

In the given circuit as shown Figure 10.22,

I = maximum current which can be measured by the ammeter

i = current which gives full-scale deflection of the galvanometer

$I - i$ = current passing through the shunt

R_G = resistance of the galvanometer

r = resistance of the shunt

The voltages across the galvanometer and shunt resistance are equal due to the parallel nature of their connection.

Therefore,

$$i R_G = (I - i) r,$$

$$r = \frac{i}{I-i} R_G \qquad (10.11)$$

The value of r can be obtained using the above equation.

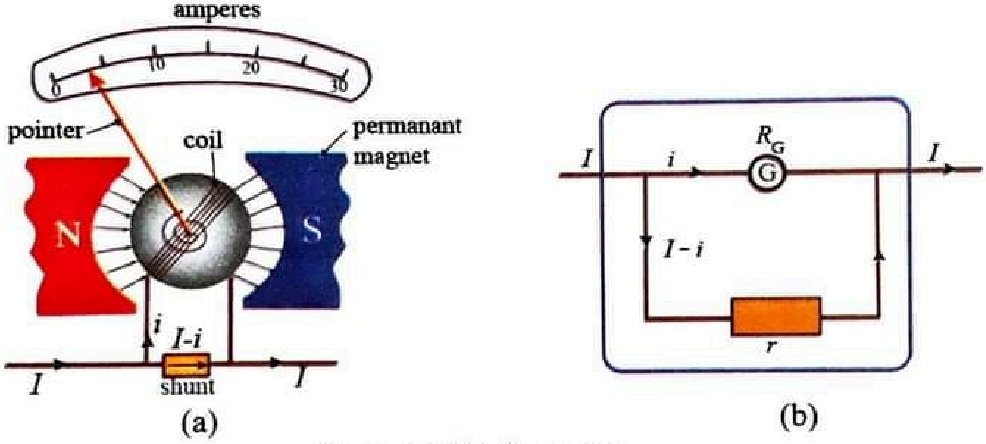


Figure 10.22 Ammeter

Modification of Galvanometer to Voltmeter

A galvanometer is converted into a voltmeter by connecting it in series with high resistance resistor (multiplier resistor). A suitable high resistance is chosen depending on the range of the voltmeter.

In the given circuit as shown Figure 10.23,

V = maximum voltage which can be measured by the voltmeter

i = current which gives full-scale deflection of the galvanometer

R_G = resistance of the galvanometer

R = value of maximum resistance

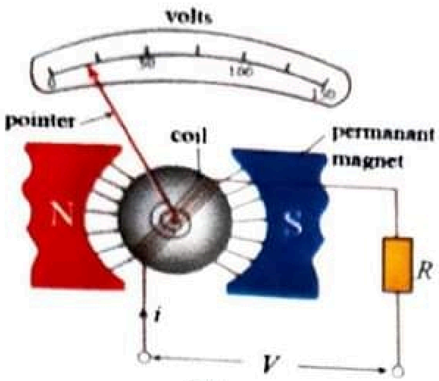
When current i passes through the series combination of the galvanometer and high resistance R, the total voltage drop across the voltmeter is given by

$$V = i R_G + i R$$

$$V = i (R_G + R)$$

$$R = \frac{V}{i} - R_G \qquad (10.12)$$

The value of R can be obtained using the above equation.



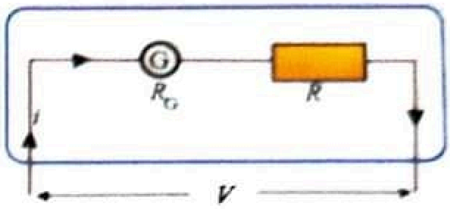


Figure 10.23 Voltmeter

Differences between an Ammeter and a Voltmeter

Ammeter	Voltmeter
(i) Ammeter is a device for measuring current.	(i) Voltmeter is a device for measuring potential difference (voltage).
(ii) Ammeter is a low resistance device.	(ii) Voltmeter is a high resistance device.
(iii) Ammeter is connected in series with circuit component.	(iii) Voltmeter is connected in parallel with circuit component.
(iv) A galvanometer is converted into an ammeter by connecting it in parallel with a wire of low resistance called shunt resistance.	(iv) A galvanometer is converted into a voltmeter by connecting it in series with high resistance resistor.

Example 10.8 A galvanometer has a resistance of 2 Ω and gives a full scale deflection when a current of 1 mA flows through it. How can it be converted for use as (i) an ammeter reading up to 10 A, and (ii) a voltmeter reading up to 50 V?

(i) $R_G = 2\ \Omega$, $i = 1\text{ mA} = 1.0\times10^{-3}\text{ A}$, $I = 10\text{ A}$

Let r be the resistance of the shunt,

$$r = \frac{i}{I-i} R_G$$

$$r = \frac{1.0\times10^{-3}}{10-1.0\times10^{-3}} \times 2 = 2\times10^{-4}\ \Omega$$

The shunt of resistance ($r = 2.0\times10^{-4}\ \Omega$) is to be connected in parallel with coil of galvanometer of resistance R_G.

(ii) $V = 50$ V

Let R be the resistance of the high resistance resistor.

$$R = \frac{V}{i} - R_G$$

$$R = \frac{50}{1\times10^{-3}} - 2$$

$$R = 49998\ \Omega \simeq 50\text{ k}\Omega$$

The high resistance resistor ($R \simeq 50\text{ k}\Omega$) is to be connected in series with coil of galvanometer of resistance R_G.

Reviewed Exercise

- Why is it necessary for the shunt of an ammeter to have a very low resistance?

Key Words: moving coil galvanometer, ammeter, voltmeter, torque, resistance

SUMMARY

The amount of charge passing through a cross-sectional area of a conductor in one second is called an **electric current**.

At a given temperature, the **resistance of a conductor** is the ratio of the potential difference between two ends of a conductor to the current flowing through it.

Resistivity of conductor is defined as the resistance of a conductor having one unit cross-sectional area and one unit length.

A **solenoid** is a cylindrical coil of wire, which has a magnetic field inside and in its vicinity when a current flows through it.

A steel bar is placed inside the solenoid of insulated wire. When a large current flows through the solenoid the steel bar becomes magnetized permanently. Such a magnet is called a **permanent magnet.** If a soft iron bar is placed inside the solenoid of insulated wire and a current flows through it, the bar becomes magnetized. It is demagnetized when the current stops. A soft iron core surrounded by a coil of wire, which acts as a magnet when a current flows through the coil is called a temporary magnet (or) an **electromagnet**.

EXERCISES

1. (a) What is an electric current ? (b) How is an electric current defined? (c) Is an electric current a scalar quantity or a vector quantity? (d) Write down the unit of electric current.

2. (a) State Ohm's law. (b) Using Ohm's law define the resistance of a conductor. (c) What is resistivity of a conductor? Write down the unit of resistivity.

3. A current of 4 A flows through a conductor of resistance 20 Ω for 5 min (i) How much charge will pass through a cross-sectional area of the conductor? (ii) How many electrons will pass through that area?

4. Choose the correct answer from the following.
   An electric iron draws a current of 15 A when connected to a 120 V power source. Its resistance is
   A. 0.125 Ω C. 16 Ω
   B. 8 Ω D. 1800 Ω.

5. When the length of a wire is doubled and its diameter is halved will the resistance of the wire be the same as before?

6. A certain three pieces of copper has length l and cross-sectional area A; l and A, $2l$ and $A/2$, and $l/2$ and $2A$ respectively. Which piece of copper will have minimum resistance? Explain.

7. A copper wire and a silver wire have the same length and the same potential difference across their ends. If the currents through the wires are the same, find the ratio of the radii of the wires. The resistivity of copper is $1.72 \times 10^{-8}\ \Omega$ m and that of silver is $1.62 \times 10^{-8}\ \Omega$ m.

8. A wire of length 100 m is made of silver of resistivity $1.62 \times 10^{-8}\ \Omega$ m, and has a radius of 1 mm. (i) Find the resistance of the wire. (ii) A second wire is made from the same mass of silver but has double the radius. Find its resistance.

9. A wire of 10 Ω is stretched to double its original length. If the resistivity and density of the wire do not change, find its resistance after stretching.

10. If the ratio of the resistances of a tungsten wire at 100 °C and 150 °C is 6/7 what is the temperature coefficient of the wire?

11. A silver wire 2 m long is to have a resistance of 0.5 Ω. What should its diameter be? The resistivity of silver is $1.62 \times 10^{-8}\ \Omega$ m.

12. (a) What resistances can be obtained by using three 1 Ω resistors? (b) When the parallel combination of two resistors having different resistances is connected to a battery, which resistor will draw a greater current?

13. Why is a compass needle placed near a current carrying wire deflected?

14. How will you know which is the north and which is the south pole of a current carrying solenoid?

15. How must a moving coil galvanometer be modified to convert it into a voltmeter?

16. A moving coil galvanometer of resistance 20 Ω gives a full scale deflection when a current of 5 mA passes through it. What modification must be made to it so that it will give a full scale deflection for (i) a current of 1 A and (ii) a potential difference of 100 V?

17. The resistance of a moving coil galvanometer is 25 Ω and the current required for a full scale deflection is 0.02 A. Find the resistance to be used to convert it into (i) an ammeter reading up to 5 A and (ii) a voltmeter reading up to 150 V.

CONCEPT MAP

ELECTRIC CURRENT AND MAGNETIC EFFECT OF ELECTRIC CURRENT

Current and Effect of Electric Current

$I = \frac{Q}{t}, I = \frac{ne}{t}$

Ohm's law and electrical resistance

$V = IR$

$R = \frac{\rho l}{A}$

$R_t = R_0(1 + \alpha t)$

Resistors

- in series

$R = R_1 + R_2 + R_3$

- in parallel

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Magnetic Field due to an Electric Current

- electromagnet

Ammeter and Voltmeter

- ammeter

$r = \frac{i}{I - i} R_G$

- voltmeter

$R = \frac{V}{i} - R_G$

CHAPTER 11

FUNDAMENTALS OF ELECTRONICS

Electronics comprises the physics, engineering, technology and applications that deals with the emission, flow and control of electrons. Most modern appliances use a combination of electronics and electrical circuitry.

The history of electronics began to evolve separately from that of electricity in the late 19^{th} century with the identification of the electron by the English physicist Sir Joseph John Thomson and the measurement of its electric charge by the American physicist Robert A. Millikan in 1909.

Learning Outcomes

It is expected that students will

- learn about pure semiconductors, p-type semiconductors, n-type semiconductors and p-n junction diode.
- explain electronic circuits, forward biased, reversed biased and biased circuit concepts, pnp and npn transistors.
- examine analog and digital signal.
- investigate number systems and the function of electronic logic gates.
- apply basic knowledge of digital electronics.

This chapter introduces semiconductor, semiconductor devices, their applications and also digital electronics.

11.1 SEMICONDUCTORS

Materials which have an electrical resistance that lies between the high resistance values of insulators and the low resistance values of metals are called semiconductors. For example, germanium and silicon are commonly used semiconductors. Gallium arsenide (GaAs) and Cadmium Telluride (CdTe) are used as semiconductors. They are compound semiconductors.

In metals, electrons are the charge carriers. In the case of semiconductors, both electrons and positive holes are the charge carriers. The creation and movement of a positive hole can be explained with the help of the following diagram.

Of the several atoms in a semiconductor, consider three atoms A, B and C as shown in Figure 11.1. Suppose that an electron leaves the atom A as it acquires sufficient energy. The atom A now has a net positive charge which is equal in magnitude to the charge of an electron. A vacancy which is left with the atom A is called a positive hole. An electron from the atom B moves into that hole so that a positive hole is left with B. In this way, a positive hole appears to move the right while the electrons move to the left.

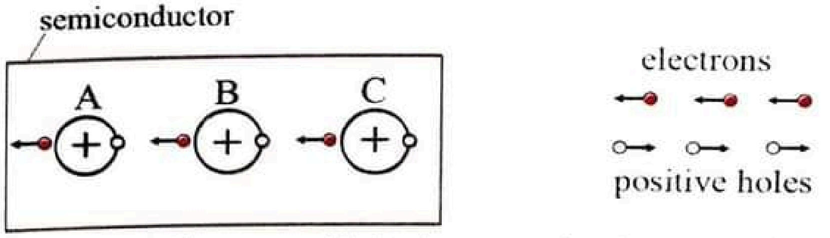


Figure 11.1 Electron and hole charge carriers in semiconductor

When the potential is applied to two ends of a semiconductor, electrons moves from lower potential to higher one, while the positive holes move in the opposite direction.

This means that the current flows due to the movement of electrons and holes in the case of semiconductors.

Pure semiconductors have equal numbers of electrons, and positive holes. Since these are relatively few in number at normal temperature, pure semiconductors have poor conductivity.

When a few impurity atoms are added to the pure semiconductor, its conductivity increases. Arsenic, aluminium and indium atoms are used as impurity atoms. As a result, two types of semiconductor are obtained.

n-type Semiconductor

When arsenic atoms having five valence electrons are added to the pure semiconductor (Ge,Si) atoms having four valence electrons, the conductivity of pure semiconductor increases. Since the number of electrons is greater than that of positive holes in this impure semiconductor it is called an n-type semiconductor ('n' stands for negative). In the n-type semiconductor electrons are the majority charge carriers of electric current.

p-type Semiconductor

When indium atoms having three valence electrons are added to pure semiconductor (Ge,Si) atoms having four valence electrons, the conductivity of pure semiconductor increases. Since the number of positive holes is greater than that of electrons in this impure semiconductor it is called a p-type semiconductor ('p' stands for positive). Thus in a p-type semiconductor that positive holes are the majority charge carriers of electric current.

Example 11.1 What is a positive hole?

An electron leaves the atom as it acquires sufficient energy. A vacancy which is left by an electron in the semiconductor atom is called a positive hole.

Reviewed Exercise

1. How can a pure semiconductor be made to increase its electrical conductivity?
2. Explain how an n-type semiconductor and a p-type semiconductor can be obtained, what are the majority carriers in the above semiconductors?

p-n junction diode

A p-n junction diode is a semiconductor device. It is one type of semiconductor diode.

A boundary or junction is formed between p-type and n-type semiconductors by semiconductor device fabrication process. This junction is called a p-n junction. A device which consists of a p-n junction is called a p-n junction diode. The p-n junction diode, structure of a p-n junction diode and its symbol are shown in Figure 11.2 (a), (b) and (c) respectively.

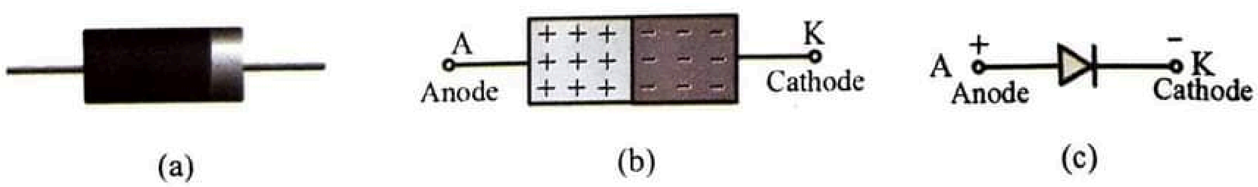


Figure 11.2 (a) The p-n junction diode (b) structure of a p-n junction diode (c) its symbol

Therefore, in the symbol of diode as shown in Figure 11.2 (c), the arrow points from anode to cathode. The current flows from anode to cathode only when the potential of anode is higher than that of cathode. This means that the diode must be forward-biased.

Forward Bias and Reverse Bias

The function of a p-n junction diode can be studied by means of electric circuits shown in Figure 11.3. When a battery is joined with its positive terminal to the p-type semiconductor and its negative terminal to the n-type semiconductor, positive holes from p-type semiconductor pass through the junction easily. Thus a current flows through the p-n junction diode. The p-n junction is now said to be forward-biased [Figure 11.3(a)].

When a battery is joined with its negative terminal to the p-type and its positive terminal to the n-type, only a very small current (reverse current due to minority carrier) flows through the p-n junction diode. In this case the p-n junction is said to be reverse-biased [Figure 11.3(b)].

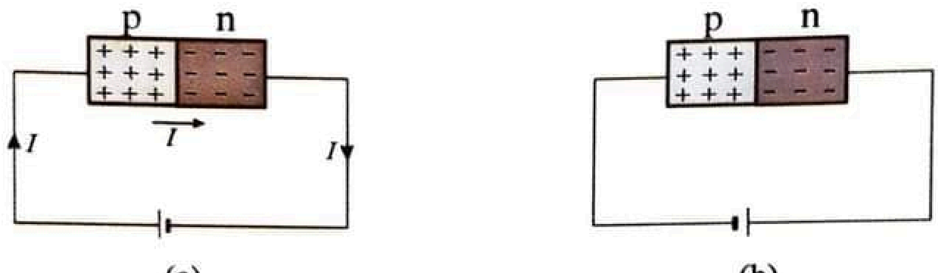


Figure 11.3 A p-n junction diode with (a) forward and (b) reverse bias

The curve of the current *I* against the voltage *V* for a junction diode shown in Figure 11.4 represents the characteristic of that diode. It means that the current flows in one direction only from the anode to the cathode in a vacuum diode and from p to n in a p-n junction diode.

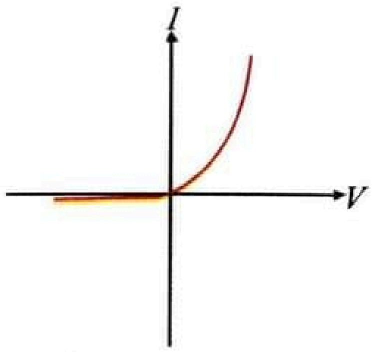


Figure 11.4 Current-voltage characteristic of p-n junction diode

Example 11.2 Are anode and cathode of a p-n junction, not in connection with a battery, at the same electric potential?

Anode and cathode of a p-n junction, not in connection with a battery, are not at the same electric potential because there is initially a potential difference across the juction.

Reviewed Exercise

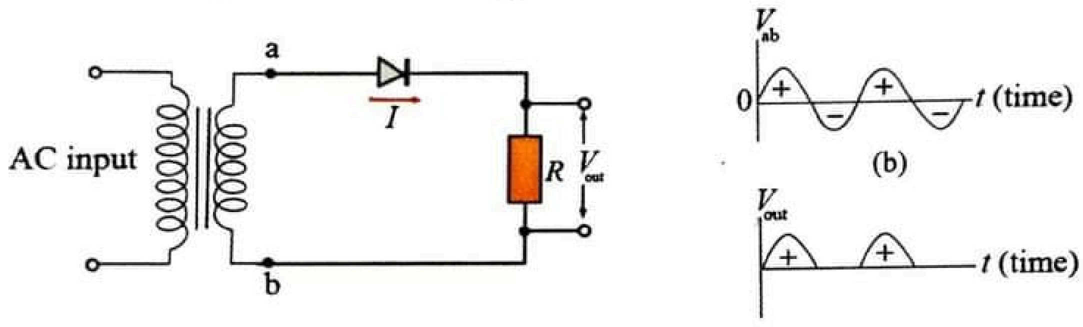
- What are meant by forward biased and reverse biased? Explain these terms using circuit diagrams.

Rectifier

A rectifier is a device which converts an alternating current (AC) into a unidirectional current or a direct current (DC). Diodes can be used as rectifiers because the current flows in one direction only from anode to cathode.

Half-wave Rectifier

The diagram shown in Figure 11.5 (a) is that of a half-wave rectifier. There is only one diode in the circuit. The variation of potential difference V_{ab} between a and b with time is shown in Figure 11.5(b).



(a) (c)

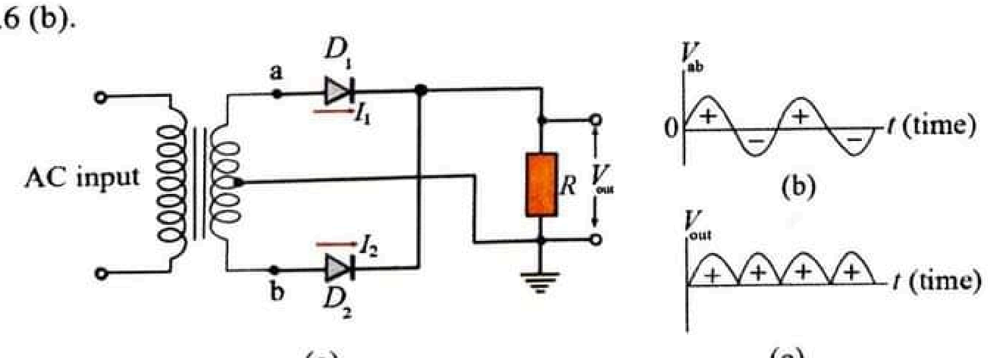
Figure 11.5 Half-wave rectifier

During the first half of the cycle, a is at a higher potential than b, so that a current flows in the circuit. During the second half of the cycle, a is at a lower potential than b, so that no current flows in the circuit. The variation of output voltage V_{out} with the time is shown in Figure 11.5(c).

As the current *I* flows in the diode for every first half of the cycle of AC (or) for every half-wave, this device acts as a half-wave rectifier.

Full-wave Rectifier

The circuit diagram shown in Figure 11.6 (a) is that of a full-wave rectifier. The circuit consists of two diodes. The variation of the potential difference V_{ab} between a and b with time t is shown in Figure 11.6 (b).



(a) (c)

Figure 11.6 Full-wave rectifier

During the first half of the cycle a is at a higher potential than b. Therefore the current I_1 flows in the diode D_1 and no current flows in the diode D_2. During the second half of the cycle b is at a higher potential than a. Therefore the current I_2 flows in D_2 and no current flows in D_1. Since D_1 and D_2 operate alternately for one cycle of an AC voltage, current always flows through the resistor R. This occurs also for other cycle of the output AC voltage. The variation of output voltage V_{out} with time is shown in Figure 11.6 (c). As the current flows through R for both half-cycle of the AC voltage (or) for a full-wave, this device is called a full-wave rectifier.

Rectifiers can also be constructed by using vacuum diodes. But the circuits must be modified properly.

Example 11.3 In what devices is a rectifier used?
The principal application of a rectifier is to convert AC to DC. Most of the electronic devices require DC power for their operation. So rectifiers are used in the power supply units of radio, television, computer, mobile phone charger and so on.

Reviewed Exercise

- Draw the diagrams to show input and output wave forms of a full-wave rectifier.

Key Words: semiconductor, electron, hole, p-n junction diode, forward bias, reverse bias, rectifier

11.2 TRANSISTOR

A transistor is a semiconductor device which works as an amplifier. In 1949, three American physicists Shockley, Bardeen and Brattain invented the transistor. From that time onwards various kinds of electronic equipment which employ transistors were designed and constructed.

A transistor is made of three layers of p- and n-semiconductors. There are two common kinds of transistors called the pnp and npn transistors. In a pnp transistor a thin layer of n-semiconductor is sandwiched between two layers of p-semiconductors. In an npn transistor a thin layer of p-semiconductor is sandwiched between two layers of n-semiconductors. An electrode is attached to each layer and hence there are three electrodes; the emitter (E), the collector (C) and the base (B) in a transistor. The various types of transistor, structure of transistor and its symbol are shown in Figure 11.7 (a), (b) and (c) respectively.

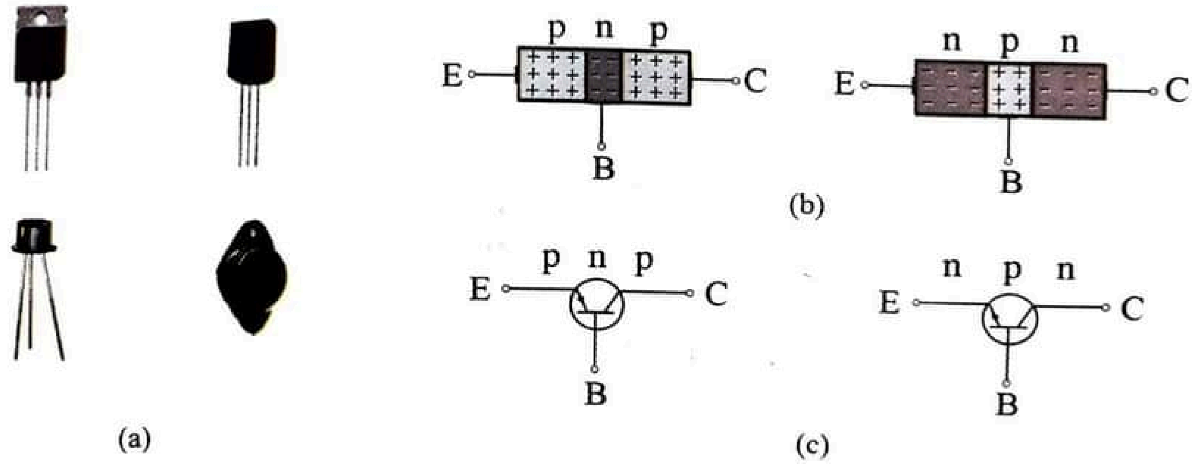


Figure 11.7 (a) Various types of transistor (b) structure of transistor (c) symbol

The advantages of transistors over the vacuum tubes can be summed up as follows:

- They do not deteriorate with time, whereas vacuum tubes do.
- They are physically much more robust than vacuum tubes.
- They waste much less electrical power than vacuum tubes.
- There is no warm-up period after switching on.
- They are very much smaller than vacuum tubes but they perform a similar function.

Transistor Biasing Circuits

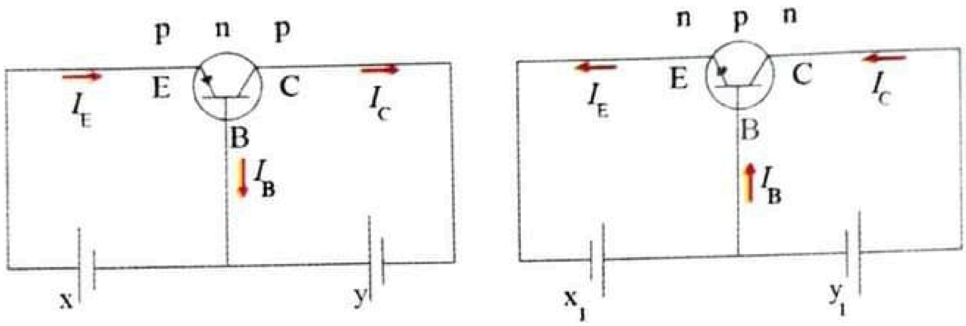


Figure 11.8 Transistor biasing circuits

In Figure 11.8, a current I_C flows in the collector circuit. The remainder of positive holes flows into the base so that the current I_B is obtained. If I_E is a current which flows across the emitter, then

$$I_E = I_C + I_B \quad (11.1)$$

However, the base is so thin that $I_B \sim 0.02\ I_E$ and $I_C \sim 0.98\ I_E$. Therefore, the small base current I_B can control a very large collector current I_C. Because of this property a transistor can be regarded as a current amplifier.

The resistance of forward-biased emitter junction is small and that of reverse-biased collector junction is large. But I_C is nearly equal to I_E. Since the electrical power is I^2R, the power in the emitter side is small whereas the power in the collector side is large. Therefore, the transistor can be regarded as a power amplifier.

Example 11.4 Why is the value of base current I_B very small with comparing I_C and I_E in a transistor biasing circuits?

Because the thickness of the base is about 10^{-3} cm and the base layer of a transistor is thinner than other two layers.

Reviewed Exercise

1. Mention two types of transistor.
2. Explain how a transistor can be used as a current amplifier and as a power amplifier.

Key Words: transistor, pnp transistor, npn transistor, emitter, collector, base

11.3 INTEGRATED CIRCUIT

Scientists have been attempting to make the electronic circuits as well as the components as small as possible. An arrangement whereby connections of electronic components such as resistors, capacitors and transistors are made is called an electronic circuit. The three groups of the electronic circuit are (i) vacuum tube circuit (ii) transistor circuit and (iii) integrated circuit (IC).

In the vacuum tube and transistor circuits it is necessary to connect the separate electronic components to form an electronic circuit. In the integrated circuit, all the electronic components and connections required for an electronic circuit are all made on one single semiconductor crystal (e.g. a silicon crystal). In the integrated circuit, the resistor, capacitor, diode and transistors are made by using various processes. Integrated circuits are used in televisions, computers and advanced electronic devices.

Analog and Digital Electronics

Electronic circuits can be divided into two categories, analog and digital. Analog electronics involves quantities with continuous values, and digital electronics involves quantities with discrete values as shown in Figure 11.9. Both analog and digital electronics are used in the control of various mechanical systems.

Most modern electronic devices such as mobile telephones, television, communication systems, radar, navigation and guidance systems, military systems, medical instrumentation, industrial process control and computers depend on digital techniques and digital electronics.

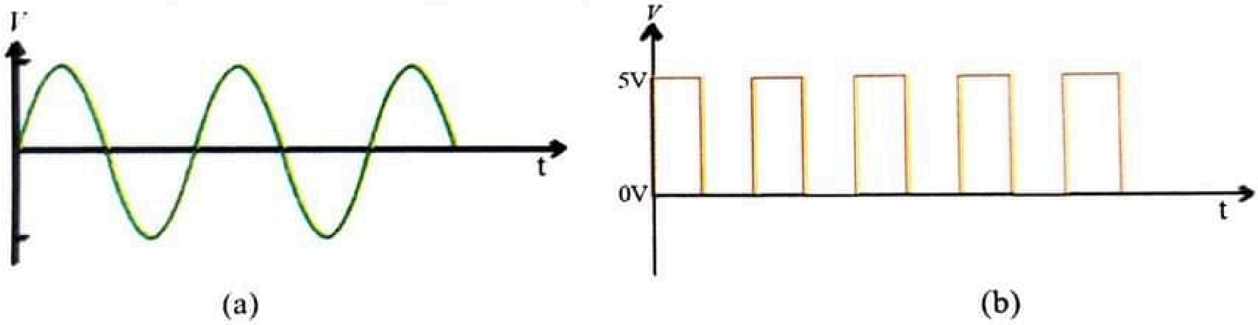


Figure 11.9 (a) Analog waveform (b) digital waveform

Number Systems

Digital electronics involves circuits and systems in which there are only two possible states. These states are represented by two different voltage levels: a HIGH (logic 1) and a LOW (logic 0). The two states can also be represented by current levels. In digital systems such as computers, combinations of the two states, called codes, are used to represent numbers, symbols, alphabetic characters, and other types of information.

The number system is a way to represent or express numbers. There are various types of number systems such as the decimal numbers system and the binary numbers system. In the context of computers and digital devices, the binary number system and digital codes are used.

Decimal Number System (Base 10)

In this number system, the digits 0 to 9 represents numbers. As it uses 10 digits to represent a number, it is also called the base 10 number system. Each digit has a value based on its position called place value. The value of the position increases by 10 times from right to left in the number. The weight is the position of each digit in a decimal number and it is positive powers of ten that increase from right to left, beginning with $10^0 = 1$.

For example, the value of 28 is

$$28_{10} = (2\times10^1) + (8\times10^0) = 20 + 8$$

Binary Number System (Base 2)

The binary number is another way to represent quantities. It is less complicated than the decimal system because it has only two digits. A computer can understand only the "on" and "off" state of a switch. These two states are represented by 1 and 0. The combination of 1 and 0 forms binary numbers.

The relation of binary number and decimal number;
for example, 1001_2 in decimal is

$$\begin{aligned}1001_2 &= (1\times2^3) + (0\times2^2) + (0\times2^1) + (1\times2^0)\\ &= (1\times8) + (0\times4) + (0\times2) + (1\times1)\\ &= 8+0+0+1\\ &= 9_{10}\end{aligned}$$

for example, 12_{10} in binary is
(divided by 2 method),

```
2 | 12
   2 | 6 + 0   LSB
      2 | 3 + 0   ↑
         2 | 1 + 1
             0 + 1MSB
```

$$12_{10} = 1100_2$$

Alternatively,

$$\begin{aligned}12_{10} &= (1\times8) + (1\times4) + (0\times2) + (0\times1)\\ &= (1\times2^3) + (1\times2^2) + (0\times2^1) + (0\times2^0)\\ &= 1100_2\end{aligned}$$

A binary number is a weighted number. The right-most bit is the LSB (least significant bit) in a binary whole number is a weight of $2^0 = 1$. The weight increases from right to left by a power of two for each bit. The left-most bit is the MSB (most significant bit); its weight depends on the size of the binary number.

Table 11.1 Decimal numbers and binary numbers

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

Measurement of Digital Data

Bit and Byte are the units to measure digital data.

Bit

The term 'bit' is abbreviation of the words 'binary' and 'digit'. It is the smallest unit of memory or instruction that can be given or stored on a computer. A bit is either a 0 or a 1.

Byte

A group of 8 bits like 10100000 is a byte (Figure 11.10). Combination of bytes comes with various names like the kilobyte. One kilobyte is a collection of 1000 bytes. A word or letter like 'A' or 'G' is worth 8 bits or one byte. One thousand bytes make up a kilobyte (one thousand letters approximately). 1024 kilobytes (kB) form a megabyte (MB), 1024 megabytes form one gigabyte (GB) and so on.

1 byte = 8 bits
1 kilobyte = 1024 bytes
1 megabyte = 1024 kilobytes
1 gigabyte = 1024 megabytes
1 terabyte = 1024 gigabytes

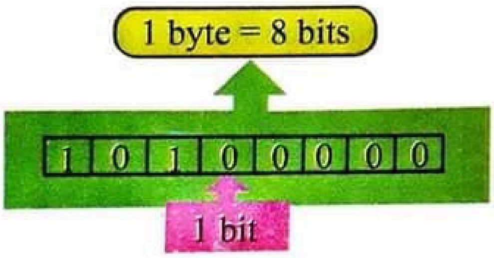


Figure 11.10 Example of a group of 8 bits (1 byte)

Reviewed Exercise

- Convert 75_{10} to its binary number.

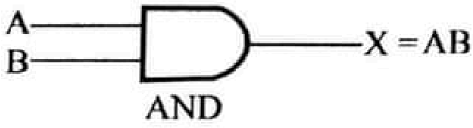
Key Words: decimal number, binary number

11.4 LOGIC GATE

A logic gate is an electronic device which takes an information from the environment, makes a decision based on that information and then gives out the result. Different types of logic gates can be built from different arrangements of electronic components. Logic gates can be built up using discrete components of resistors, diodes and transistors. However, it is more convenient to use small integrated circuits (ICs).

The five common logic gates are AND, OR, NAND, NOR and NOT (inverter) gates. Logic gate can have two or more inputs but only one output. But, NOT gate has only one input.

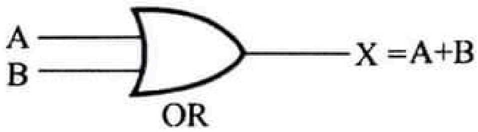
For a 2-input AND gate, output X is HIGH only when inputs A and B are HIGH; X is LOW when either A or B is LOW, or both A and B are LOW.



Input		Output
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Figure 11.11 The symbol and truth table for a 2-input AND gate

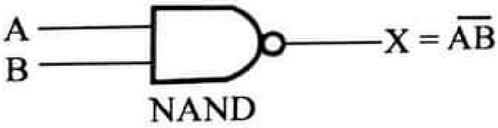
For a 2-input OR gate, output X is HIGH when either input A or input B is HIGH, or both A and B are HIGH; X is LOW only when both A and B are LOW.



Input		Output
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Figure 11.12 The symbol and truth table for a 2-input OR gate

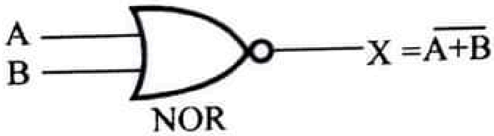
For a 2-input NAND gate, output X is LOW only when inputs A and B are HIGH; X is HIGH when either A or B is LOW, or both A and B are LOW.



Input		Output
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Figure 11.13 The symbol and truth table for a 2-input NAND gate

For a 2-input NOR gate, output X is LOW when either input A or input B is HIGH, or both A and B are HIGH; X is HIGH only when both A and B are LOW.

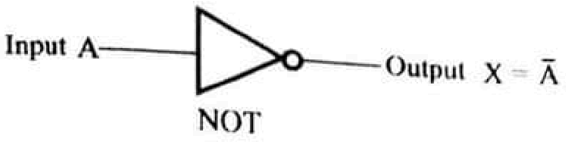


Input		Output
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Figure 11.14 The symbol and truth table for a 2-input NOR gate

The NAND gate and NOR gate are popular logic gates because they can be used as universal gates, that is, they alone can be used in combination to perform the functions of any other types of gates.

The inverter (NOT gate) performs the operation called inversion (or) complementation. The inverter changes one logic level to the opposite level. In terms of bits, it changes 1 to 0 and 0 to 1. When a HIGH level is applied to an inverter input, a LOW level will appear on its output. When a LOW level is applied to its input, a HIGH level will appear on its output.



Input A	Output X
1	0
0	1

Figure 11.15 The symbol and truth table for NOT (inverter) gate

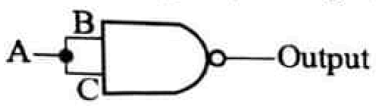
Table 11.2 Boolean expressions for five common logic gates

Logic gates	Boolean expressions
AND gate	$X = AB$
OR gate	$X = A + B$
NAND gate	$X = \overline{AB}$
NOR gate	$X = \overline{A + B}$
NOT gate	$X = \overline{A}$

Combination of Logic Gates

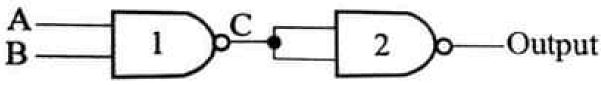
Logic gates can be combined to have necessary functions. The followings show the combination of NAND gates to form other four types of gate.

One NAND gate (NOT gate)



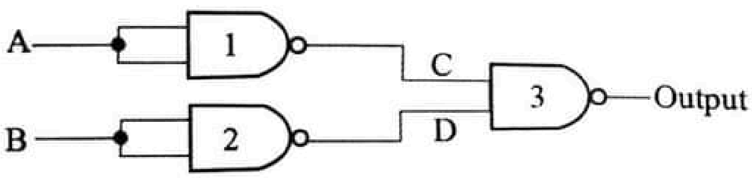
A	B	C	Output
0	0	0	1
1	1	1	0

Two NAND gates (AND gate)



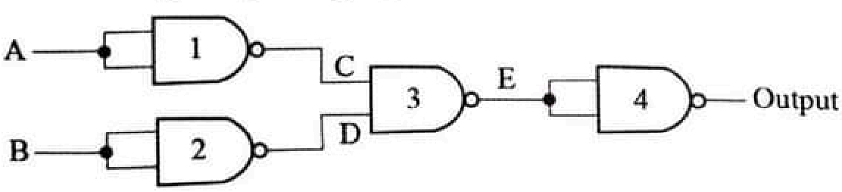
A	B	C	Output
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

Three NAND gates (OR gate)



A	B	C	D	Output
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

Four NAND gates (NOR gate)



A	B	C	D	E	Output
0	0	1	1	0	1
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	1	0

Figure 11.16 Combination of NAND gates to form other four types of gate

Example 11.5 Convert binary 110100_2 to its decimal equivalent.

$$110100_2 = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$
$$= 32 + 16 + 0 + 4 + 0 + 0$$
$$= 52_{10}$$
$$110100_2 = 52_{10}$$

Example 11.6 Converting binary fraction $(111011.101)_2$ to its equivalent decimal fraction.

$$111011.101_2 = (1\times2^5) + (1\times2^4) + (1\times2^3) + (0\times2^2) + (1\times2^1) + (1\times2^0) + (1\times2^{-1}) + (0\times2^{-2}) + (1\times2^{-3})$$
$$= (32 + 16 + 8 + 0 + 2 + 1) + (0.5 + 0 + 0.125)$$
$$= 59.625_{10}$$
$$111011.101_2 = 59.625_{10}$$

Reviewed Exercise

- A basic 2-input logic circuit has a HIGH on one input and a LOW on the other input, and the output is HIGH. What type of logic is it?

Key Words: integrated circuit, logic gate

SUMMARY

Materials which have an electrical resistance that lies between the high resistance values of insulators and the low resistance values of metals are called **semiconductors.**

A boundary or junction is formed between p- and n-type semiconductors by semiconductor device fabrication process. This junction is called a p-n junction. A device which consists of a p-n junction is called a **p-n junction diode**.

A **rectifier** is a device which converts an alternating current (AC) into a unidirectional current or a direct current (DC).

A **transistor** is a semiconductor device which works as an amplifier.

An arrangement whereby connections of electronic components such as resistors, capacitors and transistors are made is called an **electronic circuit.**

In the **integrated circuit**, all the electronic components and connections required for an electronic circuit are all made on one single semiconductor crystal (e.g. a silicon crystal).

The number system is having digit 0, 1, 2, 3, 4. 5. 6. 7. 8. 9; this number system is known as a **decimal number system** because total ten digits are involved. The base of the decimal number system is 10.

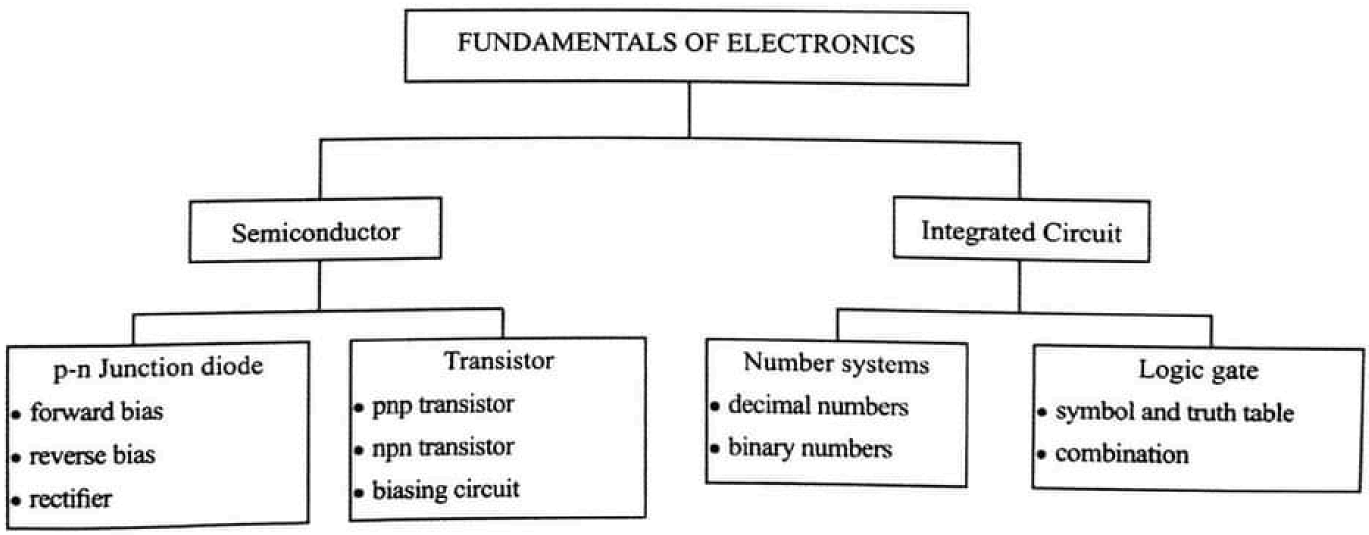
The base of **binary number system** is 2 because it has two digit 0 and 1.

A **logic gate** is an electronic device which takes in information from the environment, makes a decision based on that information and then gives out the result.

EXERCISES

1. Describe the construction of a p-n junction diode.
2. Distinguish between half-wave and full-wave rectification.
3. Why does a pure semiconductor at normal temperature have poor conductivity?
4. How can the pure semiconductor be made to increase its conductivity?
5. Why can diodes be used as rectifiers?
6. How many electrodes are there in a transistor? What are they called?
7. Convert decimal fraction 12.75_{10} to its equivalent binary fraction.
8. Define the sequence of bits (1s and 0s) represented by each of the following sequences of levels:
   (i) HIGH, HIGH, LOW, HIGH, LOW, LOW, LOW, HIGH.
   (ii) LOW, LOW, LOW, HIGH, LOW, HIGH, LOW, HIGH, LOW.
9. A logic circuit requires HIGHs on all its inputs to make the output HIGH. What type of logic is it?
10. Draw the circuit diagram and give truth table to illustrate how an OR gate can be obtained by using three NAND gates.

CONCEPT MAP



CHAPTER 12
MORDERN PHYSICS

At the tail end of the nineteenth century, physics was considered by many physicists to be a complete science. This illusion of a complete science is the man's lack of experience with atomic size particles and with objects that move at nearly the speed of light. However, in the space of just few years, scientists discover new experimental observations and phenomena concerning with extreme conditions, such as high velocities that are comparable to the speed of light and small distances comparable to the atomic radius. These findings lead to the development of Modern physics which is based on drastic assumptions and concepts that had no historical precedents.
In this chapter we will introduce the discovery of cathode rays, models of the atom, photoelectric effect, quantum theory and some other topics of modern physics.

Learning Outcomes

It is expected that students will

- explain the production and use of cathode rays and the function of cathode ray tube.
- describe the nature of X-rays, how we use them and the dangers associated with the use of X-rays.
- study the quantum concept and particle nature of wave.
- explain the hydrogen spectrum by applying Bohr's atomic model.
- analyse the photoelectric effect and the photon concept.
- explain the structure and roles of isotopes.
- investigate redshift, Hubble's law and the age of the universe.

12.1 CATHODE RAYS

Cathode rays are fast moving electrons emitted from the cathode of a discharge tube. They were discovered while studying the electric discharge through gases at low pressure. A glass tube is equipped with two electrodes (cathode and anode) and a high voltage is applied while the gas inside is pumped out gradually as shown in Figure 12.1. Electric discharge first occurs at about 20 mm Hg. As the pressure is reduced further, at very low pressure (~ 0.01 mm Hg) glass behind the positive electrode (anode) is observed to glow. This is due to the rays (later found to be composed of electrons) emitted from the cathode striking the walls of the glass tube. Cathode rays were first observed in 1869 by Julius Plucker and Johann Hittorf, and were named in 1876 by Eugen Goldstein as cathode rays.

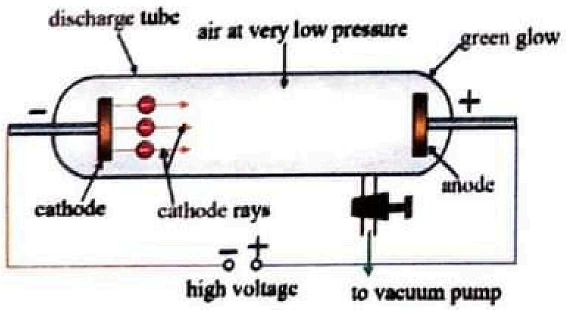
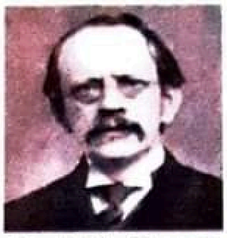


Figure 12.1 Production of cathode rays

In 1897, British physicist Joseph John Thomson showed that cathode rays composed of a previously unknown negatively charged particle, which was later named the electron. He measured the charge to mass ratio (e/m) of electron and found to be -1.759×10^{11} C kg^{-1}. Combining with the charge of the electron -1.602×10^{-19} C (measured by Robert Andrews Millikan in 1906), the mass of the electron is 9.107×10^{-31} kg. Both Thomson and Millikan were honoured with Nobel Prize in Physics.


Joseph John Thomson

Properties of cathode rays

From the experiment with cathode rays it is found that

- cathode rays travel in straight lines (Figure 12.2),
- cathode rays have momentum and kinetic energy (Figure 12.3),
- cathode rays can be deflected by both electric and magnetic fields (Figure 12.4),
- cathode rays can ionize the gases through which they pass, and
- cathode rays can produce fluorescence.

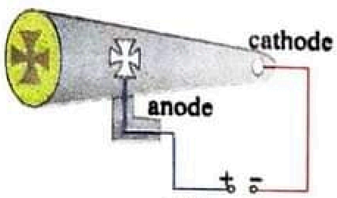


Figure 12.2 Cathode rays travel in straight line

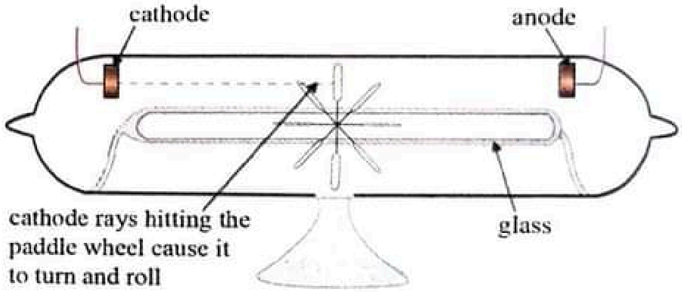


Figure 12.3 Cathode rays have momentum and kinetic energy

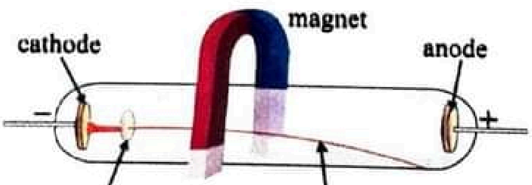


Figure 12.4 Cathode rays can be deflected by a magnetic field

Cathode Ray Tubes

The cathode ray tube (CRT) is a vacuum tube that contains an electron gun, a deflection system and a fluorescent screen, and is used to display images as shown in Figure 12.5. It accelerates the electrons emitted from the cathode, deflects the electron beam and focused onto the fluorescent screen to create the images. Before late 2000 AD, CRTs are widely used in cathode ray oscilloscopes (CROs), television sets and computer monitors.

Since then CRTs have been largely superseded by newer flat-panel display technologies such as liquid crystal display (LCD), plasma display, and organic light-emitting diode (OLED) displays, which have lower manufacturing costs and power consumption, as well as significantly less weight and bulk. Flat-panel displays can also be made in very large sizes; whereas 38 in to 40 in (97 cm to 102 cm) was about the largest size of a CRT television, flat panels are available in 85 in and even larger sizes.

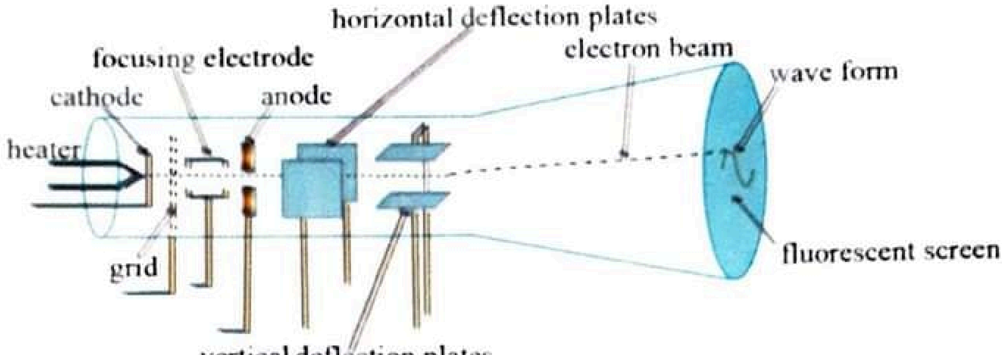


Figure 12.5 Cathode ray tube (CRT)

Example 12.1 What will happen to the cathode ray when it passes through the electric field?

When the cathode ray passes through the electric field, it experiences the electric force by electric field. Therefore its path is deflected from original direction.

Example 12.2 Are cathode rays visible?

Cathode rays are invisible. However, when cathode rays struck the glass wall of a tube or a fluorescent screen they can cause a glow called fluorescence.

Reviewed Exercise

1. How can it be known that cathode rays are electrically charged particles?
2. Name the instruments which employ cathode ray tube.

Key Words: cathode rays, electron, cathode ray tube, vacuum pump

12.2 X-RAYS

Wilhelm Conrad Röentgen

X-rays are electromagnetic waves like light, but their wavelengths are much shorter than those of light. Wilhelm Conrad Röentgen discovered X-rays in 1895 while experimenting with cathode rays. He accidentally found a penetrating radiation which would pass through most substances but leave shadows of solid objects. Because he did not know what the rays were, he called them X-rays, meaning unknown rays.

Later experiments revealed that X-rays are electromagnetic waves which have a wavelength in the range of 0.01 to 10 nanometre (nm). They are shorter in wavelength than ultraviolet (UV) rays and longer than gamma rays.

X-rays of energy about 0.1 to 10 keV (wavelength 10 to 0.10 nm) are classified as soft X-rays, and of energy about 10 to 100 keV (wavelength 0.10 to 0.01 nm) as hard X-rays, due to their penetrating abilities. (1 electron volt (eV) = 1.602×10^{-19} J is an energy unit used in atomic and nuclear physics)

X-ray Tube

An X-ray tube consists of a cathode and a target (anode) as shown in Figure 12.6. A high potential difference (about 10-100 kV) is applied between target and cathode to accelerate electrons emitted from the cathode. X-rays are produced when high-energy electrons strike the target which is made of a high dense material such as tungsten.

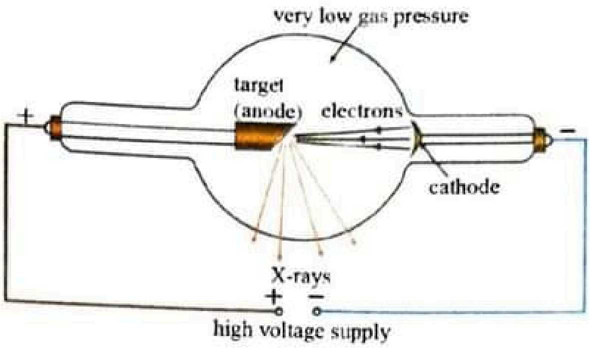


Figure 12.6 X-ray tube

Properties of X-rays

- X-rays are electromagnetic waves. They have no charge, cannot be deflected by both electric and magnetic fields.
- They travel through vacuum with the speed of light.
- They have high penetrating power. Hence, thick dense shielding, such as high density concrete or lead is necessary to protect against X-rays.
- Soft tissue (like organs and skin) cannot absorb the X-rays; however high density materials (such as bone) absorb X-rays.
- X-rays can cause ionization by stripping electrons from the atoms and can cause biological effects.
- X-rays consist of high energy photons which are energy packets of electromagnetic wave.

Uses of X-rays

- X-rays have wide medical applications. For example, soft X-rays (low penetrating power) are used to take X-ray photographs of some parts of human body (Figure 12.7). Hard X-rays (high penetrating power) are used to destroy cancer cells.
- X-rays are used in industries for finding defects in welded joints and metal castings.
- Since X-rays can exhibit wave nature, they are used to determine the structure of crystals.
- X-rays are also used in security systems to reveal hidden unlawful materials (such as weapons and drugs).

Figure 12.7 X-ray photograph of a fractured bone

Two types of X-rays

X-rays can be classified as two types: (i) characteristic X-rays (intense sharp lines) and (ii) continuous X-rays (or) white X-rays.

(i) Characteristic X-rays: When a high energy electron emitted from the cathode bombard the target material, it may knock an electron in the inner-shell of an atom completely out of its orbit. Hence, a vacancy is created. When outer-shell electrons, having higher energy, fill the vacancy in the inner shell, the energy is released in the form of X-rays. These X-rays are called characteristic X-rays because they have specific energies characteristic of the element used as the target. These discrete energy of characteristic X-rays can be explained based on Bohr's atomic model as shown in Figure 12.9.

(ii) Continuous X-rays: When an electron striking the target passes through the atoms of the target material, it will suffer a retarding force in the Coulomb field of the nuclei of the target.

When this occurs, the electron will lose some of its kinetic energy and the energy is released in the form of X-rays. The energy of X-ray photons can take a value from zero to the maximum kinetic energy of the incident electrons. These X-rays have continuous distribution of wavelengths; hence, bear the name continuous X-rays (or) white X-rays. Continuous X-rays are also called bremsstrahlung radiation (braking radiation) according to the phenomenon they are produced.

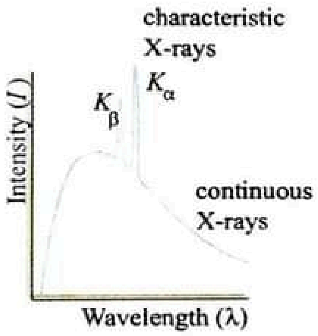


Figure 12.8 A continuous X-ray spectrum with two characteristic lines superimposed

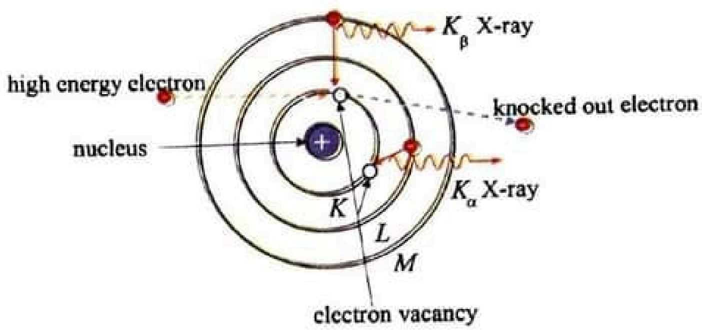


Figure 12.9 Emission of characteristic X-rays

Example 12.3 In which approximate wavelength region of the electromagnetic spectrum do X-rays lie? Find the maximum energy of X-ray photon.

X-rays are electromagnetic waves which have a wavelength in the range of 10 to 0.01 nm approximately.

$h = 6.626 \times 10^{-34}$ Js

Photon energy $E = h\nu = \frac{hc}{\lambda}$

For maximum energy, $\lambda = 0.01$ nm $= 0.01 \times 10^{-9}$ m $= 1 \times 10^{-11}$ m

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{10^{-11}} = 1.99 \times 10^{-14}\text{ J}$$

$$= \frac{1.99 \times 10^{-14}}{1.6 \times 10^{-19}} = 1.24 \times 10^{5}\text{ eV}$$

Example 12.4 How are X-rays produced?

X-rays are commonly produced in X-ray tubes by accelerating electrons through a potential difference (a voltage drop) and directing them onto a target material (tungsten).

Reviewed Exercise

1. Mention the uses of X-rays.
2. State similarity and difference between light and X-rays.

Key Words: X-rays, electron, X-ray tube, characteristic X-rays, continuous X-rays, energy packet, discrete energy, continuous energy

12.3 BOHR'S HYDROGEN ATOM AND ATOMIC SPECTRA

Bohr's Atomic Model

Rutherford's model states that an atom has a central nucleus and an electron or electrons revolve around it. However, according to electromagnetic theory, accelerating charged particles, i.e., the orbiting electrons will emit radiation and finally they will collapse into the nucleus.

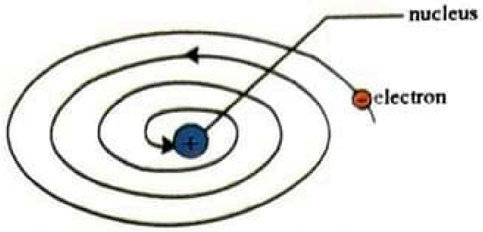


Figure 12.10 Drawback of Rutherford's atomic model

In order to overcome this drawback of Rutherford's atomic model, Niels Bohr proposed a new model.

Postulate I

Bohr postulated that in an atom, an electron can move only in allowed circular orbits. If an electron is moving in such an orbit, it does not absorb (or) emit radiation. The atom can exist in certain stable states with a definite total energy.

Postulate II

An electron revolves around the nucleus in circular orbits with angular momentum L which is an integral multiple of $\hbar$, (i.e, $L = n\hbar$).

where $\hbar = \dfrac{h}{2\pi}$, h = Planck's constant and n = 1, 2, 3, ... integer numbers.

This postulate is the concept of angular momentum quantization.

Postulate III

An electron can undergo transition from an orbit to another by absorbing (or) emitting energy which is equal to the energy difference between the two states (Figure 12.11).

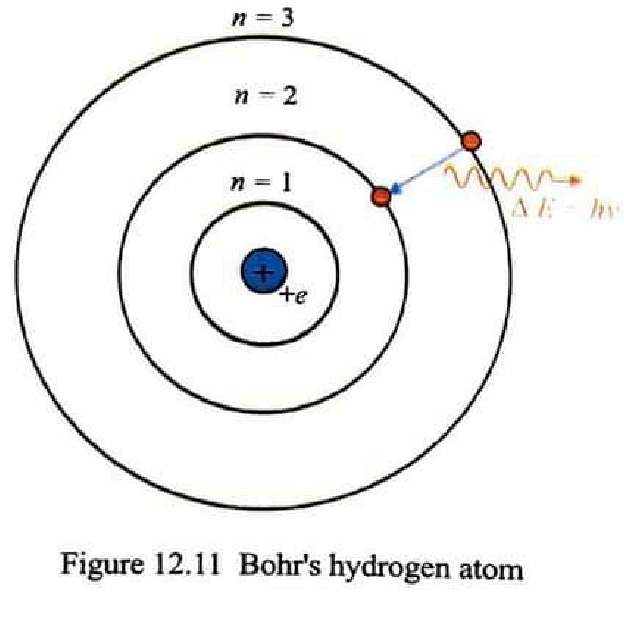


Figure 12.11 Bohr's hydrogen atom

Energy and Radius of the Atomic States

An electron of charge *e* and mass *m* moving in a circular orbit of radius *r* experiences the centripetal force balanced by the attractive Coulomb force between electron and proton (nucleus).

In electrostatic unit (esu)

$$\frac{mv^2}{r} = \frac{e^2}{r^2} \qquad (12.1)$$

According to the Bohr's postulate II, $L = mvr = n\hbar$ (12.2)

From Eq. (12.1) and (12.2) we get

$$r = \frac{n^2\hbar^2}{me^2} \qquad (12.3)$$

The total energy of hydrogen atom is

E = kinetic energy KE + potential energy PE.

$$E = \frac{1}{2}mv^2 - \frac{e^2}{r}$$

$$E = -\frac{1}{2}\frac{me^4}{n^2\hbar^2} \qquad (12.4)$$

According to Eq. (12.4), all the energy values are negative which means electron is bound to the nucleus.

Here, we would introduce an important constant, which is fine structure constant α (a dimensionless constant).

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$$

where $\hbar c = 197.3$ MeV fm (or) 197.3×10^6 eV fm

The ground state energy of hydrogen atom with $n = 1$ could be obtained as follows, where mass of electron must be given in MeV/c^2 (or) eV/c^2 units and radius r in fm.

mass of electron m = 0.511 MeV/c^2 (or) 0.511×10^6 eV/c^2

fine structure constant $\alpha = \frac{1}{137}$

Eq. (12.4) is rewritten as

$$E = -\frac{1}{2}\frac{me^4}{n^2\hbar^2} = -\frac{1}{2}\frac{mc^2e^4}{n^2\hbar^2c^2} = -\frac{1}{2}\frac{mc^2}{n^2}\alpha^2$$

Energy of hydrogen atom in the n[th] state $E = -\frac{13.6}{n^2}$ eV $(n = 1, 2, 3, \ldots)$

Radius of hydrogen atom $r = \frac{n^2\hbar^2}{me^2}$ = 52 896 n^2 m = 0.528 96 n^2 Å

For $n = 1$, the radius of hydrogen atom is called Bohr's radius (r_B),

$$r_B = 0.528\ 96 \text{ Å}$$

Hydrogen Spectrum

A hydrogen discharge tube is a slim tube containing hydrogen gas at low pressure with an electrode at each end. If a high voltage is applied, the tube lights up with a bright pink glow. If the light is passed through a prism, it is split into various colors as shown in Figure 12.12.

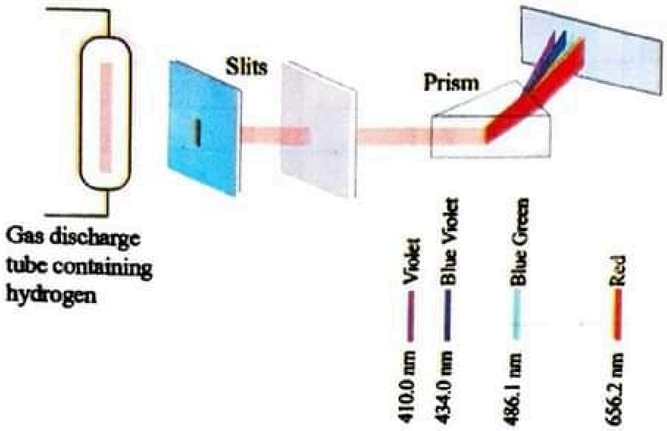


Figure 12.12 Hydrogen atomic spectrum [CREDIT: Source from the Internet]

Bohr's model explains the spectral lines of the hydrogen atomic emission spectrum. When the atom gains energy, the electron jumps from the ground state to another state with higher energy. Energy levels are designated with an integer n which is called principal quantum number. The ground state is $n = 1$, the first excited state is $n = 2$, and so on.

$$E = -\frac{13.6}{n^2}\ \text{eV} \qquad (12.5)$$

The energy gained by the atom is equal to the difference in energy between the two energy levels.

When the electron returns back from the higher energy state to a lower energy state, it releases energy that is equal to the difference in energy of the two orbits. The energy ΔE is released in the form of an electromagnetic radiation with frequency v.

Radiated energy $\Delta E = h\nu = \frac{hc}{\lambda} = E_{n_i} - E_{n_f}$, and $n_i > n_f$

where E_{n_i} and E_{n_f} are energies of initial state and final state, respectively.

ν and λ are the frequency and the wavelength of radiation, propagating with the velocity of light c.

As the energy increases further and further from the nucleus, the spacing between the levels gets smaller and smaller, see Figure 12.13. Table 12.1 expresses the principal quantum number and energy of hydrogen atom.

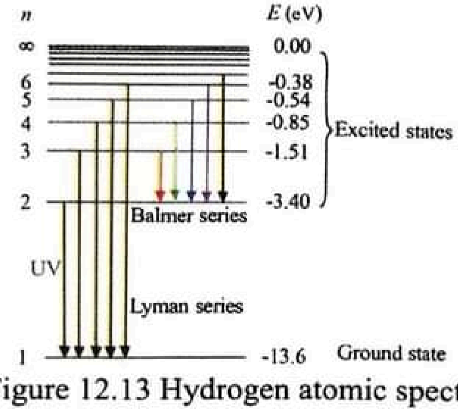


Figure 12.13 Hydrogen atomic spectrum

Table 12.1 Energy of a hydrogen atom

Principal quantum number (n)	Energy $E=-\frac{13.6}{n^2}$ eV
1	-13.6
2	-3.4
3	-1.51
4	-0.85
5	-0.54
. .	. .
∞	0

In the emission of X-rays, the energy emitted due to electron transition from $n = 2$ energy state to $n = 1$ energy state is K_α characteristic X-rays, whereas a transition from $n = 3$ energy state to $n = 1$ energy state is K_β characteristic X-rays.

Example 12.5 A photon is emitted when a hydrogen atom undergoes a transition from the $n = 3$ state to the $n = 1$ state. Find the energy of emitted photon in electron volt, its frequency and wavelength. ($h = 6.626 \times 10^{-34}$ J s)

$n_i = 3$, $n_f = 1$, $h = 6.626 \times 10^{-34}$ J s

$$E_{n_i} = -\frac{13.6}{n_i^2} = -\frac{13.6}{3^2} = -1.51 \text{ eV}$$

$$E_{n_f} = -\frac{13.6}{n_f^2} = -\frac{13.6}{1^2} = -13.6 \text{ eV}$$

energy of photon $E = E_{n_i} - E_{n_f} = -1.51 - (-13.6) \text{ eV} = 12.09 \text{ eV}$

$$E = h\nu$$

$$\nu = \frac{E}{h} = \frac{12.09 \times 1.6 \times 10^{-19}\,\text{J}}{6.626 \times 10^{-34}\,\text{J s}} = 2.91 \times 10^{15}\,\text{Hz}$$

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2.91 \times 10^{15}} = 1.03 \times 10^{-7}\,\text{m}$$

Reviewed Exercise

1. Why does an electron moving around the nucleus not fall into the nucleus?
2. When does an atom emit the radiation?

Key Words: nucleus, electron, photon, electromagnetic radiation

12.4 PHOTOELECTRIC EFFECT AND PHOTON CONCEPT

Photoelectric Effect

When electromagnetic radiation (e.g. light) of sufficient energy incident on the metal surface then electrons are ejected from its surface. This phenomenon is known as photoelectric effect. It was observed that only certain frequencies of light are able to cause the ejection of electrons. These ejected electrons are called photoelectrons. Figure 12.14 illustrates the emission of photoelectrons as a result of the photoelectric effect.

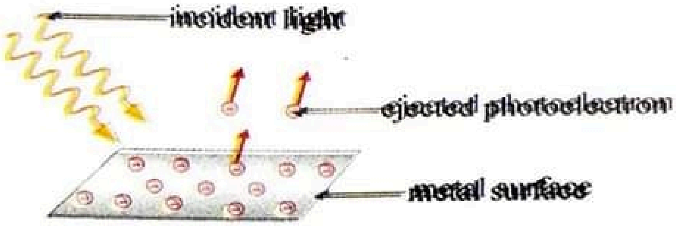


Figure 12.14 Illustration of photoelectric effect

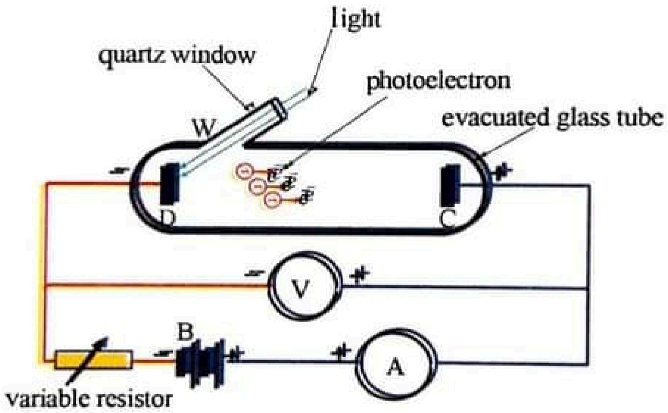


Figure 12.15 Experimental setup of photoelectric effect

Figure 12.15 shows experimental setup of photoelectric effect. In an evacuated glass tube, two zinc plates C and D are enclosed. Plate C acts as an anode and D acts as a photosensitive plate. Two plates are connected to a battery B and ammeter A. If the radiation (light) is incident on the plate D through a quartz window W, electrons are ejected out of the plate and current flows in the circuit which is known as photocurrent. Plate C can be maintained at desired potential (+ve or –ve) with respect to plate D.

It is observed that

- There are no electrons emitted below a certain frequency of incident light which is different for different metals. This frequency is called the threshold frequency ν_0.
- Photoelectric current is directly proportional to intensity of the incident light.
- The kinetic energy of photoelectrons is directly proportional to frequency of the incident light.

The Concept of Photons

The photoelectric effect cannot be explained by considering light as a wave. However, this phenomenon can be explained by the particle nature of light, in which light can be visualized as a stream of particles of electromagnetic energy. These particles are called photons. The energy of a photon is related to the frequency of light as,

$$E = h\nu = \frac{hc}{\lambda}$$

where E is the energy of the photon, h is Planck's constant, ν is the frequency of light, c is the speed of light (in a vacuum) and λ is the wavelength of the light. The above relation is known as Planck's equation.

The most important consequence of photoelectric effect leads to the concept of wave-particle dual nature of either particles or waves. That means light behaves sometimes as a particle and the other times behaves like a wave.

Example 12.6 Light of wavelength 550 nm is incident on a surface of metal. Find the energy of photon in light.

$\lambda = 550\ \text{nm} = 550 \times 10^{-9}\ \text{m}$

energy of photon $E = h\nu = \frac{hc}{\lambda}$

$$E = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} = 0.036\times10^{-17}\,\text{J}$$

$$E = \frac{0.036\times10^{-17}}{1.6\times10^{-19}} = 2.25\ \text{eV}$$

Reviewed Exercise

1. Why does a photon has no electric charge?
2. How does photoelectric current depend on intensity of the incident light?

Key Words: nucleus, photoelectron, photon, intensity, threshold frequency

12.5 ISOTOPES

Isotopes are the atoms of the same element with different masses. That is, they have the same number of protons but have different number of neutrons. In other words, isotopes have the same atomic number but have different mass number. Since isotopes of an element have same number of electrons, they have same electron configuration and thereby have the same chemical properties. They have different physical properties as they vary in their atomic mass. Some isotopes are stable, that is, they do not decay (or transform) into other elements.

However, some are unstable (or radioactive), that is, decay into other elements. Stable isotopes and some radioactive isotopes (radioisotopes) exist naturally. Isotopes can also be produced artificially using nuclear reactors and particle accelerators.

Every chemical element has one or more isotopes. For example, copper has two stable isotopes: $^{63}_{29}Cu$ and $^{65}_{29}Cu$. $^{63}_{29}Cu$ have 29 protons, 29 electrons and 34 neutrons; whereas, $^{65}_{29}Cu$ have 29 protons, 29 electrons and 36 neutrons. In nature, stable isotopes of an element occur in different abundance. For instance, of all the stable atoms of copper, 69% abundance is $^{63}_{29}Cu$ and 31% is $^{65}_{29}Cu$. Three isotopes of hydrogen are: $^{1}_{1}H$ (hydrogen), $^{2}_{1}H$ (deuterium), and $^{3}_{1}H$ (tritium). Tritium is radioactive. It transforms into helium isotope $^{3}_{2}He$. Of over 1000 isotopes known thus far, the most abundant one in the entire universe is the hydrogen isotope $^{1}_{1}H$.

Reviewed Exercise

- What are the mass numbers and atomic numbers of the following elements?

  (i) $^{206}_{82}Pb$ (ii) $^{235}_{92}U$ (iii) $^{238}_{92}U$

Key Words: atomic number, mass number, isotopes, physical properties, chemical properties

12.6 REDSHIFT, HUBBLE'S LAW AND AGE OF THE UNIVERSE

In studying cosmology, we get the information of celestial objects by the light they emit. Their wavelengths are measured to obtain the astronomical spectra. From these information, we get distance, velocity, temperature, luminosity, energy (or) power of these celestial objects. There are billions of stars (suns), clusters and galaxies in the universe.

Redshift

When a source of light waves is travelling away from the observer, the observed wavelength will be slightly longer than the wavelength of the stationary (rest) source. In astronomy, this shift toward longer wavelengths is referred to as a redshift since red is at the long-wavelength end of the visible spectrum. Conversely, a shift toward shorter wavelength, which occurs when the source is moving toward the observer, is called a blueshift.

The main reasons for redshift are as follows.

- Doppler effect i.e., the movement of objects either closer or apart from each other in space.
- Strong gravitational force leads to gravitational redshift.
- Cosmological redshift is such that light stretches (longer wavelength) as space expands.

Hubble's Law

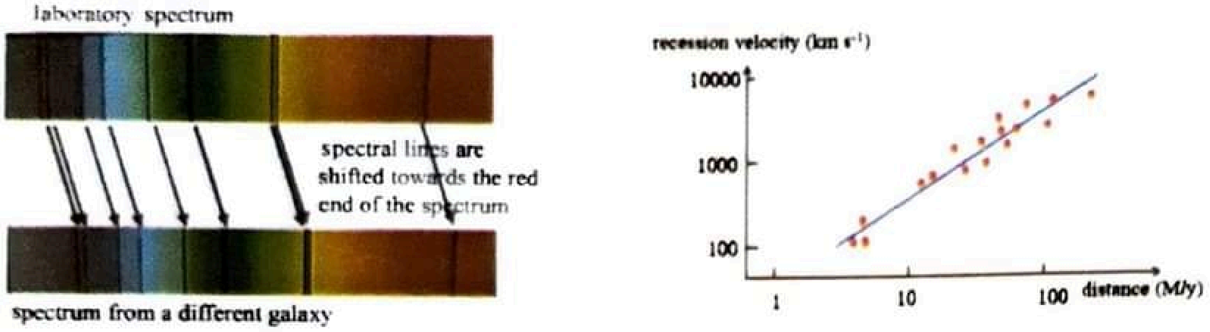
In 1929, Edwin Hubble examined a series of velocity measurements for different galaxies. The distances to the galaxies had also been measured. He noticed that the farther away the galaxy, the faster it moved, and that the relationship was linear: a galaxy twice as far away as another will move twice as fast, and a galaxy five times as far away will move five times as fast. This relationship between the velocity of a galaxy and its distance has come to be known as the Hubble law. It provided the first evidence that the Universe is expanding, a result that was rather startling at that time, as it had long been assumed that the Universe was static and unchanging.

Hubble's Law Formula

Hubble's law formula is given as follows:

$$v = H_0 d \qquad (12.6)$$

where, v is the velocity of the galaxy in km s^{-1}, H_0 is the Hubble constant in km s^{-1}(Mpc)$^{-1}$, d is the distance between the observer and the galaxy in Mpc.



(a) (b)

Figure 12.16 (a) Redshift spectrum (b) Graphical representation of the data for many galaxies (red dots)

Figure 12.16 (b) shows the diagram which is a graphical representation of the data for many galaxies (red dots).

This diagram does not have real measured data on it, but gives the qualitative picture of Hubble's law.

Limitations of Hubble's Law

By determining the redshift in observed light, one can determine the distance of the galaxy from us using Hubble's equation after measuring the recession velocity. Following are the limitations of Hubble's law which makes the challenges in measurement.

- Because of the intrinsic motion of galaxies, observed velocity gets influenced.
- Galaxy orbiting due to gravitational movements.

Hubble Constant

Hubble constant is defined as the unit of measurement, which is used for describing the expansion of the universe. Hubble constant is 72 km s^{-1}(Mpc)$^{-1}$. In Figure 12.17 the measured value of H_0 is subject to vary for different velocity measurements of galaxies.

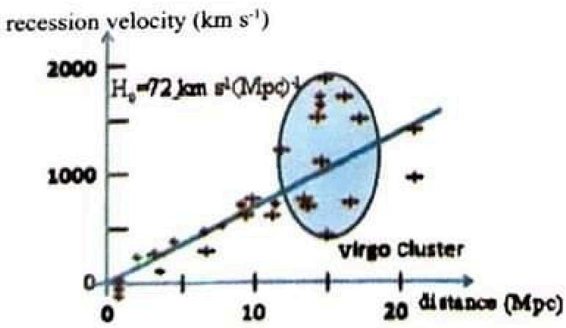


Figure 12.17 The relation between recession velocity and distance for different galaxies

The nature of the above graph interprets the linear relation between the redshift $\frac{\Delta\lambda}{\lambda}$ and distance *d*.

Redshift Formula

The redshift formula is given as, $z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

where, z is the redshift, $\Delta\lambda$ is the shift in wavelength in the spectra, λ is the wavelength, v is the recession velocity of space object, c is the speed of light.

Example 12.7 The velocity of a certain cluster from the earth is measured as $v = 10^3$ km s^{-1}. What is the distance from earth? Assume $H_0 = 72$ km s^{-1}(Mpc)$^{-1}$, 1 Mpc = 3.08×10^{19} km

$v = 10^3$ km s^{-1}, $H_0 = 72$ km s^{-1}(Mpc)$^{-1}$

The formula used is

$$d = \frac{v}{H_0}$$

$$d = \frac{10^3 \text{kms}^{-1}}{72 \text{ kms}^{-1}(\text{Mpc})^{-1}} = 13.888 \text{ Mpc}$$

$$d = 13.888 \times 3.08 \times 10^{19} \text{km} = 4.2775 \times 10^{20} \text{ km}$$

Units derived from Hubble constant

There are three units that are derived from Hubble constant, which are shown below:

(i) Hubble Time or Age of the Universe

The Hubble time or age of the universe is defined as the reciprocal of the Hubble constant. Hubble time is given as follows:

$$t_H = \frac{1}{H_0} = \frac{1}{72 \text{ km s}^{-1}(\text{ Mpc })^{-1}} = \frac{1}{72 \times \frac{1}{3.08 \times 10^{19}}} = 4.28 \times 10^{17}\ s$$

$$t_H = 1.36 \times 10^{10} \text{ years} \qquad (1\text{year} = 3.15 \times 10^7 \text{ s})$$

$$t_H = 13.6 \text{ billion years}$$

(ii) Hubble Length

The Hubble length is defined as the product of the speed of light and the Hubble time. It is also known as the Hubble distance. The product obtained is about 13.6 billion light-years which depends on the value of Hubble constant. This length is the radius of the observable universe (visible universe).

ct_H(or) cH_0^{-1} is the Hubble length.

(iii) Hubble Volume

The Hubble volume is defined as the volume of the observable universe (visible universe). It is also defined as the volume of sphere whose radius is cH_0^{-1}.

Reviewed Exercise

- What is Hubble's law used for?

Key Words: Hubble's Law, Hubble constant, Hubble length, Hubble volume

SUMMARY

Cathode rays are fast moving electrons emitted from the cathode of a discharge tube.

X-rays are electromagnetic waves like light, but their wavelengths are much shorter than those of light.

When electromagnetic radiation (e.g. light) of sufficient energy incident on the metal surface then electrons are ejected from its surface. This phenomenon is known as **photoelectric effect**.

Isotopes are the atoms of the same element with different masses.

When a source of light waves is traveling away from the observer, the observed wavelength will be slightly longer than the wavelength of the stationary (rest) source. In astronomy, this shift toward longer wavelengths is referred to as a **redshift** since red is at the long-wavelength end of the visible spectrum.

EXERCISES

1. What are cathode rays? Mention the properties of cathode rays.
2. Explain characteristic X-rays and continuous X-rays.
3. Give properties of X-rays.
4. An electron of excited hydrogen atom at $n = 5$ energy level makes a transition to $n = 2$ level. What is the wavelength of the emitted photon? Is this wavelength in the visible region?
5. In what situations does Hubble's law not apply?
6. Why is Hubble's law so important?
7. Why is Hubble's constant uncertain?
8. The data set of distance against recession velocity of the celestial objects is given below. Plot a graph using the given data. What is the nature of the graph? Find the slope of the graph and estimate the age of the universe. Give relevant comments regarding the graph.

Distance in mega parsecs (Mpc)	Recession velocity in km s^{-1}
29	2020
24	1900
39.1	2640
19	1200
9.5	590
4.2	300
41	2800
20	1120
12	870
10	750
9	680
22	1510
35	2300
20.8	1490