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CHAPTER 1

ROTATIONAL MOTION

In Grade 11, the angular displacement, angular velocity and angular acceleration of an object in circular motion have been learnt. In this chapter, the concepts and equations for rotational motion under constant angular acceleration and centripetal acceleration will be studied.

Learning Outcomes

It is expected that students will

- differentiate between the circular motion and rotational motion.
- examine rotational motion under constant angular acceleration.
- deduce the relations between angular and linear quantities.
- understand the centripetal acceleration.
- distinguish between the uniform circular motion and nonuniform circular motion.

1.1 ROTATIONAL MOTION UNDER CONSTANT ANGULAR ACCELERATION

In a circular motion, the object just moves in a circle. For example, artificial satellites going around the earth at a constant height.

In rotational motion, the object rotates about an axis. Some of the examples of rotational motion are; rotation of earth about its own axis which creates the cycle of day and night, motion of the blades of a fan, and motion of a Ferris wheel in an amusement park as shown in Figure 1.1 (a), (b) and (c).

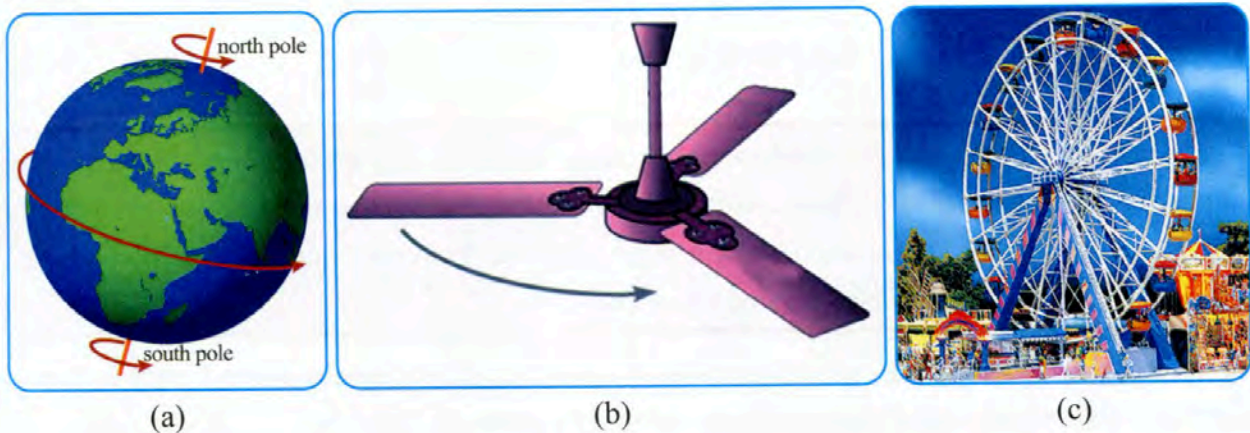


Figure 1.1 Some examples of rotational motion

A rigid body is a body that does not deform or change shape. That is, no matter how the body moves, the distance between any two particles within the body remains constant. If a rigid body is

in rotational motion, all the particles constituting it undergo circular motion about a common axis. When an object (rigid body) rotates, it can speed up (or) slow down. During these time intervals, its angular velocity is changing therefore it has an angular acceleration. If its angular velocity changes at a constant rate, then we can say that the angular acceleration is constant and the motion is called the rotational motion under constant angular acceleration.

Rotational variables; such as angular displacement, angular velocity, and angular acceleration have been introduced previously. In this section, these variables are applied to analyze the rotational motion for a rigid body about a fixed axis under a constant angular acceleration.

For an object that rotates with a constant angular acceleration, the kinematic equations for rotational motion can be derived in terms of the angular displacement, angular velocity and angular acceleration of the object at any instant of time.

If the initial angular speed (magnitude of angular velocity of the object at $t = 0$) is ω_0 , its angular speed ω at time t can be derived from the following constant angular acceleration.

$$\alpha = \frac{\omega - \omega_0}{t}$$

We get,

$$\omega = \omega_0 + \alpha t \quad (1.1)$$

Average angular speed is,

$$\bar{\omega} = \frac{\omega + \omega_0}{2} \quad (1.2)$$

Angular displacement for time interval t is $\theta = \bar{\omega}t$. Using Eq. (1.1) and Eq. (1.2) angular displacement θ and angular speed ω can be expressed by the following equations.

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad (1.3)$$

$$\omega^2 = \omega_0^2 + 2 \alpha \theta \quad (1.4)$$

In Grade 10, we derived the kinematic equations that relate acceleration, velocity, distance, and time for linear motion with constant acceleration. We can see that the equations of motion for constant angular acceleration are the same as those for linear motion, with the angular quantities replaced with corresponding linear ones.

Therefore, the equations for constant angular acceleration in rotational motion are analogous to equations for constant acceleration in linear motion as shown in Table 1.1.

Table 1.1 Analogies between kinematic equations of linear and angular motion

Linear motion	Angular motion
$v = v_0 + a t$	$\omega = \omega_0 + \alpha t$
$s = v_0 t + \frac{1}{2} a t^2$	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2 a s$	$\omega^2 = \omega_0^2 + 2 \alpha \theta$
$\bar{v} = \frac{v + v_0}{2}$	$\bar{\omega} = \frac{\omega + \omega_0}{2}$

Example 1.1 The Ferris wheel starts from rest and reaches an angular velocity of 1.5 rad s^{-1} over a 10 s period under the constant angular acceleration. (i) Find the angular acceleration of Ferris wheel. (ii) How many revolutions does it make during 10 s?

Initial angular velocity $\omega_0 = 0$, angular velocity $\omega = 1.5 \text{ rad s}^{-1}$, $t = 10 \text{ s}$

(i) The angular acceleration of Ferris wheel can be calculated by using equations of motion for constant angular acceleration.

$$\omega = \omega_0 + \alpha t$$

$$1.5 = 0 + \alpha \times 10$$

$$\alpha = 0.15 \text{ rad s}^{-2}$$

(ii) To get number of revolutions in 10 s, first we must find angular displacement.

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$= 0 + \frac{1}{2} \times 0.15 \times 100 = 7.5 \text{ rad}$$

Since 1 rev = 2π rad,

$$\theta = \frac{7.5}{2\pi} = 1.19 \text{ rev}$$

Reviewed Exercise

- Under what condition the angular acceleration of a rotating object is equal to zero?
- A ball is whirled with constant angular acceleration. From rest, it attains an angular velocity of 25 rad s^{-1} after traversing an angular displacement of 41 rad. What is angular acceleration of the ball?

Key Words: rotational motion, angular displacement, angular velocity, angular acceleration

1.2 RELATIONS BETWEEN LINEAR AND ANGULAR QUANTITIES

The relations between linear quantities (s , v , a) and angular quantities (θ , ω , α) for circular motion have been studied in Grade 11. In this section, the relation between these quantities for rotational motion will be studied. All particles of an object rotating around the axis of rotation move in a circular path.

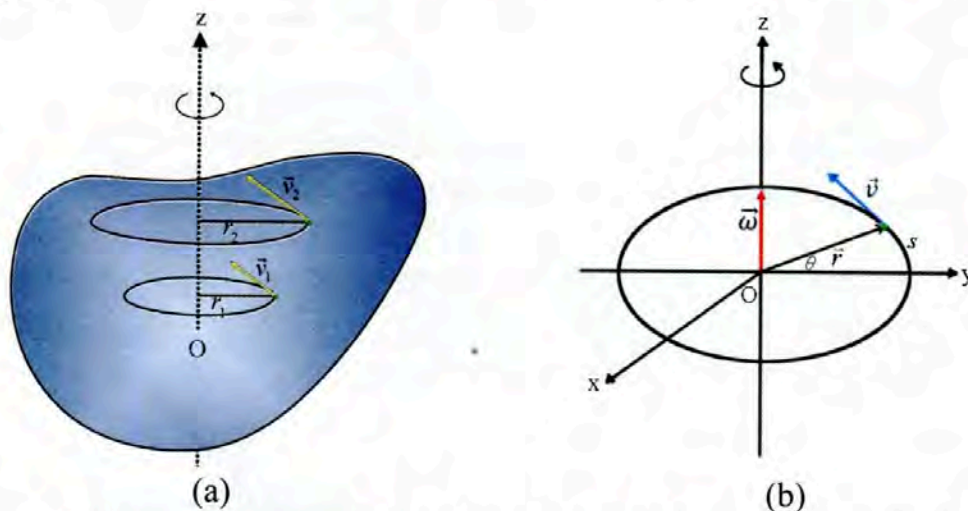


Figure 1.2 (a) Rotational motion of an object (b) circular motion of a particle of the object

Let us consider a randomly shaped rigid body undergoing a rotational motion as shown in Figure 1.2 (a). All particles of the body move in a circle lying on a plane that is perpendicular to the axis, such that the centre of rotation lies on the axis as shown in Figure 1.2 (b).

If the linear distance of a particle of rotating object measured along the arc is s and the arc subtends angle θ at the origin, the relation between arc length s and angular displacement θ , measured in radian, is $s = r\theta$ where r is the radius of the circle.

The instantaneous angular velocity ω of rotational object is the same for all the particles comprising the object. The relation between the magnitude of the linear velocity v and the angular velocity ω of a particle is $v = r\omega$, where ω is measured in radian per second. This relation applies to every particle of the rotational object. Since the direction of linear velocity is tangent to the path, the linear velocity is also called tangential velocity.

If the magnitude of the angular velocity of a rotating object changes, the magnitude of linear velocity will also change and the object will have a tangential acceleration. The direction of tangential acceleration is always tangent to the circular path. The tangential acceleration a_T relates to angular acceleration α (measured in radian per second squared) by the equation $a_T = r\alpha$.

The tangential acceleration of a rotating object is a measure of how fast the tangential velocity changes.

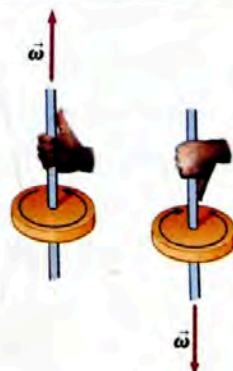
For the rotational motion, Table 1.2 expresses the relation between the linear and rotational quantities.

Table 1.2 The relation between the linear and rotational quantities

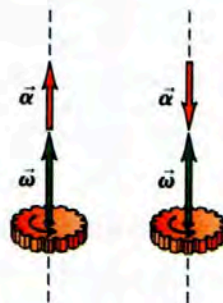
Quantity	Linear quantities	Rotational quantities	Relation
displacement	s	θ	$s = r \theta$
tangential velocity	v	ω	$v = r \omega$
tangential acceleration	a_T	α	$a_T = r \alpha$

The direction of the angular velocity vector is perpendicular to the plane of rotation, and along the axis of rotation.

The direction of angular velocity is usually specified by the right-hand rule. Wrap the right hand around the axis of rotation so that the fingers are pointing in the direction of rotation. The thumb points in the direction of the angular velocity vector.



When the angular velocity is increasing, the angular acceleration vector points in the same direction as the angular velocity. When the angular velocity is decreasing, the angular acceleration vector points in the opposite to the angular velocity.



Example 1.2 A boy steps on a merry-go-round which has a radius of 5 m and is at rest. It starts accelerating at a constant rate up to an angular velocity of 5 rad s^{-1} in 20 s. What is the distance traversed by the boy?

Radius of merry-go-round $r = 5 \text{ m}$, initial angular velocity $\omega_0 = 0$,
angular velocity in 20 s, $\omega = 5 \text{ rad s}^{-1}$, $t = 20 \text{ s}$

The angular displacement θ that the boy passes through is,

$$\begin{aligned} \theta &= \bar{\omega} t = \left(\frac{\omega + \omega_0}{2} \right) t \\ &= \left(\frac{5 + 0}{2} \right) \times 20 = 50 \text{ rad} \end{aligned}$$

The distance traversed by the boy is,

$$\begin{aligned} s &= r \theta \\ &= 5 \times 50 = 250 \text{ m} \end{aligned}$$

Example 1.3 The wheel of a car with radius 20 cm starts moving. The angular acceleration provided by the engine is 12 rad s^{-2} . What is the tangential acceleration of rim of the wheel?

Radius of wheel $r = 20 \text{ cm} = 0.2 \text{ m}$, angular acceleration $\alpha = 12 \text{ rad s}^{-2}$

The tangential acceleration of rim of the wheel is,

$$\begin{aligned} a_T &= r\alpha \\ &= 0.2 \times 12 = 2.4 \text{ m s}^{-2} \end{aligned}$$

The direction of tangential acceleration is tangent to rim of the wheel.

Reviewed Exercise

1. In circular motion, how is a tangential acceleration produced?
2. On a rotating carousel, a child sits on a horse near the outer edge and another child sits on a lion halfway out from the centre. (i) Which child has the greater linear velocity? (ii) Which child has the greater angular velocity?

Key Words: linear velocity, tangential acceleration

1.3 CENTRIPETAL ACCELERATION

An object that moves in a circle at constant speed v is said to experience uniform circular motion. The magnitude of the velocity remains constant in this case, but the direction of the velocity continuously changes as the object moves around the circle as shown in Figure 1.3 (a). The change in direction of velocity constitutes an acceleration. Thus, an object revolving in a circle is continuously accelerating, even when the speed remains constant ($v_1 = v_2 = v$). We now investigate this acceleration quantitatively.

Average acceleration has been defined as

$$\bar{a} = \frac{\bar{v}_2 - \bar{v}_1}{\Delta t} = \frac{\Delta \bar{v}}{\Delta t} \quad (1.5)$$

where $\Delta \bar{v}$ is the change in velocity during the time interval Δt . Consider the situation in which Δt approaches zero and thus the instantaneous acceleration can be obtained. During the time interval Δt , the object moves from point A to point B, covering a distance Δs along the arc which subtends an angle $\Delta \theta$. The change in the velocity vector is $\bar{v}_2 - \bar{v}_1 = \Delta \bar{v}$, and is shown in Figure 1.3 (b).

If Δt is very small (approaching zero), then Δs and $\Delta \theta$ are also very small, and, \bar{v}_2 will be almost parallel to \bar{v}_1 . $\Delta \bar{v}$ will be essentially perpendicular to both of them as shown in Figure 1.3 (c) and points toward the centre of the circle. Acceleration \bar{a} is in the same direction as $\Delta \bar{v}$, it also points toward the centre of the circle. Therefore, this acceleration is centripetal acceleration or radial acceleration (since it is directed along the radius towards the centre of the circle), and denoted by \bar{a}_c .

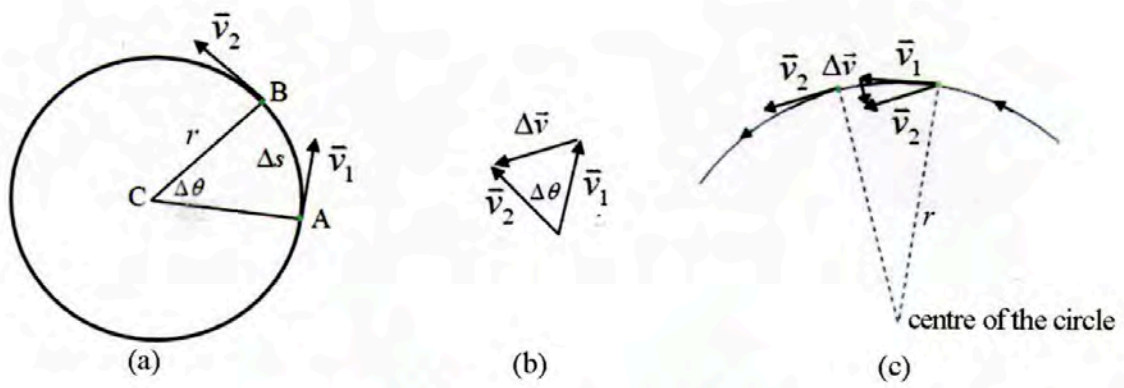


Figure 1.3 (a) Circular motion of an object at constant speed
 (b) vector diagram for change in velocity
 (c) vector diagram for Δt approaching zero

The magnitude of the centripetal (radial) acceleration can be determined as follow.

The triangle CAB of Figure 1.3(a) is geometrically similar to vector triangle of Figure 1.3(b).

Since $v_1 = v_2 = v$

$$\frac{\Delta v}{v} \approx \frac{\Delta s}{r}$$

When Δt approaches zero, the above equation can be expressed as an equality, so that

$$\Delta v = \frac{v}{r} \Delta s \quad (1.6)$$

The centripetal acceleration a_c can be obtained as

$$a_c = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{v}{r} \frac{\Delta s}{\Delta t} = \frac{v}{r} \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$$

But $\lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$ is the linear speed v of the object moving in circular motion.

Therefore,

$$a_c = \frac{v^2}{r} \quad (1.7)$$

Substituting $v = r\omega$ in Eq.(1.7), we get

$$a_c = r \omega^2 \quad (1.8)$$

For uniform circular motion, the centripetal acceleration vector points towards the centre of the circular path while the linear velocity vector is tangential to the path.

Hence, the centripetal acceleration and linear velocity are perpendicular to each other at every point in the path as shown in Figure 1.4.

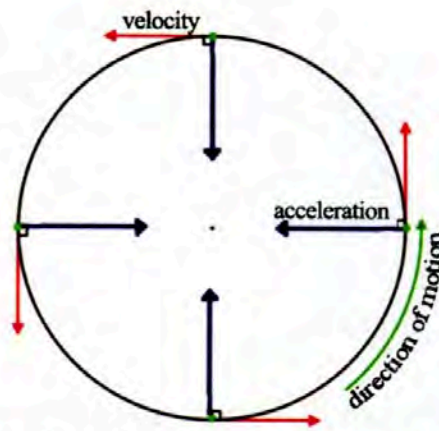


Figure 1.4 Directions of centripetal acceleration and linear velocity for uniform circular motion

Nonuniform Circular Motion

In nonuniform circular motion, an object is moving in a circular path with a varying speed (magnitude of velocity). Therefore, the angular speed of the object is changing and the object experiences the angular acceleration. This angular acceleration gives rise to the tangential acceleration which is tangential to the circle as described previously. In this case, there is tangential acceleration in addition to centripetal acceleration.

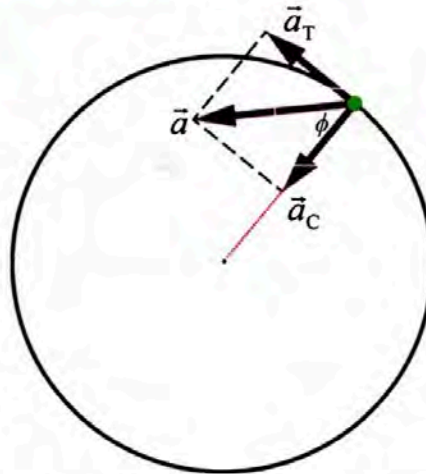


Figure 1.5 Resultant acceleration of nonuniform circular motion

Thus, an object in nonuniform circular motion has a resultant acceleration that is the vector sum of the centripetal and tangential accelerations as shown in Figure 1.5.

$$\vec{a} = \vec{a}_c + \vec{a}_T \quad (1.9)$$

Unlike tangential acceleration, centripetal acceleration is present in both uniform and nonuniform circular motion.

Note that the two acceleration vectors \vec{a}_C and \vec{a}_T are perpendicular to each other, with \vec{a}_C in the radial direction and \vec{a}_T in the tangential direction. The resultant acceleration points at an angle between \vec{a}_C and \vec{a}_T .

The magnitude of resultant acceleration \vec{a} is, $a = \sqrt{a_C^2 + a_T^2}$. (1.10)

The direction of resultant acceleration \vec{a} is, $\tan \phi = \frac{a_T}{a_C}$. (1.11)

Example 1.4 The platter of the hard drive of a computer uniformly rotates at 7200 rpm. If the reading head of the drive is located 3 cm from the rotational axis, what is the linear speed and centripetal acceleration of the point on the platter just below the reading head? If a single bit requires 0.5 μm of length along the direction of motion, how many bits per second can the writing head write when it is 3 cm from the axis?

Distance of reading head from the rotation axis $r = 3 \text{ cm} = 0.03 \text{ m}$,

Angular speed $\omega = 7200 \text{ rpm} = 7200 \times \frac{2\pi}{60} = 240 \pi \text{ rad s}^{-1}$

The linear speed v of the reading head located 3 cm from the rotation axis is,

$$\begin{aligned} v &= r \omega \\ &= 0.03 \times 240 \pi \\ &= 22.62 \text{ m s}^{-1} \end{aligned}$$

The centripetal acceleration a_C is,

$$\begin{aligned} a_C &= \frac{v^2}{r} \\ &= \frac{(22.62)^2}{0.03} = 1.71 \times 10^4 \text{ m s}^{-2} \end{aligned}$$

The distance moved by the writing head in 1 second = 22.62 m

The number of bits written in 1 second = $\frac{22.62}{0.5 \times 10^{-6}} = 4.52 \times 10^7$ bits

Example 1.5 A centrifuge has a radius of 20 cm and decelerates from a maximum rotational rate of 7.2×10^3 rpm to rest in 30 s under a constant angular acceleration. It is rotating counter clockwise. What is the magnitude of the resultant acceleration of a point at the tip of the centrifuge at 10 s? What is the direction of the resultant acceleration vector?

Radius of a centrifuge $r = 20 \text{ cm} = 0.2 \text{ m}$,

Initial angular velocity $\omega_0 = 7.2 \times 10^3 \text{ rpm} = 7.2 \times 10^3 \times \frac{2\pi}{60} = 240 \pi \text{ rad s}^{-1}$,

angular velocity $\omega = 0$, $t = 30 \text{ s}$

The angular acceleration of centrifuge is,

$$\begin{aligned}\alpha &= \frac{\omega - \omega_0}{t} \\ &= \frac{0 - 240\pi}{30} = -8\pi \text{ rad s}^{-1}\end{aligned}$$

The minus sign means that the centrifuge slows down to be rest.

A point at the tip of centrifuge has both centripetal acceleration and tangential acceleration.

The angular velocity of centrifuge at 10 s,

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ &= 240\pi + (-8\pi) \times 10 = 160\pi \text{ rad s}^{-1}\end{aligned}$$

The centripetal acceleration is,

$$\begin{aligned}a_c &= r\omega^2 \\ &= 0.2 \times (160\pi)^2 = 5.05 \times 10^4 \text{ m s}^{-2}\end{aligned}$$

The tangential acceleration is,

$$\begin{aligned}a_T &= r\alpha \\ &= 0.2 \times (-8\pi) \\ &= -5.03 \text{ m s}^{-2}\end{aligned}$$

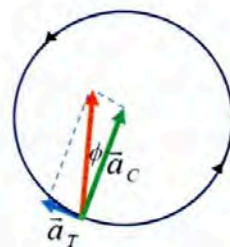
The magnitude of resultant acceleration is,

$$\begin{aligned}a &= \sqrt{a_c^2 + a_T^2} \\ &= \sqrt{(5.05 \times 10^4)^2 + (-5.03)^2} \approx 5.05 \times 10^4 \text{ m s}^{-2}\end{aligned}$$

Since $a_c \gg a_T$, the magnitude of resultant acceleration is approximately equal to the centripetal acceleration.

The direction of the resultant acceleration vector is,

$$\begin{aligned}\tan \phi &= \frac{a_T}{a_c} \\ &= \frac{5.03}{5.05 \times 10^4} \\ \phi &= 0.0057^\circ\end{aligned}$$



The resultant acceleration vector makes angle 0.0057° with \vec{a}_c .

Reviewed Exercise

1. Suppose a piece of food is on the edge of a rotating microwave oven plate. Does it experience tangential acceleration, centripetal acceleration, or both when (i) the plate starts to spin faster? (ii) the plate rotates at constant angular velocity? (iii) the plate slows down to stop?

2. A stone tied to a string is moving in a circle of radius 1.5 m at a constant speed 8 m s^{-1} . Calculate the magnitude of the centripetal acceleration of the stone.

Key Words: uniform circular motion, nonuniform circular motion, centripetal acceleration, tangential acceleration

SUMMARY

If the angular velocity of the rotating object changes at a constant rate, then we can say that the angular acceleration is constant and the motion is called the **rotational motion under constant angular acceleration**.

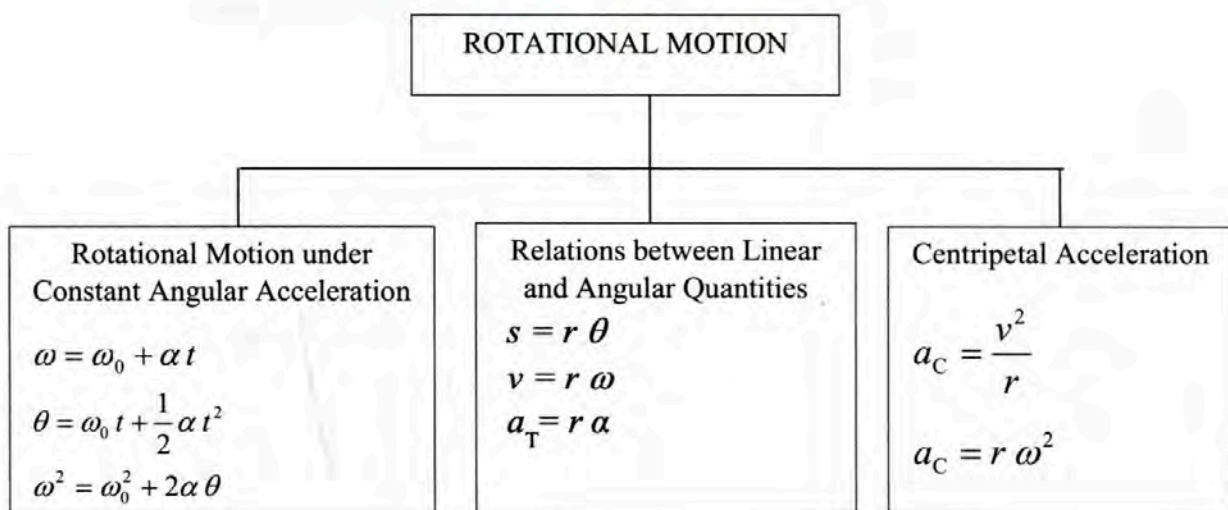
For uniform circular motion, the centripetal acceleration vector points towards the centre of the circular path while the linear velocity vector is tangential to the path. This acceleration is called **centripetal acceleration**.

EXERCISES

1. Consider a rotating circular plate of a lathe machine. Are the angular and tangential speeds at any point on the plate the same? Explain.
2. Why is centripetal acceleration produced in a circular motion?
3. Choose the correct answer. The angular acceleration is equal to zero when
A. there is no change in angular speed C. there is no change in angular velocity
B. there is no change in direction of the rotation
4. Which of the following correctly defines angular velocity?
A. Angular velocity is the rate of rotation with respect to the angular speed.
B. Angular velocity is the rate of rotation with respect to displacement.
C. Angular velocity is the rate of change of angular displacement with respect to time.
5. The angular velocity of a rotating rigid body increases from 500 rpm to 1500 rpm in 2 min. (i) What is the angular acceleration of the body? (ii) What angle does it turn through in this 2 min?
6. A centrifuge rotor is accelerated from rest to 20 000 rpm in 30 s. (i) What is its average angular acceleration? (ii) Through how many revolutions has the centrifuge rotor turned during its acceleration period, assuming constant angular acceleration?
7. A figure skater is spinning with an angular velocity of 15 rad s^{-1} . She then comes to a stop over a brief period of time. During this time, her angular displacement is 5.1 rad. Assuming constant angular acceleration, find (i) her angular acceleration and (ii) the time during which she comes to rest.

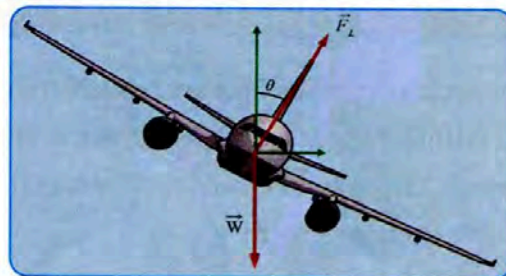
8. A tire has a radius of 0.33 m, and its centre moves forward with a linear speed of 15 m s^{-1} .
 (i) What is angular velocity of the wheel? (ii) Relative to the axle, what is linear speed of a point located 0.175 m from the axle?
9. A child spins a toy top, applying a force to the peg in the middle. The force applied results a tangential acceleration of the peg. If the radius of the peg is 0.5 cm and the tangential acceleration applied is 0.54 m s^{-2} , what is the angular acceleration of the top?
10. A 150 g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.6 m. The ball makes 2 revolutions in a second. What is its centripetal acceleration?
11. A carousel is initially at rest. It is given a constant angular acceleration 0.06 rad s^{-2} , which increases its angular velocity for 8 s. At $t = 8 \text{ s}$, determine the magnitude of the following quantities: (i) the angular velocity of the carousel, (ii) the linear velocity of a child located 2.5 m from the centre, (iii) the tangential acceleration of that child, (iv) the centripetal acceleration of the child, and (v) the total linear acceleration of the child.

CONCEPT MAP



The banked angle θ of the wings of the plane is,

$$\begin{aligned}\tan \theta &= \frac{v^2}{rg} \\ &= \frac{(222.22)^2}{2 \times 10^3 \times 9.8} \\ \theta &= 68.35^\circ\end{aligned}$$



Example 2.2 A 60 cm rope is tied to the handle of a bucket which is then whirled in a vertical circle. The mass of the bucket is 3 kg. If the tension of the rope at the lowest point in its path is 50 N, find the speed of the bucket at that point.

Mass of bucket, $m = 3$ kg

Length of rope = radius of vertical circle, $r = 60$ cm = 0.6 m

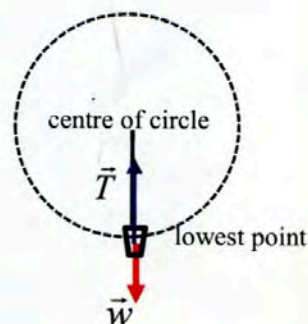
Force acting on bucket are;

- (i) Tension force on bucket $T = 50$ N, and
- (ii) Weight of bucket $w = m g = 3 \times 9.8 = 29.4$ N

The net force, $F_{\text{net}} = T - w = 50 - 29.4 = 20.6$ N

Since F_{net} provides the centripetal acceleration,

$$\begin{aligned}F_{\text{net}} &= F_C = \frac{mv^2}{r} \\ 20.6 &= \frac{3v^2}{0.6} \\ v &= 2.03 \text{ ms}^{-1}\end{aligned}$$



Reviewed Exercise

- What is the banking angle for an expressway off-ramp curve of radius 50 m at a limiting speed of 50 km h⁻¹?
- Choose the correct answer.

A ball attached to a string is whirled around a vertical circle. At the bottom of the arc,

- A. it has no net force.
- B. it has a net force acting upward.
- C. it has a net force acting downward.

Key Words: centripetal acceleration, centripetal force, banking angle

CHAPTER 2

FORCES IN CIRCULAR MOTION

According to Newton's second law, an object that is accelerating must have a net force acting on it. An object moving in a circle, such as a ball on the end of a string, must therefore have a force applied to it to keep it moving in the circle. This force is known as the centripetal force. In this chapter, centripetal force as well as the concepts of centrifugal force will be studied.

Learning Outcomes

It is expected that students will

- examine the centripetal force causing centripetal acceleration.
- investigate the concepts of centrifugal force.
- understand the applications of centripetal and centrifugal forces.

2.1 FORCES CAUSING CENTRIPETAL ACCELERATION

It has been shown in the previous chapter that for uniform circular motion (speed $v = \text{constant}$), the acceleration is centripetal acceleration which is directed towards the centre of the circle at any moment. According to Newton's second law, an object in a circular motion requires a net force directed towards the centre of the circle as shown in Figure 2.1. This net force is called centripetal force. In the absence of this force, the object would not move along a circle but move in a straight line, according to Newton's first law.

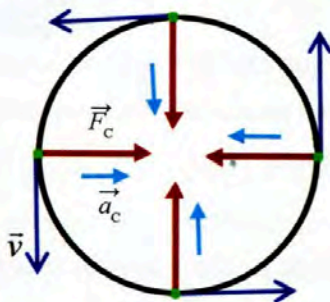


Figure 2.1 Directions of centripetal force, centripetal acceleration and linear velocity in uniform circular motion

Using Newton's second law, $F_{\text{net}} = ma$, the magnitude of the centripetal force is

$$F_C = m a_c = \frac{m v^2}{r} \quad (2.1)$$

Substituting $v = r\omega$ in Eq. (2.1),

$$F_C = m r \omega^2 \quad (2.2)$$

For an object to move in a circle, a force is needed to deviate from its straight line path and accelerate towards the centre of the circle. For example, to swing a ball on the end of a string in a circle, we pull on the string and the string exerts the force (tension) on the ball. The tension in the string provides the required centripetal force for the ball to keep moving in circular path.

Some examples of the centripetal force are the gravitational force of the earth on the moon orbiting the earth, the gravitational force of the sun on the planets orbiting the sun, the electric force acting on an electron orbiting the nucleus as illustrated in Figure 2.2.

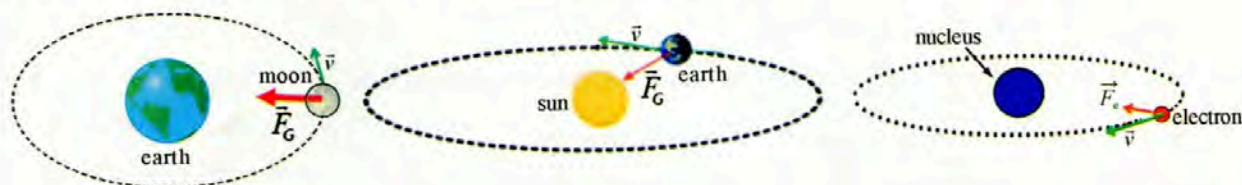


Figure 2.2 Some examples of the centripetal forces

Different Forms of Centripetal Force

There are different forces which can provide centripetal force depending on the motion of bodies under various conditions. A few more examples of the centripetal forces are described as follows.

(i) Tension force (T)

When whirling a body attached to a string, a tension force is originated. If this tension force is normal to the direction of motion of a body moving at constant speed, this force causes the body to move in a circular path. Thus, the tension force behaves as a centripetal force.

(ii) Gravitational force (F_G)

An attractive force exists between the sun and the earth in a direction perpendicular to the path of the earth. This force causes the earth to move in a circular path around the sun. In this case, the gravitational force provides as a centripetal force.

(iii) Electric force (F_e)

The Coulomb attractive force between the nucleus and the electron acts as the required centripetal force for the electron to move around the nucleus.

(iv) Frictional force (F_f)

When a car turns in a circular path or a curve, a frictional force exists between the road and the car tires. This frictional force acts normally to the direction of motion of the car, causing the car to move in a curved path. In this case, the frictional force behaves as a centripetal force.

(v) Normal force (F_N)

When a car moves in a circular path which is inclined to the horizontal, the reaction force of the road acts normally to the car. Resolving this reaction force (normal force) into two components,

2.2 CENTRIPETAL AND CENTRIFUGAL FORCES

In circular motion, there are two different forces that are encountered. They are centripetal force and centrifugal force. Centripetal force is the force acting towards the centre of the circular path around which the object is moving whereas the centrifugal force appears to act in a direction pointing away from the centre.

Consider, for example, a person whirls a ball on the end of a string in a horizontal circle around his head. The person pulls the string to exert an inward force on the ball. This force is the required centripetal force by the ball to move around the circular path. According to Newton's third law the ball exerts a reaction force on the hand as shown in Figure 2.5.

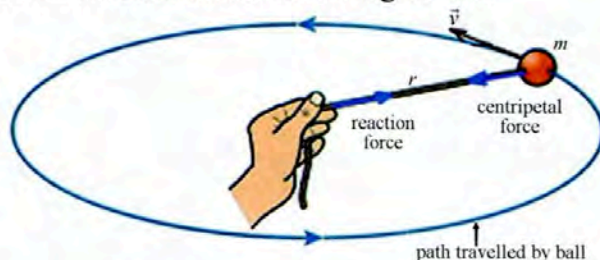


Figure 2.5 Centripetal force on the ball and its reaction force on the hand

There is a misconception that this reaction force is the outward-directed centrifugal force acting on the ball. Except centripetal force, there is no other force acting on the ball along the string.

As another example, consider a person on a moving car. When the car is moving with constant velocity along a straight road, there is no net force on the person. Hence, the person feels no force acting on him. However, when the car moves around a curve the person feels an outward force exerted on him. This is the centrifugal force caused by the effect of inertia.

Actually, the centrifugal force is not a real force. It is an apparent or fictitious force which is equal and opposite to the centripetal force. Although centrifugal force is not a real force, the concept of centrifugal force can be applied in rotating devices, such as centrifuges, washing machines, centrifugal pumps, centrifugal governors and centrifugal clutches.

Centrifuge

A centrifuge is a piece of equipment that puts an object in rotation around a fixed axis (spins it in a circle).

In a laboratory centrifuge that uses sample test tubes, the radial acceleration causes denser particles to settle to the bottom of the tube, while low density substances rise to the top. This effect is known as sedimentation principle.



the horizontal component acts towards the centre of the circle and helps the car to move in a curved path. The horizontal component of normal force is the required centripetal force.

(vi) **Upward thrust (Lifting Force) on a plane (F_L)**

The upward thrust (lifting force) acts normally to an aeroplane body. When the aeroplane tilts, the horizontal component acts normally to the motion of the aeroplane, causing the aeroplane to turn in a curve. This means that the horizontal component of the lifting force on the aeroplane provides the centripetal force.

Banking of Roads

When an automobile drives round a curve, it must have a force exerted towards the centre of the curve. On a flat road (unbanked road), this force is supplied by friction between the tires and the road. As a car makes a turn on unbanked road, the frictional force F_f acting upon the tires of the car provides the required centripetal force for circular motion as shown in Figure 2.3. Hence,

$$F_f = \frac{mv^2}{r} \quad (2.3)$$

where m is mass of the car, v is speed of the car and r is radius of the path.

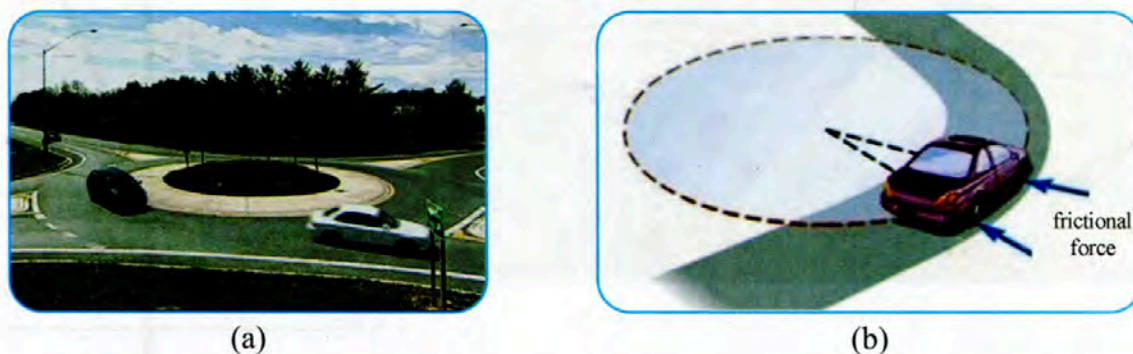


Figure 2.3 (a) Unbanked road (b) Frictional force provides centripetal force

If the frictional force is not sufficient, such as under wet and slippery conditions, the car will skid out of the circular path. To prevent this, the road must be banked to provide the necessary centripetal force to the automobiles so that they take a safe turn without skidding.

Banking of roads is raising the outer edge of the curved road higher than the inner edge. The angle that the inclined track makes with horizontal is called angle of banking. The normal force F_N exerted by a banked road is perpendicular to the road. Horizontal component of this normal force provides the required centripetal force. For every banked curve, there is a limiting speed at which the entire centripetal force is supplied by the horizontal component of the normal force, so that frictional force between the tires and the road is not required.

In this case, the horizontal component of the normal force, $F_N \sin \theta$, towards the centre of the curve of radius r just equals the centripetal force ($\frac{mv^2}{r}$) as shown in Figure 2.4.

$$F_N \sin \theta = \frac{mv^2}{r} \quad (2.4)$$

Since there is no vertical motion, the net force on car in y direction is zero.

$$F_N \cos \theta + (-mg) = 0$$

$$F_N \cos \theta = mg$$

Hence,

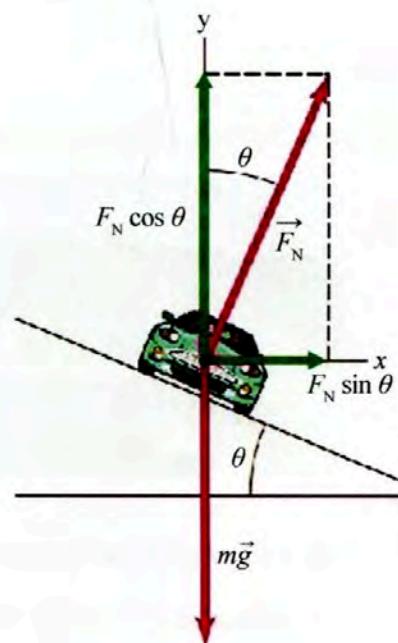
$$\frac{F_N \sin \theta}{F_N \cos \theta} = \frac{mv^2/r}{mg}$$

$$\tan \theta = \frac{v^2}{rg} \quad (2.5)$$

According to Eq.(2.5), the banking angle of a road θ is chosen so that this condition holds for the limiting speed v .



(a)



(b)

Figure 2.4 (a) Banked Road (b) horizontal component of normal force provides centripetal force

Example 2.1 A jet plane is flying around the airport with a speed of 800 km h^{-1} along a circular path with a radius of 2 km as shown in given figure. At what angle must the wings of the plane be banked?

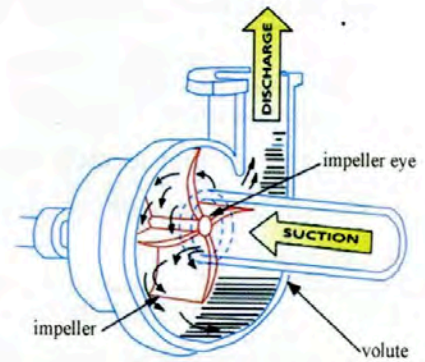
Speed of jet plane $v = 800 \text{ km h}^{-1} = 222.22 \text{ m s}^{-1}$

Radius of circular path $r = 2 \text{ km} = 2 \times 10^3 \text{ m}$

The wings of the plane must be banked an angle θ with vertical.

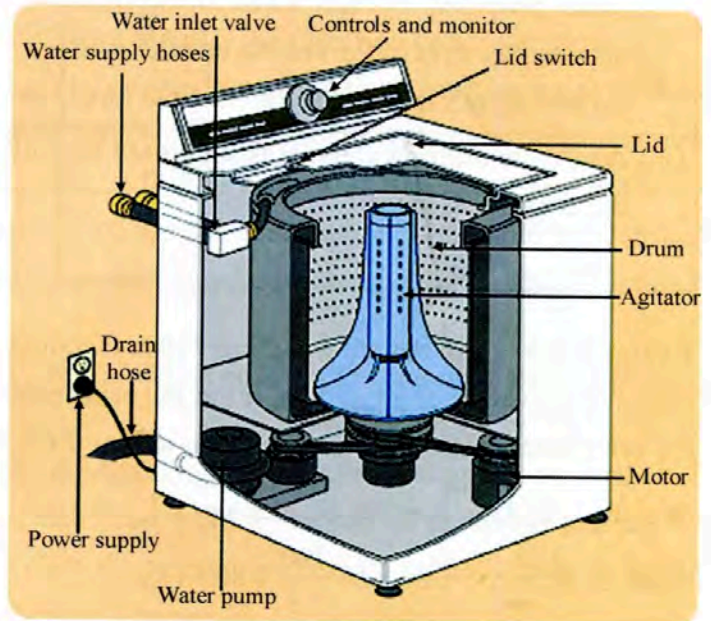
Centrifugal Pump

Centrifugal pump is a hydraulic machine which converts mechanical energy into hydraulic energy (i.e. pressure energy) by the use of centrifugal force acting on the fluid. The rotating impeller moves water or other fluids by using centrifugal force.



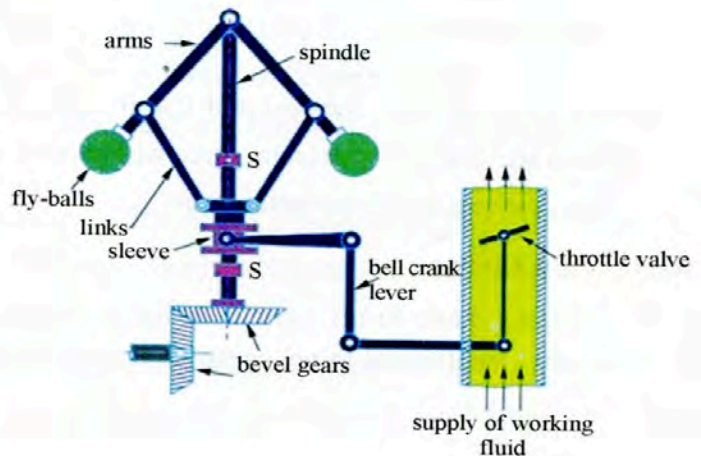
Washing Machine

The centrifugal action of a washing machine is used in spin drying the clothes. When all the clothes are completely washed, they are spun very fast and excess water is removed completely with the help of centrifugal force.



Centrifugal Governor

The centrifugal governor is a device used to measure and regulate the speed of a machine, such as an engine. The centrifugal governors consist of two balls of equal mass, which are attached to the arms. As the engine is working, these balls also revolve around the spindle at a certain speed and maintain at a position due to the centrifugal force.



Comparison Between Centripetal and Centrifugal Force

Centripetal Force	Centrifugal Force
(i) Centripetal force is the force acting on an object in circular motion which is directed towards the axis of rotation or centre of curvature.	(i) Centrifugal force in a circular motion is not a real force; it acts radially away from the centre of the circle. It results from the effect of inertia.
(ii) If a car is travelling through a curve on a circular horizontal road, the centripetal force provided by the force of friction between the tires of the vehicle and the road surface allows the car to negotiate the turn.	(ii) When a car in motion takes a sudden turn in a curved path, passengers in the car experience an outward push. This is due to the centrifugal force acting on passengers.
(iii) A satellite orbiting a planet is supported by the centripetal force.	(iii) Removing the excess water by spin drying clothes in a washing machine is with the help of centrifugal force.

Example 2.3 An athlete weighing 790 N is running a curve at a speed of 6 m s^{-1} in an arc. The radius of curvature of arc is 5 m. Find the centripetal force acting on him. Which force provides the centripetal force? What will happen to him if the radius of curvature is smaller?

Weight of athlete, $w = 790 \text{ N}$, $v = 6 \text{ m s}^{-1}$, $r = 5 \text{ m}$

Mass of athlete, $m = \frac{w}{g} = \frac{790}{9.8} = 80.61 \text{ kg}$

The required centripetal force to run a curve, $F_c = \frac{mv^2}{r}$
 $= \frac{80.61(6)^2}{5} = 580.4 \text{ N}$

Frictional force between the feet and the ground provides the centripetal force. If the radius of curvature is smaller, the centripetal force will become greater. He will slip since the frictional force is less than the required centripetal force.

Reviewed Exercise

- When a skater forms a circle, he has to lean inwards. Explain with a force diagram.

Key Words: centripetal force, centrifugal force, fictitious force

SUMMARY

Centripetal force is the force acting on an object in circular motion which is directed towards the axis of rotation (or) centre of curvature.

Centrifugal force in a circular motion is not a real force; it acts radially away from the centre of the circle. It results from the effect of inertia.

EXERCISES

1. Imagine a car driving over a hillock at a constant speed. Once the car has reached the apex of the hillock, does it have the acceleration? If so, what is the direction of acceleration?
2. Does the centripetal force and centrifugal force constitute an action - reaction pair? Explain.
3. An object is in a uniform circular motion. Which of the following statements must be true?
A. The net force acting on the object is zero. B. The velocity of the object is constant.
C. The speed of the object is constant. D. The acceleration of the object is constant.
4. For an object moving in a circle with constant angular velocity, what is the direction of acceleration of the object?
5. When a bob is swinging in the vertical circle at which point on the circular part is the tension in the rope the greatest?
6. A 0.15 kg ball on the end of 1.1 m long cord (negligible mass) is swung in a vertical circle.
(i) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (ii) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (i).
7. An 8.3 kg mass is attached to a string that has a breaking strength of 1500 N. If the mass is whirled in a horizontal circle of radius 80 cm, what maximum speed can it have?
8. An aeroplane is circling above an airport in a horizontal circle at a speed of 400 km h^{-1} . The banking angle of the wings is 20° . What is the radius of the circular path?
9. What is the tension in a 1 m string that is spinning a 0.5 kg stone in a horizontal circle with 3 rps?
10. A curved roadway has a radius of curvature of 200 m and a banking angle of 10° . What is the highest speed at which a car can round the curve safely? (Neglect the friction between tire and the road)
11. In an atom, an electron moves in a circular path around the nucleus. The speed of the electron is approximately $2.2 \times 10^6 \text{ m s}^{-1}$. Find the centripetal force acting on the electron as it revolves in a circular orbit of radius $0.53 \times 10^{-10} \text{ m}$. ($m_e = 9.1 \times 10^{-31} \text{ kg}$)

Using continuity equation, $A_{\text{pipe}} v_i = A_{\text{nozzle}} v_{\text{nozzle}}$

$$4.91 \times 101.83 = 0.5 v_{\text{nozzle}}$$

$$v_{\text{nozzle}} = 1000 \text{ cm s}^{-1}$$

Example 3.2 What area must a heating duct have if air moving 3 m s^{-1} along it can replenish the air every 15 min in a room of volume 300 m^3 ? Assume the density of air remains constant.

Let the area of heating duct be A .

The velocity of air at duct $v = 3 \text{ m s}^{-1}$

The volume of air coming in the room in 15 min = volume of room
 $= 300 \text{ m}^3$

$$\text{Volume flow rate } \frac{V}{t} = Av$$

$$\frac{300}{15 \times 60} = A \times 3$$

$$A = 0.11 \text{ m}^2$$

Example 3.3 Water runs into a fountain, filling all the pipes, at a steady rate of $0.75 \text{ m}^3 \text{ s}^{-1}$.

(i) How fast will it shoot out of a hole 4.5 cm in diameter? (ii) At what speed will it shoot out if the diameter of the hole is three times as large?

Volume flow rate $\frac{V}{t} = 0.75 \text{ m}^3 \text{ s}^{-1}$, radius of the hole $= \frac{4.5}{2} = 2.25 \text{ cm} = 0.0225 \text{ m}$

$$(i) \quad \frac{V}{t} = Av = \pi r^2 v$$

$$0.75 = 3.142 \times (0.0225)^2 v$$

$$v = 471.51 \text{ m s}^{-1}$$

(ii) Since diameter is three times larger, area is nine times larger. $A_2 = 9 A_1$

Using continuity equation,

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{v_1}{9}$$

Therefore speed of water shooting out $v_2 = \frac{v_1}{9} = \frac{471.51}{9} = 52.39 \text{ m s}^{-1}$

Example 3.4 Water tank of dimension $3 \text{ m} \times 3 \text{ m} \times 3 \text{ m}$ and the base of the tank is 10 m above the ground. It takes 1 h to fill. (i) Find the power output of the pump to fill the tank. (ii) Find the flow rate through the pipe and the speed of the water flow in the pipe which has 5 cm radius.

CONCEPT MAP

FORCES IN CIRCULAR MOTION

Centripetal Force

- centripetal force is the force acting towards the centre of the circular path around which the object is moving.

$$F_c = m a_c = \frac{m v^2}{r}$$

Centrifugal Force

- centrifugal force appears to act in a direction pointing away from the centre.

Applications of centripetal forces

- road banking
- the gravitational force on the planets
- electrical force on an electron moving around the nucleus

Applications of centrifugal forces

- rotating devices; centrifuges, washing machines, centrifugal pumps, centrifugal governors, etc.

Comparison between centripetal and centrifugal forces

- a satellite orbiting a planet is supported by the centripetal force.
- removing the excess water by spin drying clothes in a washing machine is with the help of centrifugal force.

CHAPTER 3

FLUID DYNAMICS

Fluid is a substance that offers no permanent resistance to deforming forces. The study of fluids has two parts, statics and dynamics. Fluid statics concentrates on fluids at rest, while fluid dynamics concerns with the fluid in motion.

We have already studied fluid statics in Grade 10 and Grade 11; which includes atmospheric pressure, liquid pressure, Archimedes' principle and Pascal's law. Now we are going to study fluid dynamics or fluids in motion. Motion of fluids is an important application in our daily life; such as river flow, water distribution systems, gas pipelines, aviation (travels in air) and motion underneath the surface of water.

The study of the flow of liquid is called hydrodynamics, whereas hydrostatic is the study of liquid at rest. This chapter mainly concerns with hydrodynamics; however a section on surface tension and capillarity, which are important concepts of hydrostatic, is included at the end of the chapter.

Learning Outcomes

It is expected that students will

- differentiate fluid flow as laminar and turbulent.
- study equation of continuity for a fluid motion.
- understand Bernoulli's principle as a relationship between pressure energy, kinetic energy and potential energy of a fluid in streamline (laminar) flow.
- realize some manifestations of Bernoulli's principle.
- understand viscosity and Stokes' law.
- distinguish between cohesion and adhesion and explain surface tension in liquids.
- discuss capillarity.

3.1 LAMINAR AND TURBULENT FLOW

Fluid is either liquid or gas. The volume of a gas depends on pressure and temperature obeying Boyle's law and Charles' law. However, the volume of a liquid does not depend on pressure if the temperature is constant. Hence, the liquid is incompressible (i.e. density of liquid is constant for any pressure provided that the temperature is kept constant).

In the flow of fluids, the path or flow of the fluid particles is called streamline as shown in Figure 3.1 (a). Motion of fluids can be classified by two types, laminar flow and turbulent flow.

$$V = 3 \times 3 \times 3 = 27 \text{ m}^3, h = 10 + 3 = 13 \text{ m}, t = 1 \text{ h} = 3600 \text{ s}$$

$$(i) \quad \text{Power, } P = \frac{W}{t} = \frac{m g h}{t} = \frac{\rho V g h}{t}$$

$$= \frac{1000 \times 27 \times 9.8 \times 13}{3600} = 955.5 \text{ W}$$

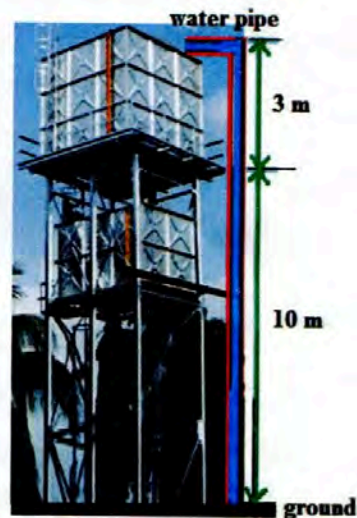
$$(ii) \quad r = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

$$\text{Flow rate } \frac{V}{t} = \frac{27}{3600} = 7.5 \times 10^{-3} \text{ m}^3 \text{ s}^{-1}$$

$$\text{Flow rate } \frac{V}{t} = A v$$

$$7.5 \times 10^{-3} = \pi (5 \times 10^{-2})^2 v$$

$$v = 0.96 \text{ m s}^{-1}$$



Reviewed Exercise

1. Why does the river flow faster in passing the defile (narrow river passage)?
2. A shower head has 20 circular openings, each with radius 1 mm. The shower head is connected to a pipe with radius 0.8 cm. If the speed of water in the pipe is 3 m s^{-1} , what is its speed as it exits the shower-head openings?

Key Words: flow rate, cross-sectional area, flow speed, continuity equation

3.3 BERNOULLI'S EQUATION

Bernoulli's equation, in fluid dynamics, relates the pressure, flow speed, and height for the steady (laminar) flow of an ideal incompressible fluid. The internal friction of the fluid flow is negligible (i.e. no viscosity). This equation was first derived in 1738 by the Swiss mathematician Daniel Bernoulli.

Bernoulli's equation can be derived using the work-energy principle to a fluid flowing in a tube. Consider a section of the fluid flowing in the tube with uneven cross section as shown in Figure 3.3. This fluid section is initially between two cross sections a and c . After a small time interval the fluid at a moves to b , and the fluid at c moves to d . The cross-sectional area, speed and pressure at two ends of the fluid section are A_1, v_1, p_1 and A_2, v_2, p_2 , respectively.



Daniel Bernoulli
(1700-1782)

1. Laminar Flow

If fluid particles are moving steady in smooth paths in layers, with each layer moving smoothly past the adjacent layers with no mixing; such a flow is called laminar flow. In this steady laminar flow, streamlines do not cross each other and every fluid particle arriving at a given point has the same velocity as illustrated in Figure 3.1 (b).

2. Turbulent Flow

If the flow or path of the fluid particles are irregular, their direction is always changing or whirling; this fluid movement is called turbulent flow. In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction as shown in Figure 3.1 (c).

Ideal laminar fluid flow considers the following facts.

- ◆ The fluid is incompressible.
- ◆ The flow has no friction or friction is neglected. (Internal friction in fluids is called viscosity; and hence, zero viscosity.)
- ◆ The fluid flow is laminar (steady) and turbulent flow is not taken into account. (For example, sea breeze is a laminar flow and storm is a turbulent flow.)

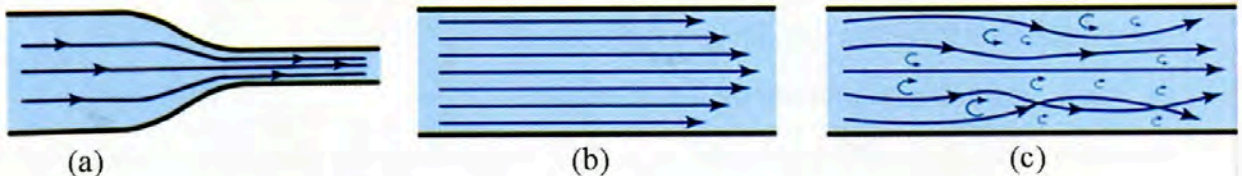


Figure 3.1 (a) Stream lines (b) laminar flow (c) turbulent flow

Reviewed Exercise

1. Is motor car brake oil compressible or non-compressible?
2. Are gases compressible or non-compressible?
3. What is the difference between laminar flow and turbulent flow?
4. At a river junction, water streams are whirling; is it laminar or turbulent?

Key Words: fluid, incompressible, laminar, turbulent, streamline, viscosity

3.2 EQUATION OF CONTINUITY FOR FLUIDS

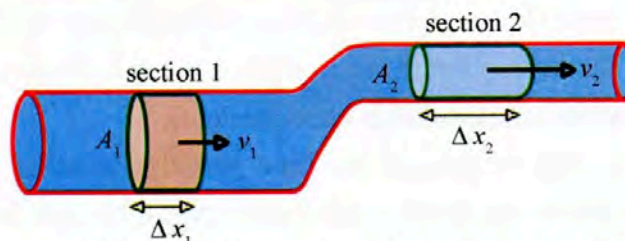


Figure 3.2 Illustrating constant volume flow rate in a pipe

Consider an ideal fluid there must be laminar flow and no viscosity. The density of fluid is constant. In Figure 3.2 all the liquid that flow through section 1 (larger cross sectional area A_1) flow through section 2 (smaller cross sectional area A_2). The mass flowing in section 1 must equal the mass flowing in section 2. Hence, volume flow rate $\frac{V}{t}$ is the same throughout the pipe.

$$\begin{aligned} m_1 &= m_2 \\ \rho V_1 &= \rho V_2 \\ \frac{V_1}{t} &= \frac{V_2}{t} \end{aligned}$$

Therefore,

Since $V_1 = A_1 \Delta x_1$ and $V_2 = A_2 \Delta x_2$

$$\begin{aligned} \frac{A_1 \Delta x_1}{t} &= \frac{A_2 \Delta x_2}{t} \\ A_1 v_1 &= A_2 v_2 \end{aligned} \tag{3.1}$$

where v_1 and v_2 are speed of liquid at section 1 and section 2 respectively.

The equation (3.1) states that the liquid entering one end of the pipe leaves from the other end with a constant volume flow rate.

This equation is known as equation of continuity for fluid flow.

The larger the area of the pipe, the slower is the speed; and the smaller the area, the faster is the speed. For example, the speed of water spraying from a hose increases by making the hose area smaller.

Example 3.1 A gardener uses a water pipe of 2.5 cm diameter. It takes 1 min to fill 30 litre bucket. What is the initial speed of water coming out of the hose? Then nozzle with an opening 0.5 cm^2 is attached to the hose. Find the water speed coming out from the nozzle.

Initial pipe area, $A_{\text{pipe}} = \pi r^2 = \pi \left(\frac{2.5}{2}\right)^2 = 4.91 \text{ cm}^2$,

Volume of water flow in 1 min (60 s) $V = 30 \text{ litre} = 30 \times 1000 \text{ cm}^3$

Volume flow rate $\frac{V}{t} = \frac{30 \times 1000}{60} = 500 \text{ cm}^3 \text{ s}^{-1}$ (this value is constant for the system)

Since volume flow rate $\frac{V}{t} = A v$,

The initial speed of water coming out of the hose $v_i = \frac{V/t}{A_{\text{pipe}}}$

$$= \frac{500}{4.91} = 101.83 \text{ cm s}^{-1}$$

When nozzle is attached, nozzle area $A_{\text{nozzle}} = 0.5 \text{ cm}^2$

According to work-energy principle, the work done on this section of fluid must equal the change in total mechanical energy (i.e. kinetic energy + potential energy).

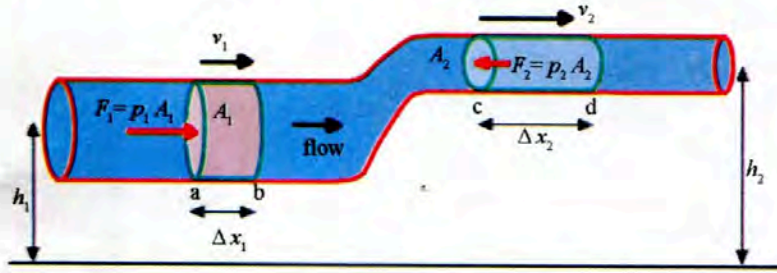


Figure 3.3 A section of a tube of flowing fluid

Since the fluid is incompressible, $A_1 \Delta x_1 = A_2 \Delta x_2 = \Delta V$

The work done on the fluid in this case is due to the pressure of the fluid.

$$\begin{aligned} \text{The work done } \Delta W &= F_1 \Delta x_1 - F_2 \Delta x_2 \\ &= p_1 A_1 \Delta x_1 - p_2 A_2 \Delta x_2 \\ &= p_1 \Delta V - p_2 \Delta V = (p_1 - p_2) \Delta V \end{aligned}$$

(The second term has a negative sign because the force at c opposes the force at a.)

The mass of the fluid between a and b $= \rho A_1 \Delta x_1 = \rho \Delta V$

The mass of the fluid between c and d $= \rho A_2 \Delta x_2 = \rho \Delta V$

$$\text{The change in kinetic energy } \Delta KE = \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2)$$

The change in potential energy $\Delta PE = \rho \Delta V g (h_2 - h_1)$

Using the work-energy principle $\Delta W = \Delta KE + \Delta PE$

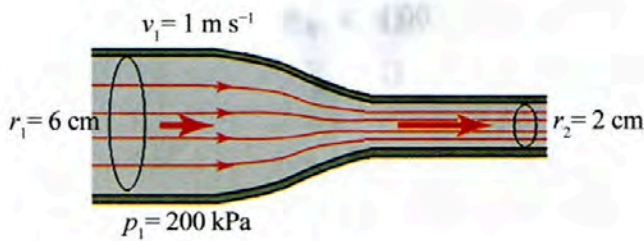
$$\begin{aligned} (p_1 - p_2) \Delta V &= \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2) + \rho \Delta V g (h_2 - h_1) \\ (p_1 - p_2) &= \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g (h_2 - h_1) \\ p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 &= p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \end{aligned}$$

$$\text{Thus, } \quad p + \frac{1}{2} \rho v^2 + \rho g h = \text{constant} \quad (3.2)$$

This equation is known as Bernoulli's equation.

In words, Bernoulli's theorem states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined laminar flow remains constant.

Example 3.5 Water flowing through a restriction in a horizontal pipe is shown in the given figure. The radius of the pipe at the left end and right end are 6 cm and 2 cm respectively. If the velocity and pressure of water at the left end of pipe are 1 m s^{-1} and 200 kPa, find the velocity and pressure of water at the right end of the pipe.



Let velocity and pressure at right end be v_2, p_2 .

Using continuity equation,

$$\begin{aligned}
 A_1 v_1 &= A_2 v_2 \\
 \pi r_1^2 v_1 &= \pi r_2^2 v_2 \\
 v_2 &= v_1 \left(\frac{r_1}{r_2} \right)^2 \\
 &= 1 \times \left(\frac{0.06}{0.02} \right)^2 = 9 \text{ m s}^{-1}
 \end{aligned}$$

Using Bernoulli's theorem,

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

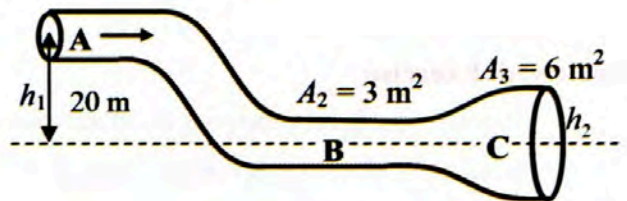
Since $h_1 = h_2$,

$$p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$$

$$\begin{aligned}
 p_2 &= p_1 - \frac{1}{2} \rho (v_2^2 - v_1^2) \\
 &= 200 \times 10^3 - \frac{1}{2} \times 1000 (9^2 - 1^2) \\
 &= 160 \times 10^3 \text{ Pa} = 160 \text{ kPa}
 \end{aligned}$$

Example 3.6 Calculate the pressure and speed of water at points B and C shown below. (density of water is 1000 kg m^{-3})

$$\begin{aligned}
 A_1 &= 3 \text{ m}^2 \\
 v_1 &= 10 \text{ m s}^{-1} \\
 p_1 &= 300 \text{ kPa}
 \end{aligned}$$



Using continuity equation,

$$A_1 v_1 = A_2 v_2$$

Since $A_1 = A_2$, $v_1 = v_2 = 10 \text{ m s}^{-1}$

Using continuity equation

$$\begin{aligned}
 A_2 v_2 &= A_3 v_3 \\
 3 \times 10 &= 6 v_3 \\
 v_3 &= 5 \text{ m s}^{-1}
 \end{aligned}$$

For points A and B, $p_1 = 300 \text{ kPa} = 300\,000 \text{ Pa}$, $h_1 = 20 \text{ m}$, $h_2 = 0$

Using Bernoulli's theorem,

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

Since $v_1 = v_2$,

$$p_1 + \rho g h_1 = p_2 + \rho g h_2$$

$$300\,000 + 1000 \times 9.8 \times 20 = p_2$$

$$p_2 = 496\,000 \text{ Pa} = 496 \text{ kPa}$$

For points B and C, $v_2 = 10 \text{ m s}^{-1}$ and $v_3 = 5 \text{ m s}^{-1}$, $p_2 = 496\,000 \text{ Pa}$

Using Bernoulli's theorem,

$$p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 = p_3 + \frac{1}{2} \rho v_3^2 + \rho g h_3$$

Since $h_2 = h_3$,

$$p_2 + \frac{1}{2} \rho v_2^2 = p_3 + \frac{1}{2} \rho v_3^2$$

$$496\,000 + \frac{1}{2} \times 1000 \times 10^2 = p_3 + \frac{1}{2} \times 1000 \times 5^2$$

$$p_3 = 533\,500 \text{ Pa} = 533.5 \text{ kPa}$$

Example 3.7 A sealed tank containing seawater to a height of 11 m also contains air above the water at a pressure of 4 atm. Water flows out from the bottom through a small hole. How fast is this water moving?

At point A (top water surface)

$$p_1 = 4 \text{ atm} = 4 \times 1.01 \times 10^5 \text{ Pa} = 4.04 \times 10^5 \text{ Pa}$$

Since cross-sectional area of water tank is much greater than area of small hole, water level in the tank drops very slowly, we take $v_1 \approx 0$, $h_1 = 11 \text{ m}$.

At point B (small hole at the bottom)

$$p_2 = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}, h_2 = 0 \text{ m}$$

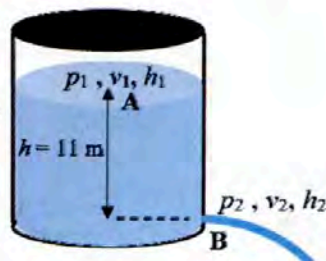
For points A and B, using Bernoulli's theorem,

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$4.04 \times 10^5 + 1000 \times 9.8 \times 11 = 1.01 \times 10^5 + \frac{1}{2} \times 1000 v_2^2$$

$$4.11 \times 10^5 = 500 v_2^2$$

$$v_2 = 28.66 \text{ m s}^{-1}$$



Reviewed Exercise

- As water in a level pipe pass from narrower cross-sectional area of pipe to a wider cross-sectional area, does the pressure change? Explain.
- A small circular hole 6.00 mm in diameter is cut in the side of a large water tank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find (i) the speed of efflux of the water, and (ii) the volume discharged per second.

Key Words: work-energy principle, incompressible, non-viscous fluid

3.4 MANIFESTATIONS OF BERNOULLI'S THEOREM

1. Lift on the Wings of an Aeroplane

The air flow moving above the top curved surface of the wing has a longer distance to travel and needs to go faster to have the same transit time as the air travelling along the lower flat surface (horizontal position). Hence, the velocity of air flow is high along the top surface and low along the bottom surface. This causes a low pressure above the wing and a high pressure below the wing which provides a lift on the wing.

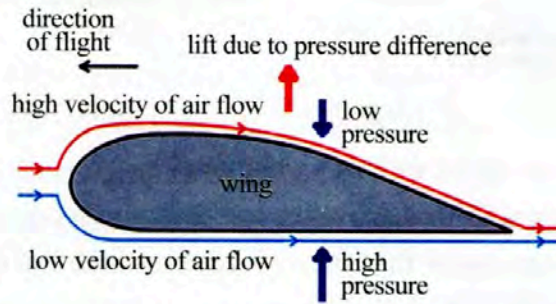


Figure 3.4 Lift on the wings of an aeroplane

2. Insect Sprayer

When the piston of the sprayer is pressed, the fast moving air causes a low pressure at the tip of the nozzle. The liquid inside the container is at a higher pressure, and therefore rushes up through the stem and out the nozzle.

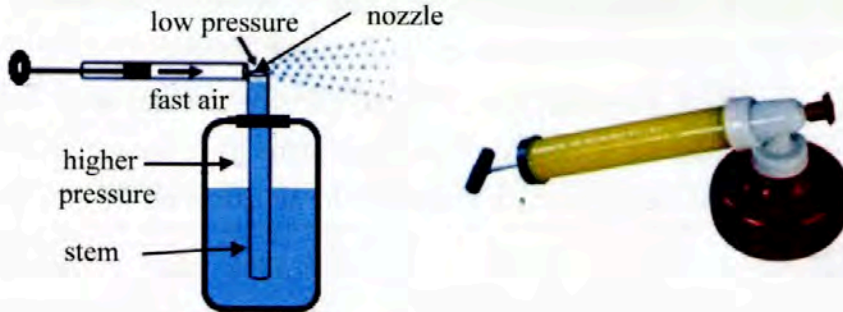


Figure 3.5 The insect sprayer

3. Magnus Effect on a Spinning Ball

The Magnus effect is a particular manifestation of Bernoulli's theorem that is commonly associated with a spinning object moving through a fluid. In the case of a ball spinning through the air, the turning ball drags some of the air around with it. This causes the difference in pressure of the air on opposite sides of the spinning ball and produces the deflection of the ball. The effect is named after the German physicist Heinrich Gustav Magnus, who described the effect in 1852.

3.6 SURFACE TENSION AND CAPILLARITY

1. Surface Tension

Surface tension is the tendency of the surface of a liquid to behave like a stretched elastic skin (or membrane). It is due to the attractive forces between liquid molecules called cohesion. Cohesion refers to forces between molecules of the same substance. For example, cohesion makes water molecules pull together making the water surface to stretch like an elastic skin.

Surface tension is the reason a water strider and it is able to stride along water surface without sinking. Paperclips, razor blades and light coins are also able to rest on water without sinking although their density is higher than that of water as shown in Figure 3.9.



Figure 3.9 Effect of surface tension

Surface tension is defined as the force per unit length that acts across any line on a surface tending to pull the surface apart.

$$\gamma = \frac{F}{l} \quad (3.6)$$

where F = force, l = length, γ = surface tension

The SI unit of surface tension is newton per metre (N m^{-1}).

Surface tension can also be referred to as a force because it is the tension between liquid molecules.

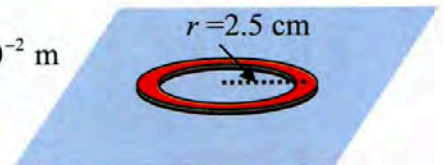
Example 3.11 A horizontal circular loop of wire has a diameter of 5 cm and is lowered in a sample of crude oil. The additional force required to pull the loop out of the oil is 0.04 N; calculate the surface tension of the crude oil.

$$F = 0.04 \text{ N}, d = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}, r = \frac{d}{2} = \frac{5}{2} = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$$

Circumference of the loop

$$l = 2 \pi r$$

$$= 2 \times 3.142 \times 2.5 \times 10^{-2} = 0.16 \text{ m}$$



Additional force F required to pull the loop is the force due to surface tension. Since the circular loop has two rims (outer and inner rims) in contact with the liquid surface,

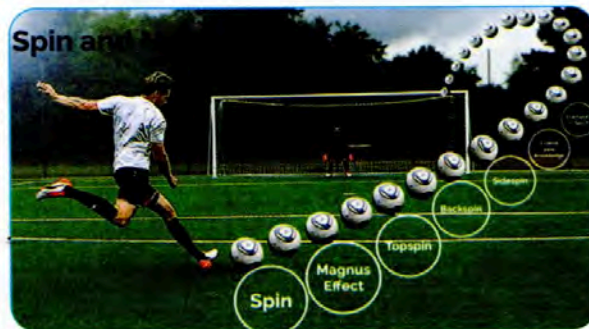
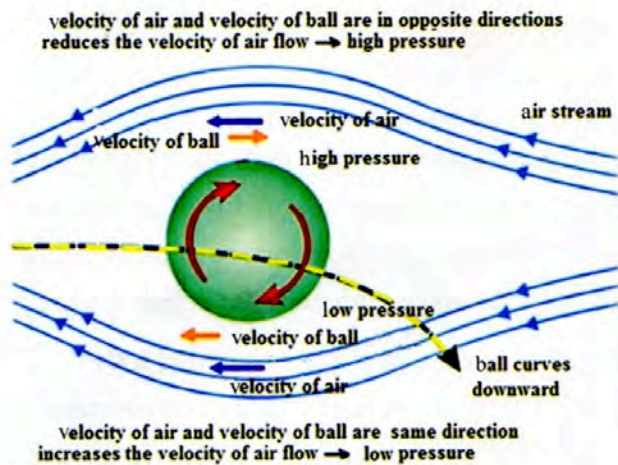


Figure 3.6 Magnus effect on a spinning ball

4. The Roof of Houses can fly away during Cyclones

During cyclones, moving air gain speed above the roof of a house causing the reduced pressure on the roof. Therefore the pressure inside the house is greater, which can raise the roof.

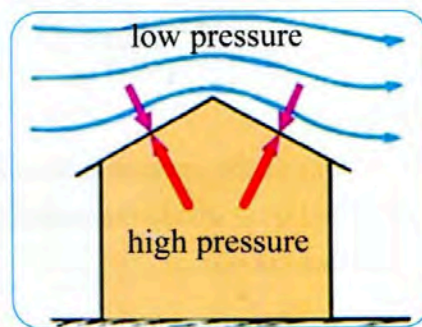


Figure 3.7 Roof blown away during cyclone

Example 3.8 Air streams horizontally past a small airplane's wings such that the speed is 70 m s^{-1} over and 60 m s^{-1} past the bottom surface. If the plane has a wing area of 16.2 m^2 on the top and on the bottom, what is the net vertical force that the air exerts on the airplane? The density of the air is 1.2 kg m^{-3} .

$$v_1 = 70 \text{ m s}^{-1}, v_2 = 60 \text{ m s}^{-1}, A = 16.2 \text{ m}^2, \rho = 1.2 \text{ kg m}^{-3}$$

For top and bottom surfaces of the wing, using Bernoulli's theorem,

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$\begin{aligned} \text{Pressure difference, } p_2 - p_1 &= \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2 & (h_1 = h_2) \\ &= \frac{1}{2} \times 1.2 [(70)^2 - (60)^2] = 780 \text{ Pa} \end{aligned}$$

$$\begin{aligned} F &= p A \\ &= 780 \times 16.2 = 12\,636 \text{ N} \end{aligned}$$

Reviewed Exercise

1. Can fluid flow from lower pressure to higher pressure?
2. Air is streaming past a horizontal aeroplane wing such that its speed is 120 m s^{-1} at the upper surface and 90 m s^{-1} at the lower surface. If the wing is 10 m long and 2 m wide and density of air is 1.3 kg m^{-3} , find the net lift on it.

Key Words: ideal fluid, kinetic energy per unit volume, potential energy per unit volume

3.5 VISCOSITY (Fluid Friction)

Most of the fluids are not ideal ones and offer some resistance to motion. This resistance to fluid motion is like an internal friction analogous to friction when a solid move on a surface. The resistance to fluid motion (or) internal resistance of fluid is called viscosity. Viscosity arises when there is relative motion between layers of the fluid. More precisely, it measures resistance to flow arising due to the internal friction between the fluid layers as they slip past one another when fluid flows.

Viscosity can also be thought of as a measure of a fluid's thickness (stickiness) or its resistance to objects passing through it. A fluid with large viscosity resists motion because its strong intermolecular forces give it a lot of internal friction, resisting the movement of layers past one another. On the contrary, a fluid with low viscosity flows easily because its molecular make up results in very little friction when it is in motion. Gases also exhibit viscosity, but it is harder to notice in ordinary circumstances.

The viscosity of liquids decreases rapidly with an increase in temperature, and the viscosity of gases increases with an increase in temperature. Thus, upon heating, liquids flow more easily, whereas gases flow more slowly. Also, viscosity does not change as the amount of matter changes, therefore it is an intensive property. The SI unit of viscosity is pascal second (Pa s) which is equivalent to newton second per metre squared (N s m^{-2}). It is sometimes referred to as the poiseuille (Pl).

There are two ways to measure the fluid's viscosity. One way is to measure fluid's resistance to flow when an external force is applied. This is known as dynamic viscosity. And the other way is to measure the resistive flow of a fluid under the weight of gravity. It is known as kinematic viscosity, which is more useful than dynamic viscosity (absolute viscosity).

Newton's Law of Viscosity

Newton's law of viscosity states that the shear stress between the two adjacent layers of fluid is directly proportional to the negative value of the velocity gradient between the same two adjacent fluid layers.

In symbols, shear stress $\tau \propto \frac{dv}{dy}$ (or) $\tau = -\eta \frac{dv}{dy}$

$$\gamma = \frac{F}{2l}$$

$$= \frac{0.04}{2 \times 0.16} = 0.13 \text{ N m}^{-1}$$

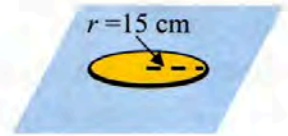
Example 3.12 Calculate the force required to pull a flat circular plate of radius 15 cm from the surface of a liquid which has a surface tension of 0.053 N m^{-1} .

Radius of circular plate $r = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$,

surface tension $\gamma = 0.053 \text{ N m}^{-1}$

circumference of the plate $l = 2\pi r$

$$= 2 \times 3.142 \times 15 \times 10^{-2} = 94.26 \times 10^{-2} \text{ m}$$



Since the circular plate has only one rim in contact with the liquid surface,

$$\gamma = \frac{F}{l}$$

$$0.053 = \frac{F}{94.26 \times 10^{-2}}$$

$$F = 0.0499 \text{ N}$$

Example 3.13 A needle has a length of 3.2 cm. When placed gently on the surface of the water in a glass, this needle will float if it is not too heavy. What is the weight of the heaviest needle? Assume that the surface tension of water is 0.073 N m^{-1} .

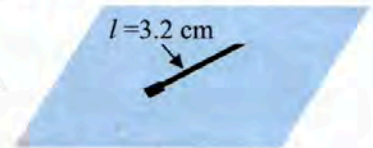
Surface tension of water $\gamma = 0.073 \text{ N m}^{-1}$, $l = 3.2 \text{ cm} = 3.2 \times 10^{-2} \text{ m}$,

Since the needle has two edges in contact with the liquid surface,

$$\gamma = \frac{F}{2l}$$

$$0.073 = \frac{F}{2 \times 3.2 \times 10^{-2}}$$

$$F = 4.67 \times 10^{-3} \text{ N}$$



This force is weight of the heaviest needle.

2. Capillary Action (Capillarity)

Capillary action is defined as the spontaneous flow of a liquid into a narrow tube or porous material. Capillary action is caused by the combination of cohesive forces of the liquid and the adhesive forces between the liquid and tube material. Cohesion and adhesion are two types of intermolecular forces. Cohesion refers to forces between molecules of same substance; whereas, adhesion refers to forces between molecules of different substances. These forces pull the liquid into the tube. In order for wicking to occur, a tube needs to be sufficiently small in diameter.

Capillarity (or) capillary action is also the rise or depression of a liquid in a narrow tube of small cross-sectional area, like the spaces between the fibres of a towel or the openings in a porous

Shear stress is the shearing force per unit area. That is $\tau = \frac{F}{A}$, where F is the shearing force acting between two adjacent layers of a fluid, η is the viscosity (coefficient of viscosity), A is the area of the fluid layer and $\frac{dv}{dy}$ is velocity gradient.

In Figure 3.8, $\Delta v = v_1 - v_2$, $\lim_{\Delta y \rightarrow 0} \frac{\Delta v}{\Delta y} = \frac{dv}{dy}$

Viscosity is defined as the ratio of shearing stress to the velocity gradient.

$$\eta = - \frac{F/A}{dv/dy} \quad (3.3)$$

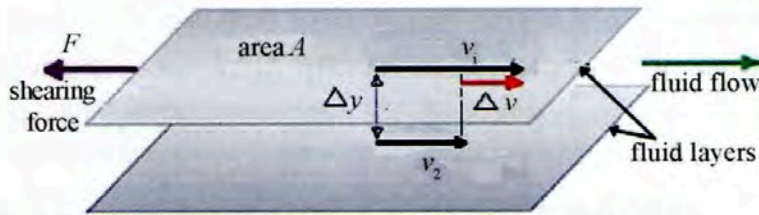


Figure 3.8 Force between two adjacent fluid layers

Determination of Kinematic Viscosity using Stokes' Law

Stokes' law, derived by the British scientist Sir George G. Stokes in 1851, expresses the drag force resisting the fall of small spherical objects through a fluid medium under the influence of gravity. It states that the drag force F acting upward in resistance to the fall of a spherical object in a viscous liquid is

$$F = 6 \pi \eta r v \quad (3.4)$$

where η is the coefficient of viscosity of the liquid, r is the radius of the sphere, and v is the constant velocity of fall called the terminal velocity.

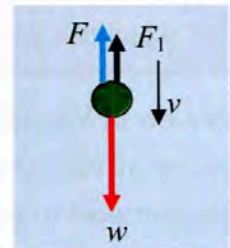
A simple way of measuring viscosity is that a small sphere ball is dropped through the fluid and time of fall and distance of fall are measured. From the distance and time of fall, the velocity can be determined.

Consider the fall of a sphere ball as shown in the diagram. The forces acting on the object are;

weight of the sphere ball $w = \frac{4}{3} \pi r^3 \rho_1 g$

upward thrust $F_1 = \frac{4}{3} \pi r^3 \rho_2 g$

drag force $F = 6 \pi \eta r v$



where ρ_1 is the density of the sphere, ρ_2 is the density of the liquid, and g is the acceleration due to gravity.

At a constant velocity of fall (terminal velocity), the upward and downward forces are in balance.

Therefore,
$$\frac{4}{3} \pi r^3 \rho_2 g + 6 \pi \eta r v = \frac{4}{3} \pi r^3 \rho_1 g$$

$$6 \pi \eta r v = \frac{4}{3} \pi r^3 (\rho_1 - \rho_2) g$$

$$\eta = \frac{2gr^2(\rho_1 - \rho_2)}{9v} \quad (3.5)$$

Example 3.9 A metal plate of area $2.5 \times 10^{-4} \text{ m}^2$ is placed on a $0.25 \times 10^{-3} \text{ m}$ thick layer of castor oil. If a force of 50 N is needed to move the plate with a velocity $3 \times 10^{-2} \text{ m s}^{-1}$, calculate the coefficient of viscosity of castor oil.

$$A = 2.5 \times 10^{-4} \text{ m}^2, \Delta y = (0.25 \times 10^{-3} - 0) = 0.25 \times 10^{-3} \text{ m},$$

$$F = -50 \text{ N (the drag force is opposite to the direction of motion),}$$

$$\Delta v = (3 \times 10^{-2} - 0) = 3 \times 10^{-2} \text{ m s}^{-1}$$

$$\text{velocity gradient } \frac{\Delta v}{\Delta y} = \frac{dv}{dy} = \frac{3 \times 10^{-2}}{0.25 \times 10^{-3}} = 120 \text{ m s}^{-1}$$

$$\begin{aligned} \eta &= -\frac{F/A}{dv/dy} \\ &= -\frac{(-50)}{2.5 \times 10^{-4}} \times \frac{1}{120} = 1.67 \times 10^3 \text{ Pa s} \end{aligned}$$

Example 3.10 Determine the radius of a rain drop falling through air with terminal velocity 9 m s^{-1} . Viscosity of air is $1.8 \times 10^{-5} \text{ Pa s}$, density of water is 1000 kg m^{-3} , density of air is 1.21 kg m^{-3} .

$$v = 9 \text{ m s}^{-1}, \eta = 1.8 \times 10^{-5} \text{ Pa s, density of water } \rho_1 = 1000 \text{ kg m}^{-3}, \text{ density of air } \rho_2 = 1.21 \text{ kg m}^{-3}$$

$$\begin{aligned} \eta &= \frac{2gr^2(\rho_1 - \rho_2)}{9v} \\ r &= \sqrt{\frac{9v\eta}{2g(\rho_1 - \rho_2)}} \\ &= \sqrt{\frac{9 \times 9 \times 1.8 \times 10^{-5}}{2 \times 9.8 \times (1000 - 1.21)}} = 2.7 \times 10^{-4} \text{ m} \end{aligned}$$

Reviewed Exercise

1. On which factors does the viscosity of a liquid depend?
2. How does temperature affect the viscosity of a fluid?

Key Words: intermolecular forces, intensive property, dynamic viscosity, kinematic viscosity

material. The rise of water underneath the ground is also the example of capillary action. Capillary action helps bringing water up into the roots.

When a glass tube with a small inside diameter is dipped into water, the water rises into the tube. For example, a glass tube with a bore of about 1 mm in diameter, the water will rise to a height of about 5 cm. The smaller the bore, the higher the water will rise as shown in Figure 3.10 (a).

Liquids that rise in small bore tubes inserted into the liquid are said to wet the tube, whereas liquids that are depressed within thin tubes below the surface of the surrounding liquid do not wet the tube. Water is a liquid that wets glass capillary tubes; mercury is one that does not, as shown in Figure 3.10 (b). Capillarity does not occur for non-wetting liquids.

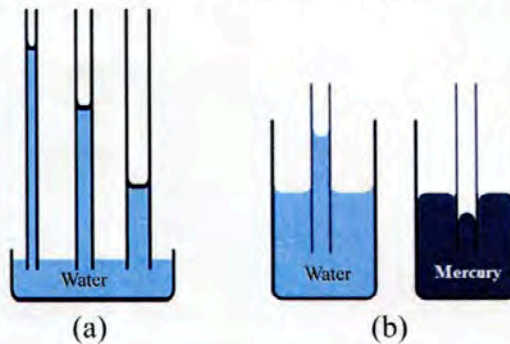


Figure 3.10 Capillary action (a) rising water into the tube (b) comparing water and mercury

When a glass tube is dipped into water, the adhesion between glass and water causes a thin film of water to be drawn up over the inner and outer surfaces of the tube. Since the adhesive force is greater than cohesive force, water level rises until there is a sufficient mass of liquid for gravitational forces to overcome these intermolecular forces.

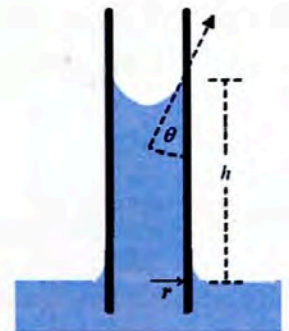
The actual amount of rise depends on the surface tension γ , the contact angle θ and the radius r of the tube. The height h to which the liquid can be lifted is given by,

$$h = \frac{2 \gamma \cos \theta}{\rho g r} \quad (3.7)$$

where ρ is density of liquid.

Example 3.14 When a capillary tube stands upright in a beaker of water, the water rises h in the tube. If the radius of the tube is 2 mm, contact angle is 30° , density of water is 1000 kg m^{-3} and surface tension is 0.072 N m^{-1} calculate h .

$$\gamma = 0.072 \text{ N m}^{-1}, \rho = 1000 \text{ kg m}^{-3}, \theta = 30^\circ, r = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$



$$h = \frac{2\gamma \cos \theta}{\rho g r}$$

$$= \frac{2 \times 0.072 \times \cos 30^\circ}{1000 \times 9.8 \times 2 \times 10^{-3}} = 0.0064 \text{ m} = 0.64 \text{ cm}$$

Example 3.15 At a certain temperature, water has a surface tension of 0.072 N m^{-1} . In a 3 mm diameter vertical tube if the liquid rises 6 mm above the liquid outside the tube, calculate the contact angle. Assume density of water $\rho = 1000 \text{ kg m}^{-3}$.

$$\gamma = 0.072 \text{ N m}^{-1}, \rho = 1000 \text{ kg m}^{-3}, h = 6 \times 10^{-3} \text{ m},$$

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}, r = \frac{d}{2} = 1.5 \times 10^{-3} \text{ m}, \theta = ?,$$

$$h = \frac{2\gamma \cos \theta}{\rho g r}$$

$$\cos \theta = \frac{h \rho g r}{2\gamma}$$

$$= \frac{6 \times 10^{-3} \times 1000 \times 9.8 \times 1.5 \times 10^{-3}}{2 \times 0.072} = 0.6125$$

$$\theta = 52.23^\circ$$

Reviewed Exercise

1. Water rises in a capillary tube, whereas mercury falls in the same tube. Explain.
2. A soap film is formed on a rectangular frame of length 7 cm side dipping into soap solution. The frame hangs from the arm of a balance. An extra weight of 0.4 g is to be placed in the opposite pan to balance the pull on the frame. Calculate the surface tension of soap solution.

Key Words: capillary tube, surface tension, viscosity, shearing force, terminal velocity

SUMMARY

If fluid particles are moving steady in smooth paths in layers, with each layer moving smoothly past the adjacent layers with no mixing; such a flow is called **laminar flow**.

If the flow or path of the fluid particles are irregular, their direction is always changing or whirling; this fluid movement is called **turbulent flow**.

Bernoulli's theorem states that the sum of pressure, kinetic energy per unit volume and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined laminar flow remains constant.

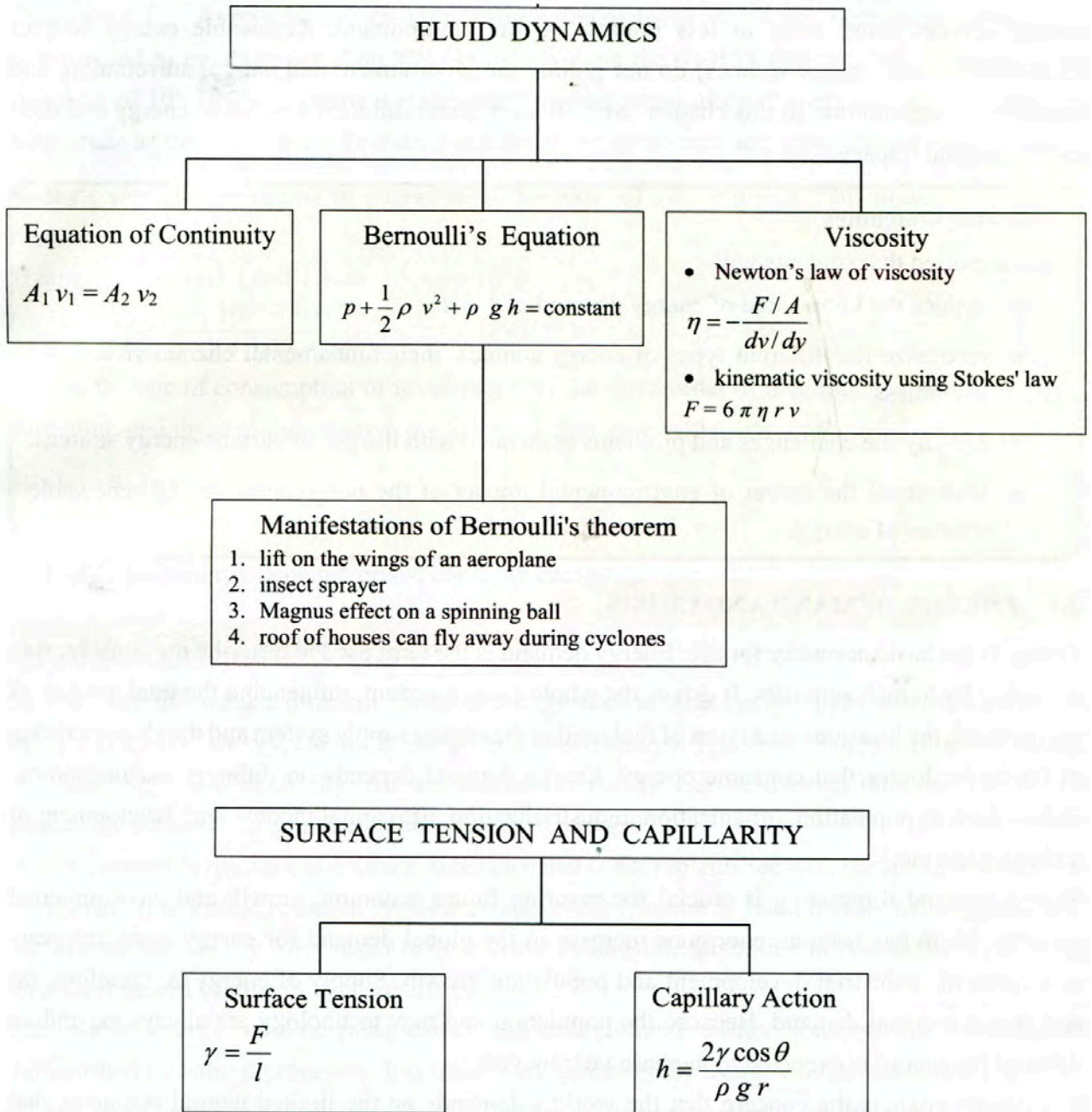
The resistance to fluid motion (or) internal resistance of fluid is called **viscosity**.

Surface tension is defined as the force per unit length that acts across any line on a surface tending to pull the surface apart.

EXERCISES

1. How are the flow rate and fluid velocity related?
2. A perfume bottle or atomizer sprays the fluid that is in the bottle. How does the fluid rise up in the vertical tube?
3. One hypodermic syringe contains medicine with density 1010 kg m^{-3} . The barrel of the syringe has a cross-sectional area $2.5 \times 10^{-5} \text{ m}^2$, and the needle has a cross-sectional area 10^{-8} m^2 . The syringe is in horizontal position and injection is forced by 2 N acting on the plunger. Find the speed of the injection that enters into the patient. (The volume of syringe is 3 cc and injection time is 2 s)
4. In an adult, the radius of aorta is normally $\sim 1.5 \text{ cm}$ and blood moves through it at an average speed of 30 cm s^{-1} . If typical capillary has a radius of $5 \times 10^{-6} \text{ m}$, and blood passes through them with a velocity of 0.1 cm s^{-1} approximately how many capillary are in the body?
5. An aeroplane's wings have a total surface area of 480 m^2 . The pressure difference between the upper and lower surfaces of each wing is 6500 Pa . Calculate the lift created.
6. Derive expression for terminal velocity when a ball of radius r is dropped through a liquid of viscosity η and density ρ .
7. The shear stress at a point in a liquid is found to be 0.03 N m^{-2} . The velocity gradient at the point is 0.15 s^{-1} . What will be its viscosity (in poise)? ($1 \text{ Pa s} = 10 \text{ poise}$)
8. A square plate 0.1 m side moves parallel to second plate with a velocity of 0.1 m s^{-1} , both plates being immersed in water. If the viscous force is 0.002 N and the coefficient of viscosity is 0.01 poise , what is the distance between the plates? ($1 \text{ Pa s} = 10 \text{ poise}$)
9. Assume that a spherical object is flowing through water. Viscosity of water is 0.001 Pa s , radius of spherical object is 2 mm and the velocity of the object at particular instant is 2 m s^{-1} . Find the drag force on the object due to the fluid. Assume that Stokes' law is valid.
10. Which force causes capillary action rise?
11. How does the temperature affect capillary action?
12. Identify which property of water allows water to move against gravity.
 - A. Capillary action allows water to move against gravity.
 - B. Surface tension allows water to move against gravity.
 - C. Cohesion allows water to move against gravity.
13. Due to capillary action, a liquid will rise in a tube, if the angle of contact is
 - A. acute
 - B. obtuse
 - C. 90°
14. What will happen to the rise of liquid in a capillary tube if the radius of the tube is (i) increased, (ii) decreased?
15. Calculate the height to which water will rise in a capillary tube of diameter 1 mm . The surface tension of water is 0.072 N m^{-1} , density of water is 1000 kg m^{-3} . (Assume angle of contact is 0° .)

CONCEPT MAP



CHAPTER 4

SOURCES OF ENERGY AND ENVIRONMENTAL IMPACTS

Energy resources are all forms of fuel used in the modern world, either for heating, generation of electrical energy, or for other forms of energy conversion processes. The different types of energy sources cause more or less impacts on the environment. Renewable energy sources (non-conventional energy sources) do not pollute the environment and natural surrounding and require less expenditure. In this chapter, we will learn about different sources of energy and their environmental impacts.

Learning Outcomes

It is expected that students will

- deduce the knowledge of energy demand and crisis.
- recognize the different types of energy sources, their fundamental characteristics and usefulness.
- identify the challenges and problems associated with the use of various energy sources.
- understand the extent of environmental impact of the non-renewable and renewable sources of energy.

4.1 ENERGY DEMAND AND CRISIS

Energy is the basic necessity for life. Energy demand is the term used to describe the consumption of energy by human activities. It drives the whole energy system, influencing the total amount of energy used; the locations and types of fuel used in the energy supply system and the characteristics of the technologies that consume energy. Energy demand depends on different socioeconomic factors such as population, urbanization, industrialization, net capital income and development of technologies, etc.

Energy demand forecasting is crucial for ensuring future economic growth and environmental security. There has been an enormous increase in the global demand for energy in recent years as a result of industrial development and population growth. Supply of energy is, therefore, far less than the actual demand. Because the population and new technology are always expanding demand for energy is expected to increase year by year.

The energy crisis is the concern that the world's demands on the limited natural resources that are used to power industrial society are diminishing as the demand rises. It is a situation in which a nation suffers from a disruption of energy supplies connected by increasing energy prices that threaten economic and national security.

With ninety percent of the world's oil reserves already discovered; people need to find new ways

to make energy. Since the non-renewable resources of the energy will not last long and cannot be regained, the world is going to face the problem of energy crisis in near future. Renewable energy has huge potential to provide solution to increase energy crisis and it is the key factor to the future of energy, food and economic security.

Example 4.1 At present day, the rate of formation of fossil fuels (coal, petroleum, natural gas) is estimated to be about 4 billion kW (4×10^9 kW) and the world's consumption of energy is of the order of 10^{17} Btu/year. Show that the rate of consumption of fossil fuel is of the same order of magnitude as the rate of their formation and these energy sources are therefore non-renewable.

Today's world consumption of energy is of the order of 10^{17} Btu/year. This number need to be converted to kW.

Using 1 Btu = 1055 J and 1 year = 3.15×10^7 s

$$10^{17} \text{ Btu/year} = \frac{10^{17} \times 1055}{1 \times 3.15 \times 10^7} \text{ J s}^{-1} = 3.35 \times 10^{12} \text{ W} = 3.35 \times 10^9 \text{ kW}$$

Hence, the rate of consumption of fossil fuel is of the same order of magnitude as the rate of their formation and these energy sources are therefore non-renewable.

Reviewed Exercise

- Why does the energy demand increase year by year?

Key Words: energy demand, crisis, consume energy

4.2 SOURCES OF ENERGY

In our daily life we use different forms of energy such as heat energy, light energy, mechanical energy, electrical energy, chemical energy and sound energy. The most common forms of energy are heat, light and electricity. All the sources of energy can be divided into two categories: renewable sources and non-renewable sources of energy.

A non-renewable resource is a natural substance that is not replenished with the speed at which it is consumed. It is a finite resource. Non-renewable energy resources include coal, natural gas, crude oil, and nuclear energy. They occur in limited and exhaustible quantities and cannot be regenerated in a short period of time or used again and again.

Renewable energy sources (also called non-conventional energy sources) are continuously replenished by natural processes. It is the energy generated by using wind, solar, hydro, tides, etc. They cannot be exhausted easily and can be generated at a constant rate for their use over and over again. They can be produced or generated through natural processes, at a rate greater than or equal to the rate of their consumption.

Non-renewable energy sources are available only in limited supply. Examples include fossil fuels (like coal, oil, natural gases, etc.) and nuclear energy. It takes a long time (often millions of years) for these resources to be replenished.

Renewable resources are replenished naturally and over short periods of time, so are in a virtually unlimited supply. Examples include the solar energy, hydro energy, wind energy and geothermal energy.



Figure 4.1 Energy resources

Reviewed Exercise

- What is the main source of energy?

Key Words: renewable sources of energy, non-renewable sources of energy

4.3 NON-RENEWABLE SOURCES OF ENERGY

1. Fossil Fuels

Fossil fuels, such as coal, oil and natural gas, are important non-renewable sources of energy. Carbon is the main constituent of these fossil fuels. A large amount of chemical energy is stored in the fossil fuels. This stored chemical energy is converted into various other forms of energy such as heat, light and mechanical energy. Fossil fuels are excellent sources of energy for our transportation needs. For example, the fossil fuel industry extracts crude oil from the ground and converts it to gasoline. Fossil fuel liquids also are refined into petrochemical products that are used as ingredients in the manufacture of literally hundreds of products from plastics and polyurethane to solvents.

Coal is formed in a way similar to the other fossil fuels, though it goes through a different process called coalification. It is made of decomposed plant matter in conditions of high temperature and pressure, and it takes a relatively shorter amount of time to form. Coal is not a uniform substance; its composition varies from deposit to deposit.

Natural gas is another major source of the energy in our country. Oil and gas fields have been found some places on the earth. These fields always contain some gases, but this natural gas (methane) does not take nearly as long to form. Natural gas is also found in independent deposits within the ground as well as from other sources too. Methane is a common gas found in swamps and is also the by-product of animals' digestive system.

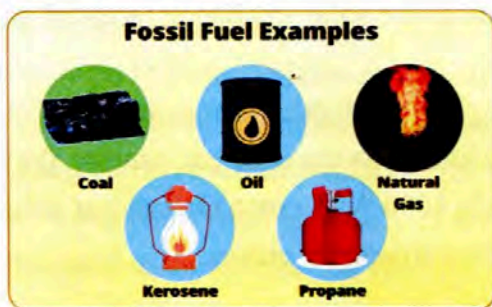


Figure 4.2 Examples for fossil fuel

2. Nuclear Energy

Nuclear energy is non-renewable as the uranium fuels used are consumed in the nuclear fission reaction and hence are non-replenishable. Nevertheless, nuclear energy has many uses:

- ◆ energy produced in a nuclear reactor can be harnessed to produce electricity,
- ◆ nuclear energy is also being used to power submarines and ship. Vessels driven by nuclear energy can sail for long periods without having to refuel,
- ◆ radioisotopes obtained as by-products in nuclear reactions are used in medicine, agriculture and research.



Reviewed Exercise

- What are the main characteristics of fossil fuels?

Key Words: fossil fuels, petrochemical products, nuclear energy

4.4 RENEWABLE SOURCES OF ENERGY

There are several alternative and renewable sources of energy which are not only environment friendly but can also be available in abundance.

The major types of renewable energy sources are: (i) solar, (ii) wind, (iii) hydropower, (iv) ocean, (v) geothermal and (vi) biomass.

1. Solar Energy

Sun is one of the most powerful renewable sources of energy. Solar energy is the radiant light and heat from the sun that has been harnessed by humans since ancient times using a range of

ever-evolving technologies. Earth's atmosphere and clouds absorb or scatter as much as 54 % of the incoming sunlight. The sunlight that reaches the ground consists of visible light, infrared, ultraviolet and other forms of electromagnetic radiation.

The total amount of radiation energy passing through per unit area and per unit time is called irradiance and is measured in W m^{-2} or kW m^{-2} . Another commonly used term for irradiance of sunlight is insolation. Insolation is the solar irradiance received on a horizontal surface such as ground.

Above the earth's atmosphere, solar radiation has an intensity of approximately 1350 W m^{-2} ($429 \text{ Btu h}^{-1} \text{ ft}^{-2}$). This value is known as the solar constant. At the earth's surface solar radiation has an intensity of approximately 1000 W m^{-2} on a clear day at solar noon in the summer months. Figure 4.3 shows the solar spectral irradiance curves.

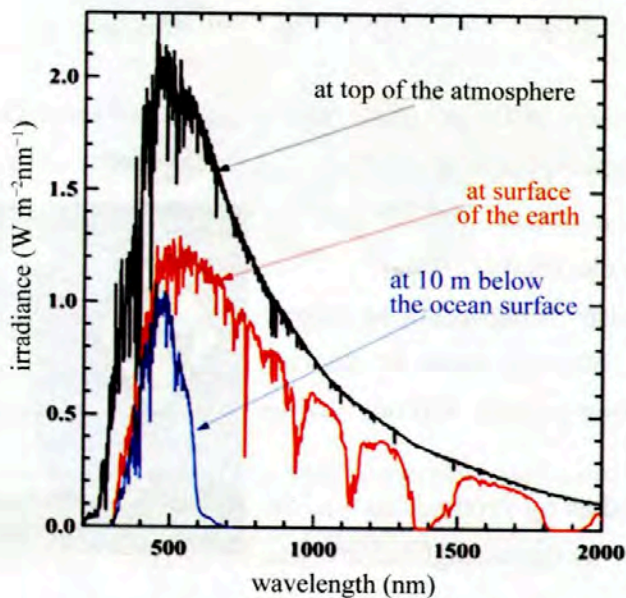


Figure 4.3 The solar irradiance curves at the top of the atmosphere and at surface of the earth

Solar Technologies

There are three main ways to harness solar energy: photovoltaic solar cell, solar heating, and concentrated solar power (CSP).

Photovoltaic Solar Cell

Conversion of light energy into electrical energy is based on a phenomenon called photovoltaic effect. When semiconductor materials are exposed to light, some of the photons of light are absorbed by the semiconductor crystal which causes a significant number of free electrons in the crystal. This is the basic reason for producing electricity due to photovoltaic effect.

A solar cell (also known as a photovoltaic cell) is defined as an electrical device that converts light energy into electrical energy through the photovoltaic effect. A solar cell is basically a p-n junction diode. When light is incident on the photovoltaic cell, the light photons can easily reach the p-n junction. The light energy, in the form of photons, supplies sufficient energy to the junction to create a number of electron-hole pairs as shown in Figure 4.4. As a result, a current flow in the external circuit connected to the p-n junction.

Individual solar cells can be combined to form modules commonly known as solar panels. When combined into a large solar panel, considerable amounts of renewable energy can be generated. Achieving the perfect perpendicular sun rays is the optimal orientation so that solar panels receive direct light throughout the day. Power conversion efficiency of the solar panel is the ratio of the electrical power delivered by the panel to the solar power received by it.

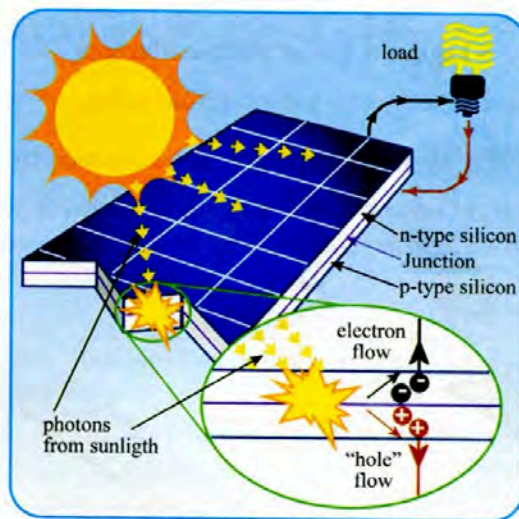


Figure 4.4 Photovoltaic solar cell

Solar Heating

Solar heating can also play an important role in reducing the impact of global warming. Solar water heating systems installed in homes are comprised of three main elements: the solar collector, insulated piping, and a hot water storage tank. The solar collector gathers the heat from solar radiation and transfers the heat to water. This heated water flows out of the collector to a hot water tank, and is used as necessary.

Flat plate collectors are the most common type of collector. Flat plate collectors have copper pipes attached to an absorber plate contained in an insulated box that is covered with a tempered glass or polymer cover plate.

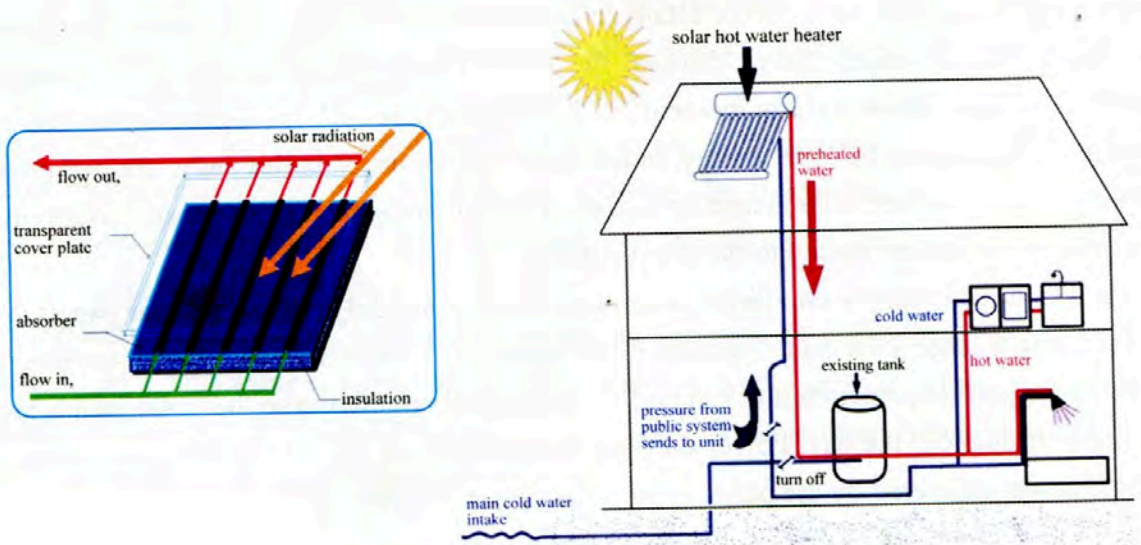


Figure 4.5 Flat plate collectors

Concentrated Solar Power

Concentrated solar power (CSP) systems generate solar power by using mirrors or lenses to concentrate a large area of sunlight onto a specific point of receiver which converts into heat.

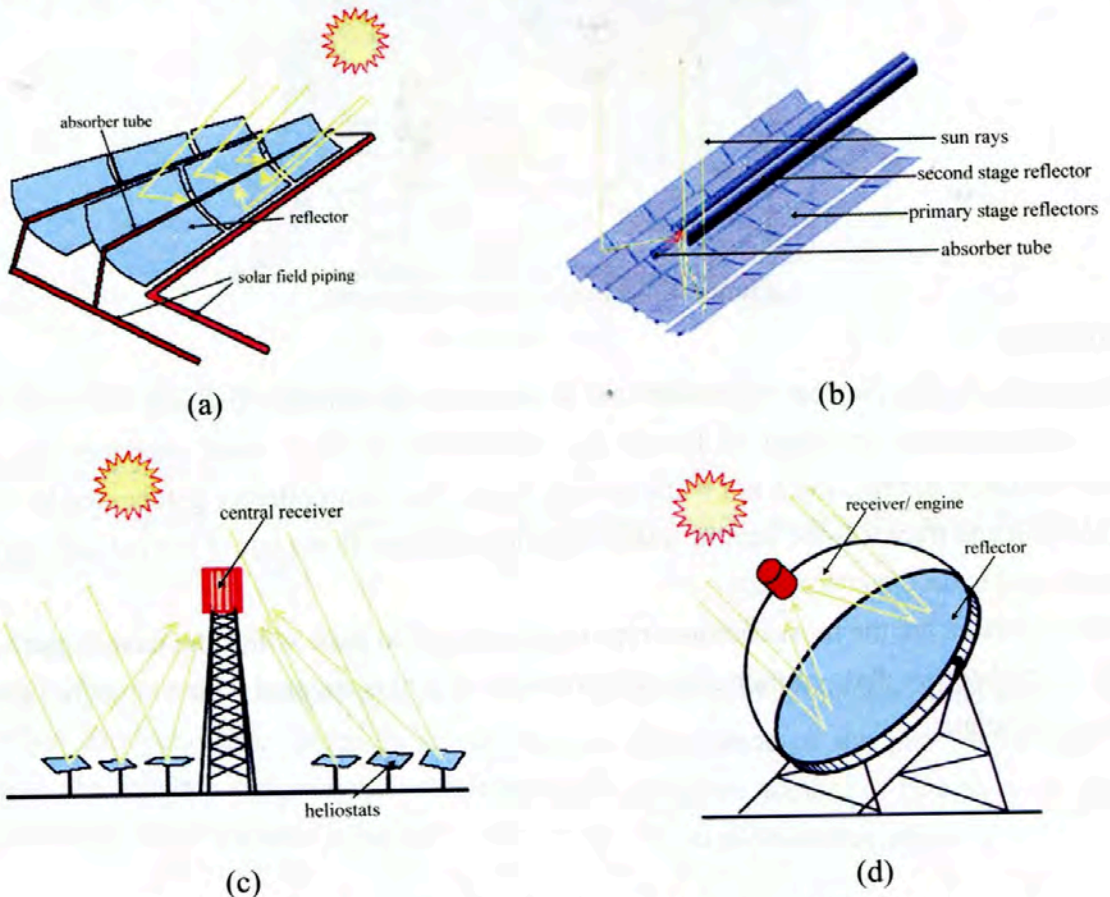


Figure 4.6 Four types of concentrated solar power (CSP) systems

The heat is then used to create steam, which drives a turbine to generate electrical power. Therefore CSP plants consist of two parts: one that collects solar energy and converts it to heat, and another that converts the heat energy to electricity.

There are four types of CSP technologies: (a) Parabolic trough systems, (b) Linear fresnel systems, (c) Power tower systems and (d) Parabolic dish systems. Figure 4.6 (a), (b), (c) and (d) show the four types of CSP technologies.

Example 4.2 A home requires 62 kWh of heat on a winter day to maintain a constant indoor temperature of 20 °C. Assume that the temperature of hot water outlet is 60 °C. (i) How much collector surface area does it need for a solar heating system that has a 20 % efficiency? (ii) How large does the storage tank have to be to provide this much energy? (The average solar radiation per day in winter is about 6.5 kWh m⁻²)

Solar radiation per day = 6.5 kWh m⁻², efficiency of system = 20 % = 0.2

$$\begin{aligned} \text{(i) Thermal energy per day} &= \text{solar radiation per day} \times \text{efficiency of system} \\ &\hspace{15em} \text{(obtained using collector)} \\ &= 6.5 \times 0.2 = 1.3 \text{ kWh m}^{-2} \end{aligned}$$

This means that for every square metre of collector surface area, 1.3 kWh of heat are produced every day.

$$\text{Collector surface area} = \frac{\text{heat required}}{\text{heat produced per unit area}} = \frac{62}{1.3} = 48 \text{ m}^2$$

(ii) Temperature difference is that between the hot fluid and the cold water going into the storage tank = 60 – 20 = 40 °C = 40 K

Since 1 kWh = 3.6 × 10⁶ J, ΔQ = 62 kWh = 62 × 3.6 × 10⁶ J

Specific heat capacity of water $c = 4184 \text{ J kg}^{-1} \text{ K}^{-1}$

$$\Delta Q = m c \Delta T$$

$$m = \frac{\Delta Q}{c \Delta T}$$

$$= \frac{62 \times 3.6 \times 10^6}{4184 \times 40} = 1333.6 \text{ kg}$$

This is equivalent to a volume of 1334 liters (or about 352.4 gallons).

Reviewed Exercise

1. The band gap of GaAs is 1.4 eV. Calculate the minimum wavelength of light for photovoltaic generation in a GaAs solar cell.
2. A 100 ft × 50 ft building has a flat roof. What is the average solar energy received by the roof in a month? (The solar radiation per day in winter is about 1600 Btu ft⁻²)

3. Calculate the solar energy received by a standard hot water collector of dimensions $1\text{ m} \times 2\text{ m}$, over one hour at around noon, if the irradiance stays fairly constant at about 800 W m^{-2} .

Key Words: irradiance, insolation, photovoltaic cell, concentrated solar power

2. Wind Energy

Wind energy is the energy generated or produced by harnessing the power of the wind. Like solar power, harnessing the wind is highly dependent on weather and location. However, it is one of the oldest and cleanest forms of energy and the most developed of the renewable energy sources. Both windmills and wind turbines harness wind energy and put it to practical use. Windmills convert wind energy directly into mechanical energy for such tasks as milling grain and pumping water. A wind turbine converts wind energy into electricity, which can then be used to power electrical equipment, stored in batteries or transmitted over power lines.

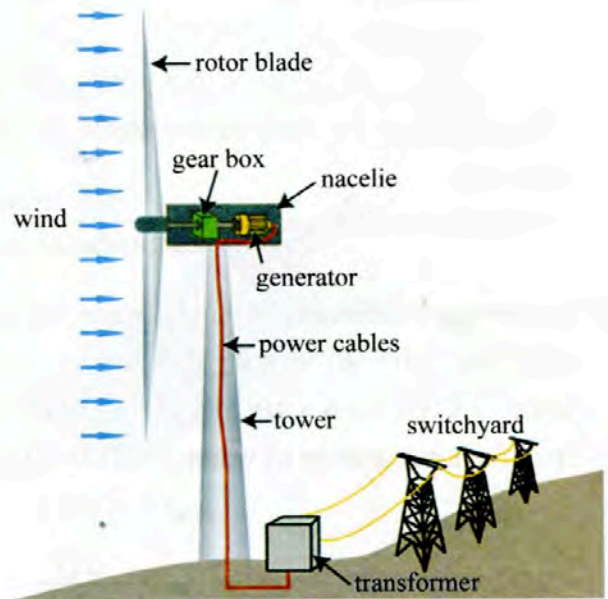


Figure 4.7 Windmill and wind turbines

In a wind turbine, wind first hits the blades of the turbine, causing them to rotate and turn the turbine connected to them. That changes the kinetic energy to rotational energy, by rotating a shaft which is connected to a generator, and thereby producing electrical energy through electromagnetism. Wind farms consist of many individual wind turbines, which are connected to the electric power transmission network.

The amount of power that can be harvested from wind depends on the size of the turbine and the length of its blades. Wind energy is the kinetic energy of air in motion. Total wind energy flowing through an imaginary surface with area A during the time t is calculated as follows,

$$E = \frac{1}{2} m v^2 = \frac{1}{2} \rho V v^2 = \frac{1}{2} \rho A v t v^2 = \frac{1}{2} \rho A t v^3$$

where ρ is the density of air, v is the wind speed, $A v t$ is the volume V of air passing through area A (which is considered perpendicular to the direction of the wind) in time t .

Power is energy per unit time, so the wind power incident on A is,

$$P = \frac{1}{2} \rho A v^3 \quad (4.1)$$

where A = the rotor area of a wind turbine.

The output is proportional to the dimensions of the rotor and to the cube of the wind speed. Theoretically, when wind speed doubles, wind power potential increases by a factor of eight.

Wind turbines, like windmills, are mounted on a tower to capture the most energy. At 100 feet (30 meters) or more above ground, they can take advantage of the faster and less turbulent wind. Turbines catch the wind energy with their propeller-like blades. Usually, two or three blades are mounted on a shaft to form a rotor.

Example 4.3 Determine the input power of the wind if the wind speed is 20 m s^{-1} and blade length is 20 m. Density of air is 1.3 kg m^{-3} .

Radius of blade of turbine $r = 20 \text{ m}$, wind speed = 20 m s^{-1} , density of air = 1.3 kg m^{-3}

Area of a wind turbine $A = \pi r^2 = 3.142 \times (20)^2 = 1256.8 \text{ m}^2$

$$\begin{aligned} \text{Wind power} \quad P &= \frac{1}{2} \rho A v^3 \\ &= \frac{1}{2} \times 1.3 \times 1256.8 \times (20)^3 \\ &= 6.535 \times 10^6 \text{ W} = 6.535 \text{ MW} \end{aligned}$$

Reviewed Exercise

- How does wind power generate electricity?

Key Words: windmill, wind turbine

3. Hydroelectric Energy

The flowing water and water stored in huge dams is also a very important source of energy which is known as hydroelectric energy. Hydropower, or hydroelectric power, is one of the oldest and largest sources of renewable energy. The source of power in a hydroelectric plant is the motion of water. It is an affordable source of electricity that costs less than most of other sources of energy. However, overdevelopment and unrestricted harnessing of water power can have a devastating effect on the local environment and habitation areas. There are three types of hydropower facilities: impoundment, diversion, and pumped storage.

Hydroelectric energy, also called hydroelectric power or hydroelectricity, is a form of energy that harnesses the power of water in motion such as water flowing over a waterfall to generate electricity.

Impoundment Facility

The most common type of hydroelectric power plant is an impoundment facility. It is typically a large hydropower system, uses a dam to store river water in a reservoir. Water is collected or stored at a higher elevation and gravity pulls the water downward through large pipes or tunnels (penstocks) to a lower elevation; the difference in these two elevations is known as the head. At the end of its passage down the pipes, the falling water causes turbines to rotate. The turbines in turn drive generators, which convert mechanical energy into electricity. Transformers are then used to convert the alternating voltage of the generators to a higher voltage suitable for long-distance transmission.

A penstock is a closed conduit that channels the flow of water to turbines with water flow regulated by gates and valves.

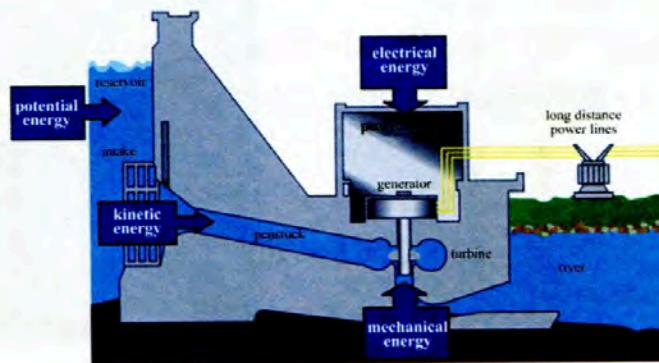
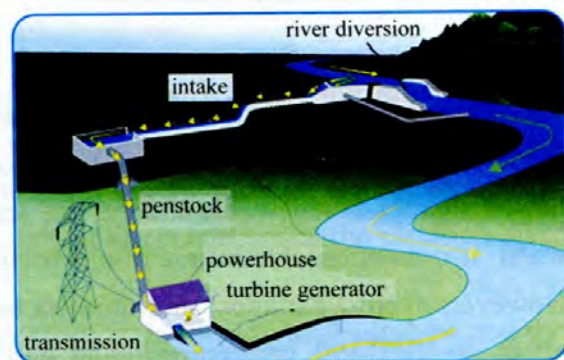


Figure 4.8 Hydroelectric power plant

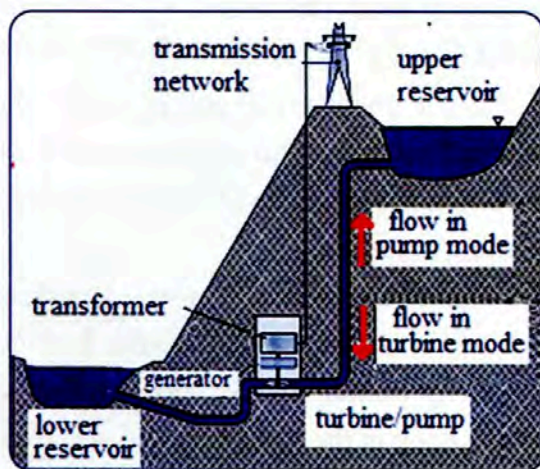
Diversion Facility (or) Run-of-river Facility

A diversion, sometimes called a run-of-river facility, channels a portion of a river through a canal or a penstock to utilize the natural decline of the river bed elevation to produce energy. A diversion may not require the use of a dam.



Pumped Storage Facility

Another type of hydropower is pumped storage hydropower (PSH). A PSH facility is able to store the electricity generated by other power sources. When the demand for electricity is low, a PSH facility stores energy by pumping water from the lower reservoir to an upper reservoir at a higher elevation. During periods of high electrical demand, the water is released back to the lower reservoir and turns a turbine, generating electricity.



Available Hydroelectric Power

A hydropower resource can be evaluated by its available power. The power available from falling water can be calculated from the flow rate and density of water, the height of fall, and the local acceleration due to gravity.

$$P = \eta \rho \dot{V} g h$$

where P is useful power output (W), η is efficiency of the turbine (dimensionless), ρ is density of water, h is difference in height between the outlet and inlet and $\dot{V} = \frac{V}{t}$ = volumetric flow rate. It is measured in $\text{m}^3 \text{s}^{-1}$.

Most conventional hydroelectric plants include most common parts, (i) dam (raises the water level of the river and controls flow of water) (ii) turbine (a water turbine converts the kinetic energy of falling water into mechanical energy) (iii) generator (connected to the turbine by shafts and converts the mechanical energy from the turbine to electrical energy) (iv) transformer (v) transmission system (transport electricity from the hydropower station to consumers).

Example 4.4 At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is $100 \text{ m}^3 \text{ s}^{-1}$. If the turbine generator efficiency is 60 %, estimate the electric power available from the plant. ($g = 9.8 \text{ m s}^{-2}$, $\rho = 1000 \text{ kg m}^{-3}$)

$h = 300 \text{ m}$, Volume of water flowing per second $\dot{V} = 100 \text{ m}^3 \text{ s}^{-1}$, $\eta = 60 \% = 0.6$

$$\begin{aligned} \text{Available electric power} \quad P &= \eta \rho \dot{V} g h \\ &= 0.6 \times 1000 \times 100 \times 9.8 \times 300 \\ &= 17.64 \times 10^7 \text{ W} = 176.4 \text{ MW} \end{aligned}$$

Reviewed Exercise

- ♦ The hydroturbine receives water from a reservoir at an elevation of 100 m above it. What is the minimum water flow in kg s^{-1} to produce a steady turbine output of 50 MW if the turbine generator efficiency is 75 %?

Key Words: dams, hydroelectric, hydropower, reservoir, pumped storage, efficiency

4. Energy from the Ocean

Tidal Energy

Due to the gravitational pull of mainly the moon on the spinning earth, the level of water in the sea rises and falls. This phenomenon is called high and low tides. The tidal energy is produced by the difference in sea levels. It is harnessed by constructing a dam (a tidal barrage) across a narrow opening to the sea.

A tidal barrage works similarly to a large hydropower reservoir dam but it is placed at the entrance to a bay or estuary. A turbine fixed at the opening of the tidal barrage converts tidal energy to electricity. Similar to river current technologies, turbines anchored to the ocean floor or suspended from a buoy in the path of an ocean current could also be used to generate power.

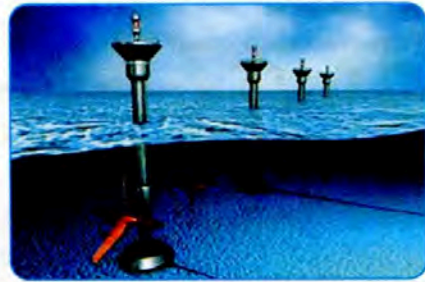
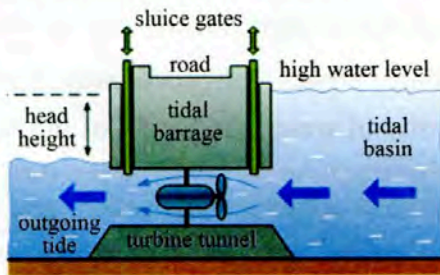


Figure 4.9 Ocean tidal power plant

Wave Energy

The kinetic energy of huge waves near the seashore can be trapped to generate electricity. The waves are generated by strong winds blowing across the sea. A variety of technologies are being tested to convert wave energy into electricity. Most systems capture energy on the surface of waves or use pressure differences just below the surface.

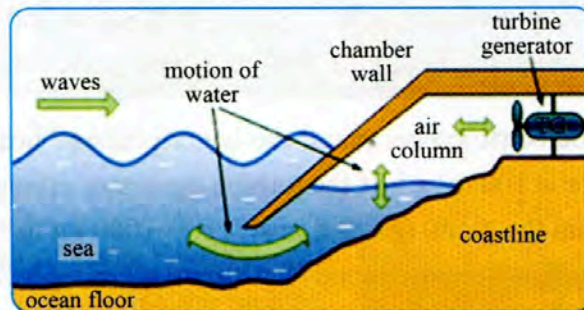


Figure 4.10 Conversion of wave energy into electricity

Ocean Thermal Energy

The water at the surface of the sea or ocean is heated by the sun while the water in deeper sections is relatively cold. This difference in temperature is exploited to obtain energy in ocean-thermal-energy conversion plants. These plants can operate if the temperature difference between the water at the surface and water at depths up to 2 km is 20 K (20 °C) or more. The warm surface-water is used to boil a volatile liquid (e.g. ammonia, boiling point ~ -33.34 °C). The vapours of the liquid (ammonia) are then used to run the turbine of generator. The cold water from

the depth of the ocean is pumped up and condense vapour again to liquid. The energy potential from the sea (tidal energy, wave energy and ocean thermal energy) is quite large, but efficient commercial exploitation is difficult.

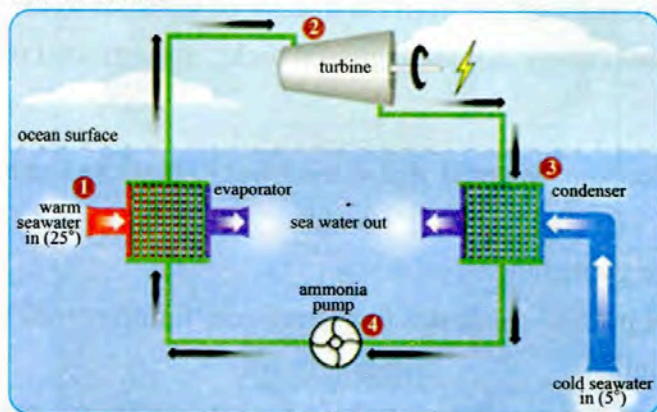


Figure 4.11 Conversion of ocean thermal energy into electricity

Reviewed Exercise

1. Why are wind and solar energies more popular than ocean energy?
2. What type of working fluids do closed-cycle ocean thermal energy conversion systems use?

Key Words: ocean thermal energy, tidal energy, wave energy

5. Geothermal Energy

Geothermal energy is a type of renewable energy taken from the Earth's core (the internal heat of the earth) and it is one of the oldest types of natural sources of heat. However, it is limited to regions of the world where there are hot underground rocks. Thermal energy from magma can be used to generate electricity. Magma is extremely hot liquid and semi-liquid rock located under Earth's surface.

Wells of up to a mile deep or more are drilled into underground reservoirs to tap into the geothermal resources. These resources can be exploited from naturally occurring heat, rock and water permeability. Cooled water is pumped into heated rocks where it is turned into steam, which then rises up to drive the turbines and generate electricity. Condensed water is cooled and the process is repeated.

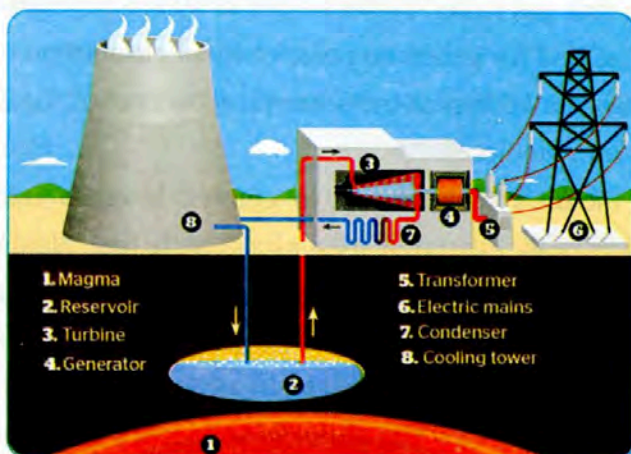


Figure 4.12 Electricity generated from geothermal energy

Reviewed Exercise

1. What is geothermal energy?
2. How does geothermal heat get from the interior of the earth to the surface?

Key Words: geothermal energy, hot underground rocks, underground reservoirs, magma

6. Biomass

Biomass is renewable organic material that comes from plants and animals. Biomass contains stored chemical energy.

Biomass sources for energy include:

- ◆ Wood and wood processing wastes (e.g. firewood, furniture mill sawdust and waste from pulp and paper mills).
- ◆ Agricultural crops and waste materials (e.g. corn, soybeans, sugar cane, woody plants, algae, rice husk and food processing residues).
- ◆ Biogenic materials in municipal solid waste (e.g. paper, cotton, wool products, food and wood wastes).
- ◆ Animal manure and human sewage.

Biomass is converted to energy through various processes, including: direct combustion (burning) to produce heat, thermochemical conversion to produce solid, gaseous, and liquid fuels, chemical conversion to produce liquid fuels, biological conversion to produce liquid and gaseous fuels.

Direct combustion is the most common method for converting biomass to useful energy. All biomass can be burned directly for heating buildings and water, for industrial heat process, and for generating electricity in steam turbines.

Thermochemical conversion of biomass includes pyrolysis and gasification. Both are thermal decomposition processes in which biomass feedstock materials are heated in closed, pressurized vessels called gassifiers at high temperatures. Biomass pyrolysis produces fuels such as charcoal, bio-oil, renewable diesel, methane, and hydrogen. Biomass gasification produces a carbon monoxide and hydrogen rich gas called synthesis gas or syngas. Syngas can be used as a fuel for diesel engines, for heating, and for generating electricity in gas turbines.

Chemical conversion process known as transesterification is used for converting vegetable oils, animal fats, and greases into biodiesel.

Biological conversion includes fermentation to convert biomass into ethanol and anaerobic digestion to produce natural gas. Ethanol is used as a vehicle fuel. Natural gas, also called biogas or biomethane, is produced in anaerobic digesters at sewage treatment plants and at dairy and livestock operations. It also forms in and may be captured from solid waste landfills. Properly treated renewable natural gas has the same uses as fossil fuel natural gas.



Figure 4.13 Biomass sources for energy

Reviewed Exercise

1. What is biomass?
2. Give the types of biomass energy sources.

Key Words: Biomass, direct combustion, biological conversion, chemical conversion

4.5 ENVIRONMENTAL IMPACTS AND CONSEQUENCES

Apart from the problem of energy resources getting depleted, energy harnessing and utilization cause an immense amount of environmental damage. There are environmental problems associated with the use of almost all forms of energy. The environmental problems directly related to energy production and consumption include air pollution, climate change, water pollution, thermal pollution, and solid waste disposal. It is important to understand the environmental impacts associated with producing electrical power from both non-renewable and renewable sources.

1. Environmental Impacts of Conventional Sources

The problems that relate to the use of fossil fuels are: global warming, acid rains, dangers posed by leaded fuels, oil spills, gas leaks and explosions, air pollution and water pollution caused by poorly managed coal mines.

Global warming refers to the gradual increase in the average temperature of the earth's surface and its atmosphere which has been attributed to the accumulation of greenhouse gases. The main greenhouse gases are carbon dioxide (CO_2), methane (CH_4), water vapour, oxides of nitrogen (NO_x) and chlorofluorocarbons (CFCs). CFCs also destroy the earth's protective ozone layer which shields the earth from harmful ultraviolet (UV) rays from the sun.

Acid rains are caused mainly by the release of sulphur dioxide (SO_2) and oxides of nitrogen (NO_x) when fossil fuels burn. The oxides combine with water vapour in the air to form acids, which

return to the ground as acid rain. Because carbonic acid is a relatively weak acid, the ability of carbon dioxide alone to generate true acid rain is very limited. It is important to note that acidified clouds could travel great distances before releasing the acid rain. The problems posed by acid rains include corrosion of the built environment, soil degradation, water pollution and depletion of forests.

The oil industry adds lead (Pb) to petrol (gasoline) to help engines run more smoothly. Vehicles that burn leaded gasoline pour out leaded fumes that contaminate the air. The World Health Organisation (WHO) has established that smoke from the combustion of leaded fuels in vehicles causes cancer and high blood pressure in adults; and in children, it impairs mental development, reduces intelligence thus hindering learning ability and causes behavioural disorders.

The leakage of fuel oil from storage vessels, oil tankers, pipelines and tanker trucks is unavoidable. Oil spills seriously damage the land, vegetation, and water bodies, including the oceans. In addition, spilled oil can damage the ecosystem and its habitats.

Gas leaks and explosions sometimes accompany the harnessing and utilization of fossil fuels especially in the coal mines and storage plants. The explosions are sometimes accompanied by fire outbreaks.

Excavated areas that have been strip mined for coal but are not filled and revegetated cause water pollution as surface water runoff from the mined area can flush sediments and sulphur-bearing compounds into nearby streams and rivers. This could endanger human life, plant and wildlife communities.

Emissions from vehicles, thermal power plants and factories contain unburned hydrocarbons, particulates, carbon dioxide, carbon monoxide and oxides of nitrogen and sulphur that contribute to the lowering of the quality of air. These substances in the air could irritate the eyes, throat and the lungs.



Figure 4.14 Environmental impacts of conventional sources

2. Environmental Impacts of Non-Conventional Sources

Environmental Impacts of Solar Power

Photovoltaics is now a proven technology which is inherently safe, as opposed to some dangerous

electricity generating technologies. Over its estimated life a photovoltaic module will produce much more electricity than was used in its production. A solar module will prevent the emission of CO₂ produced by other conventional energy sources. Photovoltaic systems make no noise and cause no pollution while in operation. PV cell technologies have relatively lower environmental risks compared to other types of electric sources. However, chemicals used in PV cells in the manufacturing facility, the installation site, and the disposal or recycling facility could be released to air, surface water, and groundwater.

Environmental Impacts of Wind Power

A wind farm, when installed on agricultural land, has one of the lowest environmental impacts of all energy sources. It occupies less land area per kilowatt hour (kWh) of electricity generated than any other energy conversion system. Greenhouse gas emissions and air pollution produced by its construction are very tiny and declining.

Modern wind turbine designs have significantly reduced the noise from turbines. Low frequency sound and infrasound (i.e. usually beneath the threshold of human hearing) are everywhere in the environment. Modern turbine designs which locate the blades upwind instead of downwind have significantly reduced the level of infrasound. Wind turbines may create shadow flicker on nearby residences when the sun passes behind the turbine. However, this can easily be avoided by locating the wind farm to avoid unacceptable shadow flicker, or turning the turbine off for a few minutes of the day when the sun is at the angle that causes flicker.

Environmental Impacts of Hydroelectric Power

Although hydropower has no air quality impacts, construction and operation of hydropower dams can significantly affect natural river systems as well as fish and wildlife populations. It has major effect on river transportation, agriculture, fertile lands, etc. Assessment of the environmental impacts of a specific hydropower facility requires case-by-case review.

Negative impacts of dams are as follows:

- ◆ in flat basins large dams cause flooding of large tracts of land, destroying local animals and habitats,
- ◆ villages and people have to be displaced causing change in life style,
- ◆ large amounts of plant life are submerged and decay anaerobically,
- ◆ the migratory pattern of river animals are affected,
- ◆ dams restrict sediments that are responsible for the fertile lands downstream,
- ◆ large dams are breeding grounds for mosquitoes and cause the spread of disease,
- ◆ dams serve as a heat sink, water in dams is hotter than the normal river water and when this warm water released into the river downstream can affect animal life.

Environmental Impacts of Geothermal Energy

Geothermal power is a relatively benign (less harmful) source of energy. For the most part, the impacts of development are positive. Worldwide geothermal energy utilization increases yearly because it is an attractive alternative to burning fossil fuels. Electricity generation from geothermal resources involves much lower greenhouse gas emission rates than that from fossil fuels.

However, geothermal development could have certain negative impacts if appropriate mitigation actions and monitoring plants are not in place. Any large-scale construction and drilling operation will produce visual impacts on the landscape, create noise and wastes and affect local economies.

Environmental Impacts of Biomass

Biomass power plants share some similarities with fossil fuel power plants: both involve the combustion of a feedstock to generate electricity. Thus, biomass plants raise similar, but not identical, concerns about air emissions and water use as fossil fuel plants. Biomass power plants, like coal and natural gas-fired power plants, require water for cooling. Land use impacts from biomass power production are driven primarily by the type of feedstock: either a waste stream or an energy crop that is grown specifically for generating electricity.

There are global warming emissions associated with growing and harvesting biomass feedstock, transporting feedstock to the power plant, and burning or gasifying the feedstock. Transportation and combustion emissions are roughly equivalent for all types of biomass. It is understood that some biomass feedstock sources are associated with substantial global warming emissions. However, global warming emissions from the sourcing of biomass feedstock vary widely.

The energy transfers which occur in each type of energy resource.

Fossil fuels: chemical energy → thermal energy → kinetic energy → electrical energy

Solar energy: light & thermal energy → electrical energy

Nuclear: nuclear energy → thermal energy → kinetic energy → electrical energy

Wind: kinetic energy → electrical energy

Bio-fuel: chemical energy → thermal energy → mechanical energy → electrical energy

Hydroelectricity: gravitational potential energy → kinetic energy → electrical energy

Geothermal: thermal energy → kinetic energy → electrical energy

Tidal: kinetic energy → electrical energy

Reviewed Exercise

- What are consequences of development on the environment?

Key Words: global warming, air pollution, water pollution, greenhouse gas

SUMMARY

A **non-renewable resource** is a natural substance that is not replenished with the speed at which it is consumed.

Renewable energy sources (also called non-conventional energy sources) are continuously replenished by natural processes.

Solar energy is the radiant light and heat from the sun that has been harnessed by humans since ancient times using a range of ever-evolving technologies.

Conversion of light energy into electrical energy is based on a phenomenon called **photovoltaic effect**.

Wind energy is the energy generated or produced by harnessing the power of the wind.

The flowing water and water stored in huge dams is also a very important source of energy which is known as **hydroelectric energy**.

Geothermal energy is a type of renewable energy taken from the Earth's core (the internal heat of the earth) and it is one of the oldest types of natural sources of heat.

Fossil fuels, such as coal, oil and natural gas, are important non-renewable sources of energy.

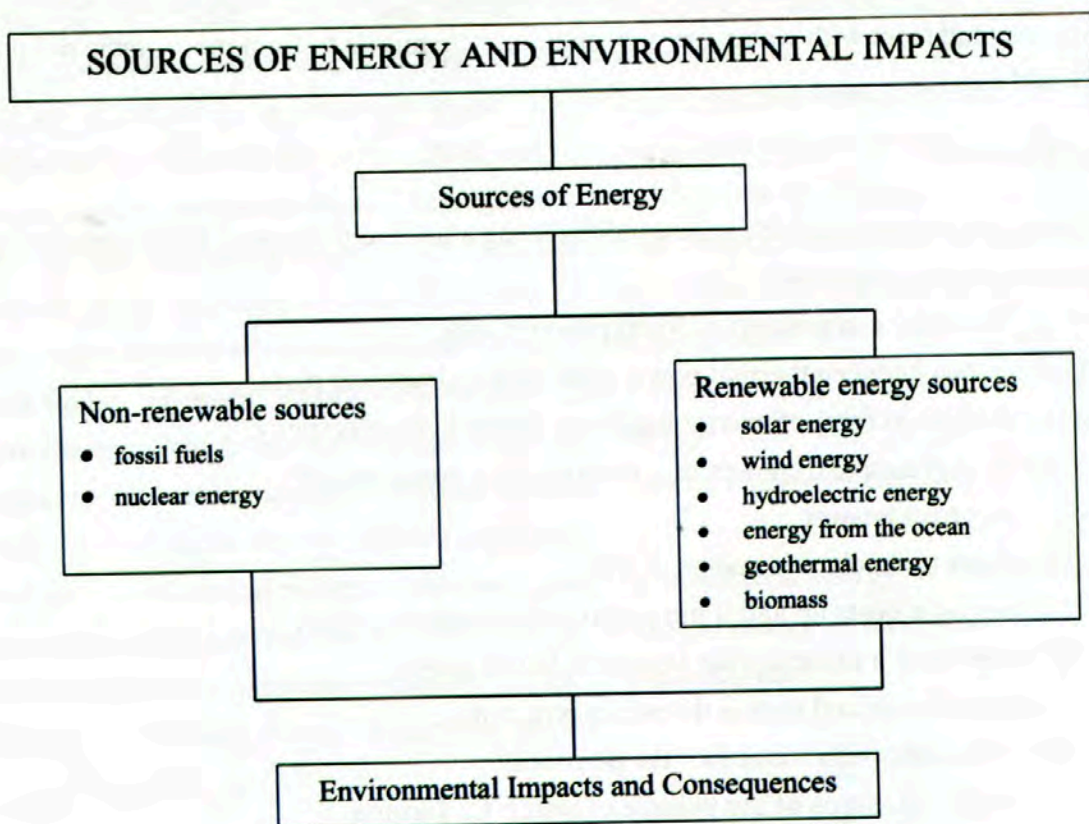
Acid rain is caused by a chemical reaction that begins when compounds like sulfur dioxide and nitrogen oxides are released into the air. These substances can rise very high into the atmosphere, where they mix and react with water, oxygen, and other chemicals to form more acidic pollutants, known as acid rain.

EXERCISES

1. Can we generate electricity from waterfall by using a turbine-generator? If it is possible, what are the necessary conditions?
2. What are the major components of hydropower plants?
3. Write difference between thermal power plant and hydropower plant.
4. State the changes in form of energy while producing hydroelectricity.
5. What is the main source of power of a hydroelectric power plant?
6. Choose the correct answer.
 - (i) How does hydroelectric energy work?
 - A. The water heats up and it turns into water vapor to power.
 - B. Water turns a piece similar to a propeller to power.
 - C. Water freezes and then is thrown to power it.
 - (ii) What principle pulls water into the penstock?
 - A. Gravity
 - B. Force of the volume of water
 - C. Turbine
 - (iii) Which is true for hydroelectric energy?
 - A. It does not costs to build.
 - B. Water everywhere, it can be used everywhere.
 - C. It rains all the time
7. Why is geothermal energy considered as a renewable resource?
8. How is geothermal electricity produced?

9. What are the advantages of geothermal energy compared to other energies?
10. An important hydroelectric plant has a head of 100 m and water volumetric flow rate of $10\,000\text{ m}^3\text{ s}^{-1}$. The turbine generator efficiency is 60 %. What is the maximum power that the plant can produce? (Density of water = 1000 kg m^{-3})
11. A dam which maximum head of 200 m produces to 2000 MW electrical power. What is the rate of falling water on the turbines? The turbine generator efficiency is 50 %. (Density of water = 1000 kg m^{-3})
12. The wind is blowing at 10 m s^{-1} , how much total power produce by the wind turbine if the blades are 45 m long? Density of air is 1.3 kg m^{-3} .
13. Why is it better to use more renewable energy resources rather than non-renewable resources?

CONCEPT MAP



CHAPTER 5

HEAT TRANSFER AND THERMODYNAMICS

Heat is the form of energy that can be transferred from one system to another as a result of temperature difference. In the previous levels, we have already learnt the concepts of heat, temperature, law of heat exchange and phase change. In this chapter, we are going to study heat transfer processes and the basis of thermodynamics. Heat transfer and thermodynamics are two complementary branches of science. Heat transfer is a discipline of thermal engineering that concerns the generation, use, conversion, and exchange of thermal energy between physical systems. Thermodynamics deals with equilibrium states by outlining the relationship between energy and work.

Learning Outcomes

It is expected that students will

- explain some phenomena that involve conductive, convective, and radiative heat transfer.
- understand thermal conductivity and emissivity, total emissive power and blackbody.
- solve problems using the formulas for conduction and radiation.
- solve problems on the relationships between heat transfer, time, and rate of heat transfer.
- know the difference between internal energy and heat transfer.
- realize some uses of properties of thermodynamics.
- examine heat engines and their efficiency.

5.1 HEAT TRANSFER

One of the basic requirement for heat transfer is the presence of a temperature difference. There can be no net heat transfer between two systems that are at the same temperature. There are three basic ways of heat transfer; namely, heat conduction, heat convection and heat radiation. In fluids, heat is mainly transferred by convection, in which the motion of the fluid itself carries heat from one place to another. In solid, heat is transferred by conduction process in which heat energy is transferred among the adjacent parts of the material without the motion of the material itself. The third way to transfer heat energy is by radiation process in the form of electromagnetic waves. The three basic heat transfer processes are illustrated in Figure 5.1.

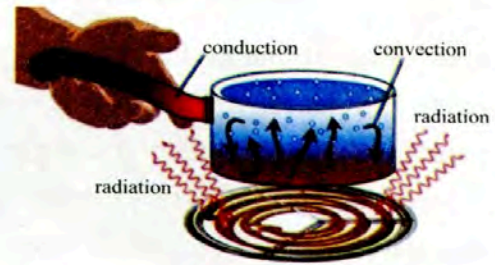
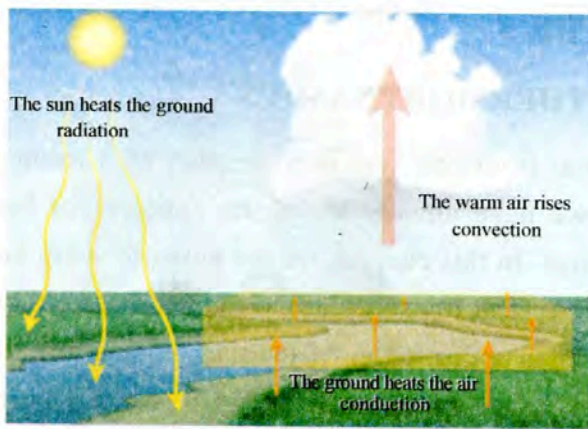


Figure 5.1 Three heat transfer processes

Heat Conduction

Heat conduction is one mode of energy transfer. The individual parts of a medium do not move as a whole in heat conduction. Heat energy is transferred by successive distribution among the adjacent parts in heat conduction.

When heat is transferred via conduction, heat energy is transferred internally by vibrations of atoms and molecules. Metals have many free electrons. Since these free electrons can carry heat, metals are generally very good conductors of heat. For example, if a tea spoon is put into a hot cup of tea, the spoon handle becomes hot. At first the end of the spoon placed in the hot tea gains heat energy. Then the handle end of the spoon becomes hot by successive distribution of heat energy among the adjacent parts. However, the spoon which acts as a medium for heat transfer, does not move at all.

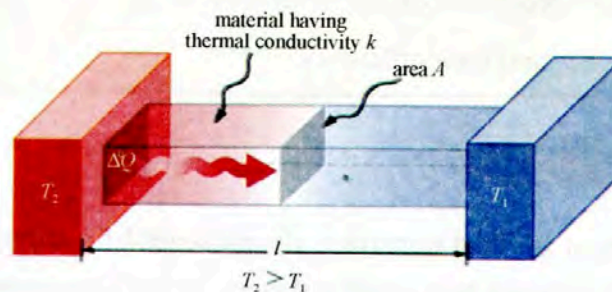


Figure 5.2 Heat conduction process

The rate of heat conduction through a material of cross-sectional area A is proportional to the temperature difference ΔT , but is inversely proportional to the thickness l of the material as shown in Figure 5.2. The heat transfer or conduction rate H_{cond} is

$$H_{\text{cond}} = k A \frac{T_2 - T_1}{l} \quad (5.1)$$

where k is the thermal conductivity of the material, which is a measure of the ability of a material to conduct heat.

In SI units, k is watt per metre per kelvin ($\text{W m}^{-1} \text{K}^{-1}$). The ratio $\frac{T_2 - T_1}{l}$ is called temperature gradient. It is also expressed in differential form as $\frac{dT}{dx}$.

In one dimensional form,

$$\frac{H}{A} \propto -\frac{dT}{dx}$$

$$\frac{H}{A} = -k \frac{dT}{dx}$$

where $\frac{H}{A}$ is heat flow per unit area and H is heat current.

This mathematical model of heat conduction was first formulated by the French physicist Jean Baptiste Joseph Fourier in his Fourier's law of heat conduction. This law states that the rate of heat flow per unit area (heat flux density) is proportional to the magnitude of temperature gradient. Negative sign indicates that temperature T must decrease as distance x increases.

In the SI units, heat flux density is watt per metre squared (W m^{-2}) and heat current is watt (W). The thermal conductivity of a material can be defined as the rate of heat transfer per unit area per unit temperature gradient. A high value for thermal conductivity indicates that the material is a good thermal conductor, and a low value indicates that the material is a poor thermal conductor or thermal insulator. The thermal conductivities of some common materials at room temperature are given in Table 5.1.

Table 5.1 The thermal conductivities of some materials at room temperature

Materials	Thermal conductivity, k ($\text{W m}^{-1} \text{K}^{-1}$)	Materials	Thermal conductivity, k ($\text{W m}^{-1} \text{K}^{-1}$)
Silver	429	Water	0.613
Copper	401	Human skin	0.37
Gold	317	Wood (oak)	0.17
Aluminum	237	Helium (gas)	0.152
Iron	80.2	Soft rubber	0.13
Mercury	8.54	Glass fiber	0.043
Glass	0.78	Air (gas)	0.026
Brick	0.72	Urethane, rigid foam	0.026

Note that materials such as copper and silver that are good electric conductors are also good heat conductors, and have high values of thermal conductivity. Materials such as rubber, wood, and styrofoam are poor conductors of heat and have low conductivity values.

Example 5.1 A person walking at a regular speed generates heat at the rate of 0.07 W. If the surface area of the body is 1.5 m^2 and heat is to be generated 0.03 m below the skin, what should be the temperature difference between the skin and interior of the body if the heat is to be conducted to the surface of the skin? ($k = 5 \times 10^{-5} \text{ W m}^{-1} \text{ K}^{-1}$)

Assume that heat is conducted through a small tissue.

$$H_{\text{cond}} = 0.07 \text{ W}, A = 1.5 \text{ m}^2, l = 0.03 \text{ m}, k = 5 \times 10^{-5} \text{ W m}^{-1} \text{ K}^{-1}$$

$$H_{\text{cond}} = kA \frac{(T_2 - T_1)}{l}$$

Thus,

$$(T_2 - T_1) = \frac{l H_{\text{cond}}}{kA}$$
$$= \frac{0.03 \times 0.07}{5 \times 10^{-5} \times 1.5} = 28 \text{ K} = 28 \text{ }^\circ\text{C}$$

This value of temperature difference is not realistic in human beings. The temperature difference in body is only a few degrees. Actually, the major heat transfer in human beings is due to the flow of warm blood.

Heat Convection

Heat convection is the flow of heat through a fluid from places of higher temperature to places of lower temperature by movement of the fluid itself.

In heat convection, the transfer of energy occurs between an object and its environment due to fluid motion. In other words, the heat is carried by a flowing fluid from one place to another. Convection currents are set up in the fluid because the hotter part of the fluid is not as dense as the cooler part, so there is an upward buoyant force on the hotter fluid, making it rise while the cooler and denser fluid sinks as shown in Figure 5.3 (a). Although some heat may be transferred by conduction in liquids and gases, a much larger quantity of heat is carried by the motion of the fluid itself. In cold regions where rooms are heated by fire, heating is done by convection process. The fluid carrying the heat is the air in the room as shown in Figure 5.3 (b).

Convection occurs only in a liquid or a gas, never in a solid.

Buoyant force is what drives natural convection, where the pressure difference between air heated by heat sources and surrounding ambient air drives the hot air upwards and away from the heat source.

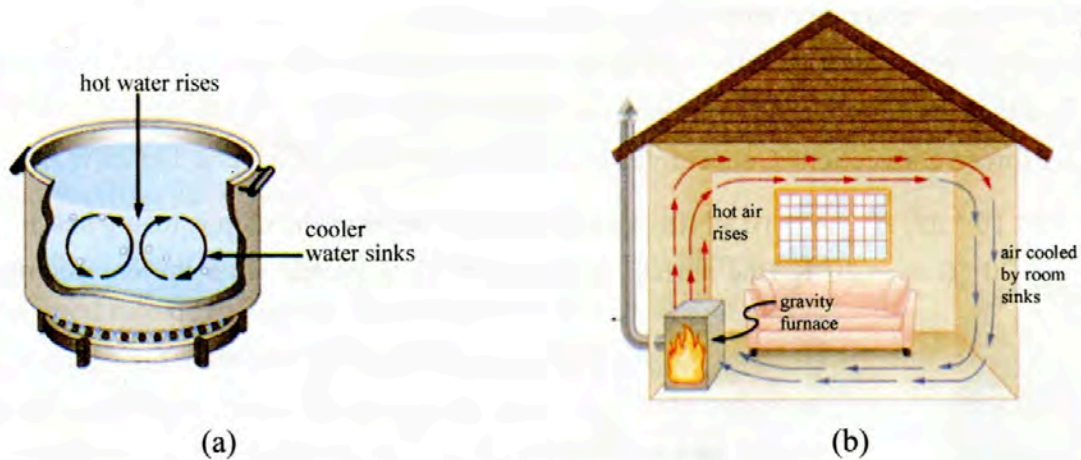


Figure 5.3 Heat convection process

The basic equation for the rate of convection heat transfer is known as Newton's law of cooling,

$$H_{\text{conv}} = q A (T_2 - T_1) \quad (5.2)$$

where H_{conv} is the rate of heat convection, q is the convective heat transfer coefficient (heat convection constant), A is the surface area of the object being cooled or heated, T_2 is the temperature of the surrounding fluid, and T_1 is the surface temperature of the object. In SI units, q is expressed watt per metre squared per kelvin ($\text{W m}^{-2} \text{K}^{-1}$).

When $T_2 > T_1$, heat is gained by the object and H_{conv} is positive. When $T_2 < T_1$, heat is lost by the object and H_{conv} is negative.

Some of the weather conditions are created by heat convection as shown in Figure 5.4. One of the reasons why the weather is fair at the base of mountain ranges, at the sea coast, lakes and ponds is that the hot air in those regions rises and is replaced by cooler air which blows across the water surface. This process occurs due to heat convection. The heat loss by convection is important for living beings.

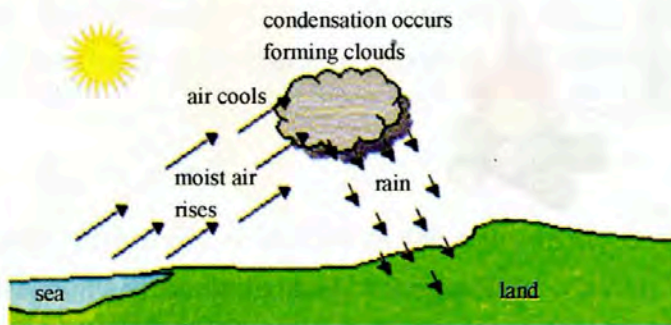


Figure 5.4 Creation of weather condition

The household hot water system is also designed based on the process of heat convection in liquid. In cold countries, many homes have a hot water tank. Hot water from the boiler rises and flows into the tank where it rises to the top. When you use a hot water tap at home, you draw hot water from the top of the tank.

Example 5.2 A new computer chip with a surface area of 1 cm^2 generates 10 W of heat. Determine the convective heat transfer coefficient of the material needed to keep the temperature of the chip less than $20 \text{ }^\circ\text{C}$ above the environmental temperature. Can the chip be cooled with air, or will it require forced convection? Convective heat transfer coefficient of air is 2.5 to $25 \text{ W m}^{-2} \text{ K}^{-1}$.

$A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$, $H_{\text{conv}} = -10 \text{ W}$ (the sign is negative because the chip is losing heat)
 $(T_2 - T_1) = -20 \text{ }^\circ\text{C} = -20 \text{ K}$ (the surface temperature T_1 is greater than the environmental temperature T_2)

$$H_{\text{conv}} = q A (T_2 - T_1)$$

$$q = \frac{H_{\text{conv}}}{A (T_2 - T_1)}$$

$$= \frac{-10}{1 \times 10^{-4} \times (-20)} = 5000 \text{ W m}^{-2} \text{ K}^{-1}$$

Since convective heat transfer coefficient of air is very much less than $5000 \text{ W m}^{-2} \text{ K}^{-1}$, the chip cannot be cooled with air. Therefore it will require the forced convection.

Heat Radiation

Heat radiation is a transfer of heat that does not require a material medium. Heat is transferred by thermal radiation which are electromagnetic waves that pass through a medium or even vacuum. Unlike conduction and convection, the transfer of energy by radiation does not require an intervening medium. In fact, energy transfer by radiation is the fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth by heat radiation as shown in Figure 5.5.



Figure 5.5 Heat radiation

Thermal radiation is the form of radiation emitted by bodies because of their temperature. All bodies at a temperature above absolute zero emit thermal radiation. All solids, liquids and gases emit or absorb radiation to varying degrees. Thermal radiation differs from other forms of electromagnetic radiation such as X-rays, gamma rays, microwave and radio wave that are not related to temperature.

Radiation is mostly considered to be a surface phenomenon for solids that are opaque to thermal radiation. The radiation emitted by the interior regions of such material can hardly reach the surface and the radiation incident on such bodies is usually absorbed within a few microns from the surface.

If an object has a good rate of emission of radiation, then it also has a good rate of absorption. The object which can absorb all the electromagnetic radiations falling upon it is called a blackbody and the radiation emitted by a blackbody is called blackbody radiation. The blackbody is not only a perfect absorber but also a best emitter of radiation.

The total emissive power is defined as the total radiant energy of different wavelengths emitted from unit area of a surface of a body in one second.

The total emissive power of a blackbody is directly proportional to the fourth power of absolute temperature.

$$\epsilon_0 = \sigma T^4 \quad (5.3)$$

This equation is known as Stefan-Boltzmann's law and σ is called Stefan's constant, $\sigma = 5.685 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

The maximum rate of radiation that can be emitted from a surface of area A (blackbody) at an absolute temperature T is given by the Stefan-Boltzmann's law.

$$H_{\text{rad}} = \sigma A T^4 \quad (5.4)$$

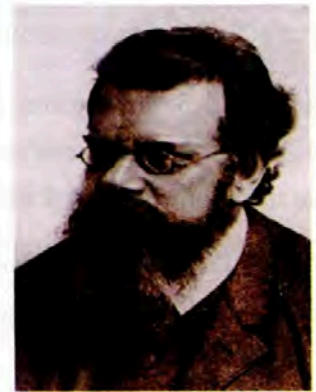
The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$H_{\text{rad}} = e \sigma A T^4 \quad (5.5)$$

where e is the emissivity. Emissivity is defined as the ratio of the total emissive power of an object to that of the blackbody, at the same temperature and wavelength. That is $e = \frac{\epsilon}{\epsilon_0}$. The total emissive power of objects other than a black body is $\epsilon = e\epsilon_0$.



Josef Stefan (1835- 1893)



Ludwig Eduard Boltzmann
(1844 - 1906)

Emissivity of an object depends to a large extent on how shiny it is. If an object reflects a lot of energy, it will absorb (and radiate) very little; if it reflects very little energy, it will absorb and radiate quite efficiently.

A black object is a good absorber and a good radiator whose emissivity is close to 1, whereas a white, clear, or silver coated object is a poor absorber and a poor radiator whose emissivity is much less than 1 as shown in Figure 5.6.

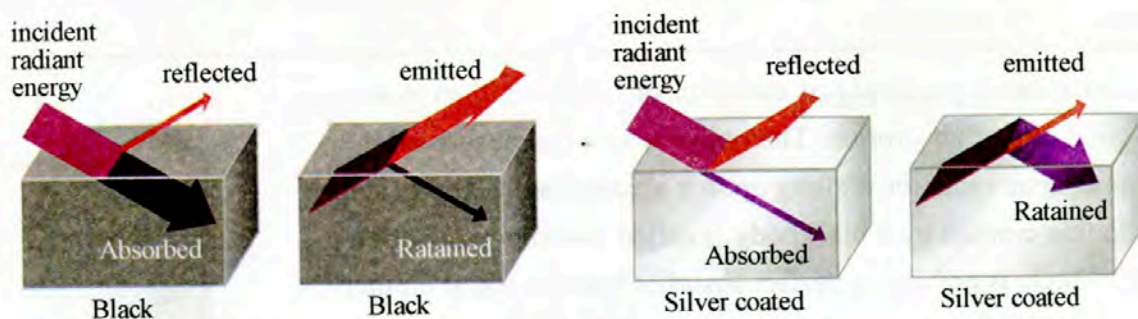


Figure 5.6 Good absorber, good radiator and poor absorber, poor radiator

Note that the emissivity of an object depends on the wavelength of radiation. A shiny object may reflect a great deal of visible light, but it may be a good absorber (emitter) of radiation of a different wavelength, such as ultraviolet or infrared light.

The net energy change is simply the difference between the radiated energy and the absorbed energy. This can be expressed as the net radiation per unit area of the surface per second is,

$$\frac{H_{\text{net}}}{A} = e\sigma(T_2^4 - T_1^4)$$

where T_2 is the temperature of the object and T_1 is the temperature of the surroundings.

Example 5.3 In a room, the temperature of water in a kettle is 80°C . The room temperature is 30°C and the surface area of the kettle is 0.02 m^2 . If the emissivity is 0.9 find the rate of heat loss due to radiation. ($\sigma = 5.685 \times 10^{-8}\text{ W m}^{-2}\text{ K}^{-4}$)

The kettle is emitting heat itself and at the same time absorbing heat radiated from the surrounding. It is emitting heat at its temperature 80°C and absorbing heat from the surrounding at the temperature 30°C .

Let the temperature of the kettle be T_2 and the surrounding's temperature is T_1 .

$$T_2 = 80 + 273 = 353\text{ K}, \quad T_1 = 30 + 273 = 303\text{ K}$$

$$A = 0.02\text{ m}^2, \quad \sigma = 5.685 \times 10^{-8}\text{ W m}^{-2}\text{ K}^{-4}, \quad e = 0.9,$$

The rate of heat radiation $H_{\text{rad}} = e\sigma AT^4$

The rate of heat radiated from the kettle at the temperature T_2 is

$$\begin{aligned} H_{\text{out}} &= e\sigma AT_2^4 \\ &= 0.9 \times 5.685 \times 10^{-8} \times 0.02 \times (353)^4 \\ &= 15.89\text{ W} \end{aligned}$$

The rate of heat absorbed by the kettle at the temperature T_1 is

$$\begin{aligned}H_{\text{in}} &= e\sigma AT_1^4 \\ &= 0.9 \times 5.685 \times 10^{-8} \times 0.02 \times (303)^4 \\ &= 8.63 \text{ W}\end{aligned}$$

Thus, the net rate of heat lost by the kettle is

$$\begin{aligned}H_{\text{lost}} &= H_{\text{out}} - H_{\text{in}} \\ &= 15.89 - 8.63 \\ &= 7.26 \text{ W}\end{aligned}$$

Reviewed Exercise

1. Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A metallic pot placed directly over a heating element will be heated more effectively. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?
2. Why is heat transfer take place between an object and its surrounding?
3. What is the relation between the total emissive power of an object and total emissive power of black body when both are at same temperature?

Key Words: heat conduction, heat convection, radiation, thermal conductivity, emissivity, blackbody

5.2 THERMODYNAMICS

Thermodynamics is the study of systems involving energy in the form of heat and work. A good example of a thermodynamic system is gas confined by a piston in a cylinder. If the gas is heated, it will expand, doing work on the piston; this is one example of how a thermodynamic system can do work.

Thermal equilibrium is an important concept in thermodynamics. When two systems are in thermal equilibrium, there is no net heat transfer between them. This occurs when the systems are at the same temperature. In other words, systems at the same temperature will be in thermal equilibrium with each other.

The Zeroth Law of Thermodynamics

The zeroth law of thermodynamics is a simple definition of thermodynamic equilibrium. It states that;

When two objects are separately in thermodynamic equilibrium with a third object, they all are in thermodynamic equilibrium with each other.

First Law of Thermodynamics

The first law of thermodynamics is an application of the conservation energy for heat and thermodynamic processes. The first law of thermodynamics relates the change in internal energy of a system to heat added and the work done by the system.

The change in internal energy ΔU of a system is equal to the net heat flow Q into the system minus the total work W done by the system.

In symbols,
$$\Delta U = Q - W \quad (5.6)$$

Q is positive if heat is added to the system, and negative if heat is removed. W is positive if work is done by the system, and negative if work is done on the system.

Internal Energy

The internal energy U of a system is defined as the sum of the energies of all the constituent particles. The internal energy of ideal gases depends only on temperature, not on volume or pressure. The internal energy of an ideal monoatomic gas was given as;

$$U = \frac{3}{2} N k_B T = \frac{3}{2} N \frac{RT}{N_A}$$

$$U = \frac{3}{2} n RT \quad (5.7)$$

where N = number of particles of an ideal gas, N_A = Avogadro's number, T = absolute temperature, R = universal (ideal) gas constant, $n = \frac{N}{N_A}$ = number of moles, and $k_B = \frac{R}{N_A}$ = Boltzmann constant.

Note that internal energy of an ideal gas is directly proportional to the absolute temperature.

Heat transfer and work are both energy in transit. Neither is stored as such in a system. However, both can change the internal energy U of a system. Internal energy is a form of energy completely different from either heat or work.

Thermodynamics Processes

A thermodynamic process may be defined as a change from one equilibrium state of a system to another. During such a process, a system starts from an initial state i (pressure p_i , volume V_i and temperature T_i) passes through various quasistatic states to a final state f (pressure p_f , volume V_f and temperature T_f).

In such a process energy may be transferred from (or) into the system and also work can be done by (or) on the system. There are various thermodynamic processes that are of importance in engineering of heat engines.

In thermodynamics, a quasi-static process (also known as a quasi-equilibrium process) is a thermodynamic process that happens slowly enough for the system to remain in internal physical thermodynamic equilibrium.

1. Isobaric Process

An isobaric process occurs at constant pressure. An example of an isobaric process is a gas, being slowly heated or cooled, confined by a piston in a cylinder. The heat Q , the work W , or the change in internal energy ΔU are nonzero in an isobaric process. The work done by the system in an isobaric process is simply the pressure multiplied by the change in volume. In an isobaric process the pressure is constant and the work is equal to the area of the shaded rectangle as shown in Figure 5.7 (pressure Vs volume graph).

In an isobaric process, the work is,
$$W = p \int_{V_i}^{V_f} dV = p (V_f - V_i) \quad (5.8)$$

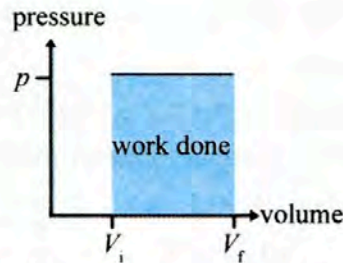


Figure 5.7 Isobaric process

2. Isochoric Process

An isochoric process occurs at constant volume. An example of an isochoric (isovolumetric) process is heating of a gas in a rigid, closed container. When the volume of a thermodynamic system is constant, no work is done by the system because $W = p \Delta V = 0$ as shown in Figure 5.8. According to the first law, the change in internal energy must equal any heat flowing into or out of the system. That is, $\Delta U = Q$.

Since $Q = m c_v \Delta T$, $\Delta U = m c_v \Delta T$ where c_v is specific heat capacity at constant volume.

However, $c_v = \frac{C_v}{M}$, where C_v is molar heat capacity at constant volume and M is molar mass of

the gas ($n = \frac{m}{M}$), n is number of moles.

The change in internal energy is expressed in terms of molar heat capacity C_v ,

$$\Delta U = Q = n C_v \Delta T \quad (5.9)$$

For an ideal monatomic gas

$$\Delta U = \frac{3}{2} n R \Delta T \quad (5.10)$$

where $C_v = \frac{3}{2} R$.

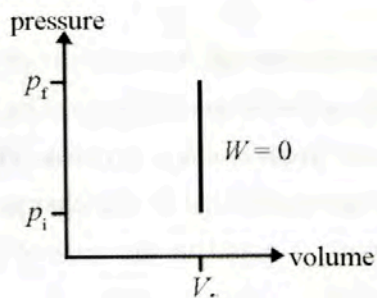


Figure 5.8 Isochoric process

3. Isothermal Processes

An isothermal process occurs at a constant temperature (i.e. $\Delta T = 0$). For a process to be isothermal, any heat flow into or out of the system must occur slowly to maintain the thermodynamic equilibrium of the system.

However, for an ideal gas, internal energy U is only a function of the temperature. In this case, internal energy is constant; therefore, ΔU is equal to zero for an isothermal process.

Since $\Delta U = 0$, according to the first law, $W = Q$.

Using the ideal gas law

$$pV = nRT$$

The work, represented by the area under curve as shown in Figure 5.9, can be calculated as follow.

$$W = \int_{V_i}^{V_f} p \, dV = \int_{V_i}^{V_f} \frac{nRT}{V} \, dV = nRT \int_{V_i}^{V_f} \frac{1}{V} \, dV$$

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) \quad (5.11)$$

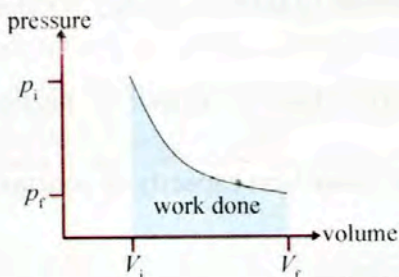


Figure 5.9 Isothermal process

If the system does the work, the energy comes from heat flowing into the system from the reservoir; if the work is done on the system, heat flows out of the system to the reservoir.

4. Adiabatic Processes

In an adiabatic process, there is no heat transfer into or out of the system, i.e. $Q = 0$. This process can be done either by enclosing the system in a thermal insulator (adiabatic container) (or) by carrying out the process so quickly that there is not enough time for heat transfer. [Figure 5.10]

The first law of thermodynamics $\Delta U = Q - W$ is thus reduced to $\Delta U = -W$.

For an ideal gas, $\Delta U = \frac{3}{2}n R \Delta T$.

Therefore, work done $W = -\Delta U = -\frac{3}{2}n R \Delta T$

$$W = \frac{3}{2}n R (T_i - T_f) \quad (5.12)$$

Using the ideal gas law in the form $pV^\gamma = K$ (constant), the work done W can also be expressed in terms of pressure and volume as,

$$W = \frac{1}{\gamma-1} (p_i V_i - p_f V_f) \quad (5.13)$$

where γ is the ratio of the heat capacities, $\gamma = \frac{C_p}{C_v}$.

C_p is molar heat capacity at constant pressure.

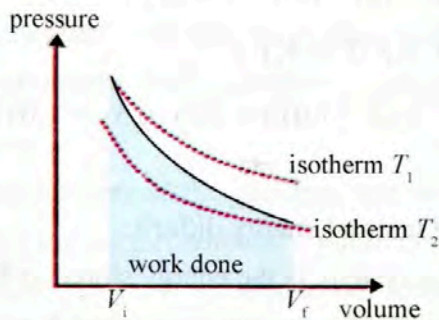


Figure 5.10 Adiabatic process compare with isothermal process

Example 5.4 Find the change in internal energy of one mole of an ideal gas when its temperature changes from 0 °C to 100 °C.

$$\Delta T = (100 - 0) = 100 \text{ °C} = 100 \text{ K}, \quad n = 1 \text{ mol}, \quad R = 8.3143 \text{ J mol}^{-1} \text{ K}^{-1}$$

The change in internal energy,

$$\begin{aligned} \Delta U &= \frac{3}{2}n R \Delta T \\ &= \frac{3}{2} \times 1 \times 8.3143 \times 100 = 1.25 \times 10^5 \text{ J} \end{aligned}$$

Example 5.5 One gram of water (1 cm³) becomes 1671 cm³ of steam when boiled at a pressure of 1 atm. The heat of vaporization at this pressure is 2265 J g⁻¹. Compute the work done and the increase in internal energy.

$$m = 1 \text{ g}, \quad p = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}, \quad L_v = 2265 \text{ J g}^{-1}$$

$$(V_f - V_i) = 1671 - 1 = 1670 \text{ cm}^3 = 1670 \times 10^{-6} \text{ m}^3$$

For isobaric process;

$$W = p(V_f - V_i) \\ = 1.013 \times 10^5 \times 1670 \times 10^{-6} = 169 \text{ J}$$

Heat required one gram of water to vaporize;

$$Q = m L_v \\ = 1 \times 2265 = 2265 \text{ J}$$

According to first law of thermodynamics

$$\Delta U = Q - W \\ = 2265 - 169 = 2096 \text{ J}$$

Example 5.6 In an isobaric process, a volume of a gas is expressed by absorption of heat at constant pressure 2 atm. If the increment in volume is 0.5 m^3 , then find the work done by the system.

$$p = 2 \text{ atm} = 2 \times 1.013 \times 10^5 \text{ Pa}$$

$$\Delta V = (V_f - V_i) = 0.5 \text{ m}^3$$

In an isobaric process,

$$W = p(V_f - V_i) \\ = (2 \times 1.013 \times 10^5)(0.5) = 1.013 \times 10^5 \text{ J}$$

Reviewed Exercise

1. How do heat transfer and internal energy differ?
2. When heat transfers into a system, is the energy stored as heat? Explain briefly.
3. Explain why the temperature of a gas increases when it is adiabatically compressed.

Key Words: thermodynamics, internal energy, isobaric process, isothermal process, isochoric process (isometric process), adiabatic processes

5.3 HEAT ENGINES

Heat engine is a system that converts heat to mechanical energy, which can then be used to do mechanical work. It operates continuously and only heat and work may pass across its boundaries. A series of processes in which a system is brought back to its initial state is called a cycle. The purpose of a heat engine is to deliver work continuously to the outside by performing the same cycle over and over again as illustrated in Figure 5.11.

Thermal efficiency for a heat engine is the ratio of work output to heat input.

$$\text{Thermal efficiency} = \frac{\text{work output}}{\text{heat input}}$$

$$\eta = \frac{W}{Q_H} \quad (5.14)$$

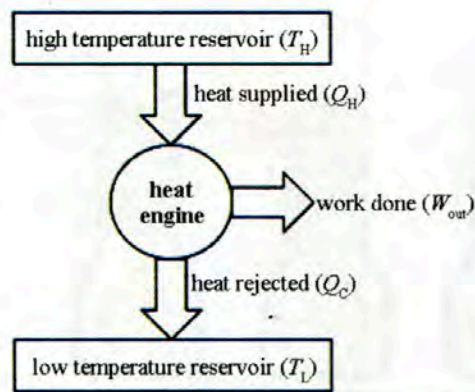


Figure 5.11 Thermodynamic cycle of heat engine

Applying the first law of thermodynamic to one complete cycle, remembering that there is no net change of internal energy ($\Delta U = 0$), we get, $Q_H - Q_C = W$.

The transformation of heat into work is usually accomplished in practice by two general types of engine; the internal combustion engine and the steam engine. In both engines, a gas or a mixture of gases contained in a cylinder undergoes a cycle. There are two types of internal combustion engine (gasoline engine and diesel engine) that generates the input heat by the combustion of fuel within the engine itself. The principle of operation of a gasoline engine is described as below.

1. Gasoline Engines

In the gasoline engines, the combustion of the gasoline and air takes place explosively by an electric spark. The cycle involves the performance of six processes, of which four require motion of the piston and are called strokes. Hence, it is called a four stroke engine.

- (i) **Intake stroke:** A mixture of gasoline vapour and air is drawn into the cylinder by the suction stroke of the piston.
- (ii) **Compression stroke:** The mixture of gasoline vapour and air is compressed until its pressure and temperature rise considerably. This is accomplished by the compression of the piston.
- (iii) **Explosion:** Combustion of the hot mixture is caused to take place very rapidly by an electric spark. The combustion products attain a very high pressure and temperature, but the volume remains unchanged. The piston does not move during this process.
- (iv) **Power stroke:** The hot combustion products expand and push the piston out, thus suffering a drop in pressure and temperature.
- (v) **Valve exhaust:** The combustion products at the end of the power stroke are still at a higher pressure and temperature than the outside. An exhaust valve allows some gas to escape until the pressure drops to that of the atmosphere. The piston does not move during this process.
- (vi) **Exhaust stroke:** The piston pushes almost all the remaining combustion products out of the cylinder by exerting a pressure sufficiently larger than that of the outside pressure.

The four strokes cycle and the p - V diagram of a gasoline engine is shown Figure 5.12 (a) and (b).

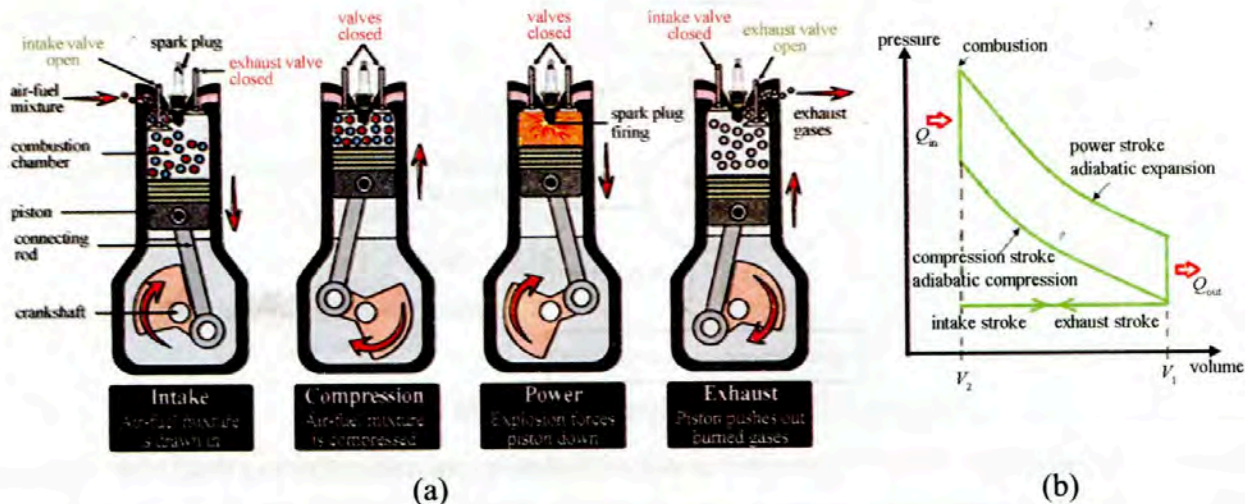


Figure 5.12 (a) The four strokes cycle and (b) p - V diagram of a gasoline engine

Theoretical efficiency of a gasoline engine is

$$\eta = 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}} \quad (5.15)$$

where $\frac{V_1}{V_2}$ = compression ratio, and $\gamma = \frac{C_p}{C_v}$ = ratio of molar heat capacities. For a typical compression ratio of 8 and with $\gamma = 1.4$, $\eta = 56\%$ for an ideal gasoline engine.

2. Diesel Engine

The diesel engine operates on either a two-stroke or four-stroke cycle; however, unlike the spark-ignition gasoline engine, the diesel engine induces only air into the combustion chamber on its intake stroke. The operation of a two-stroke diesel engine is explained below.

When the piston is at the top of its travel, the cylinder contains a highly compressed air. Diesel fuel is sprayed into the cylinder by the injector (nozzle) and immediately ignites because of the heat and pressure inside the cylinder as shown in Figure 5.13.

The pressure created by the combustion of the fuel drives the piston downward. This is the power stroke. As the piston nears the bottom of its stroke, all of the exhaust valves open. Exhaust gases rush out of the cylinder, relieving the pressure.

As the piston is at the bottom, it uncovers the air intake ports. Pressurized air fills the cylinder, forcing out the remainder of the exhaust gases. The exhaust valves close and the piston starts traveling back upward, re-covering the intake ports and compressing the fresh air. This is the compression stroke. As the piston nears the top of the cylinder, the cycle repeats.

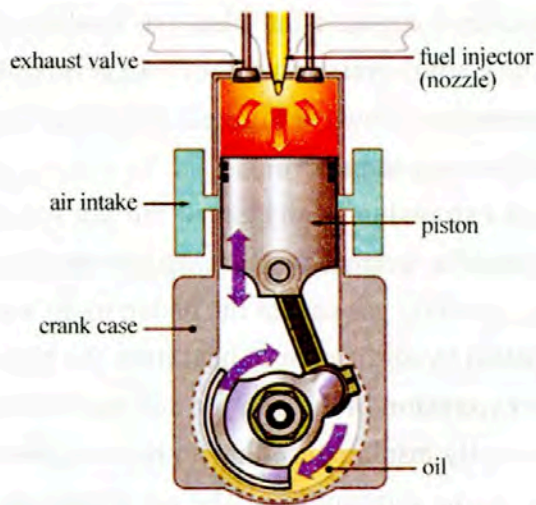


Figure 5.13 The two strokes cycle of diesel engine

Steam engine is operating between the steam temperature 373 K and outside room temperature 300 K. Gasoline engine is operating between petrol burning temperature 680 K and outside room temperature 300 K. Diesel engine is operating between the diesel burning temperature 873 K and outside room temperature.

3. Carnot Heat Engine

In 1824, Nicolas Leonard Sadi Carnot, French engineer, described a hypothetical engine known as Carnot engine. It is a heat engine operates on an ideal reversible cycle (Carnot cycle) between two heat reservoirs.

Carnot engine sets an upper limit on the efficiency of all other real engines, such as steam engines, gasoline and diesel engines. All real engines are less efficient than Carnot engine because they do not operate through a reversible cycle, and efficiency is further reduced due to friction and energy losses by conduction.



Nicolas Leonard Sadi Carnot
(1796 -1832)

In a reversible process, the system undergoing the process can be returned to its initial condition along the same path on the p - V diagram, and every point along this path is in an equilibrium state. A process that does not satisfy these requirements is an irreversible process.

The most important condition for a process to be reversible in thermodynamics is that the process should be carried out infinitesimally slowly and the driving force should be small enough to maintain a state of equilibrium.

The working substance of a Carnot engine is an ideal gas contained in a cylinder fitted with a movable piston at one end. The Carnot cycle consists of four reversible processes; two isothermal processes and two adiabatic processes shown in Figure 5.14. When it operates as a heat engine, the Carnot cycle consists of the following steps.

Step 1: Reversible isothermal expansion of the gas at the hot temperature T_H (isothermal heat addition). The gas is placed in thermal contact with heat reservoir at temperature T_H . During this step (A to B) the expanding gas causes the piston to do work on the surroundings. The gas expansion is propelled by absorption of heat from the high temperature reservoir.

Step 2: Reversible adiabatic expansion of the gas, for this step (B to C). The piston and cylinder are assumed to be thermally insulated, so that no heat is gained or lost. The gas continues to expand, doing work on the surroundings. The gas expansion causes it to cool to the cold temperature T_C .

Step 3: Reversible isothermal compression of the gas at the cold temperature T_C . (isothermal heat rejection) (C to D). The gas is in thermal contact with low reservoir at temperature T_C . Now the surroundings do work on the gas, compressing it and causing heat to flow out of the gas to the low temperature reservoir.

Step 4: Reversible adiabatic compression of the gas (D to A), once again the piston and cylinder are assumed to be thermally insulated. During this step, the surroundings do work on the gas, compressing it and causing the temperature to rise to T_H . At this point the gas is in the same state as at the start of step 1.

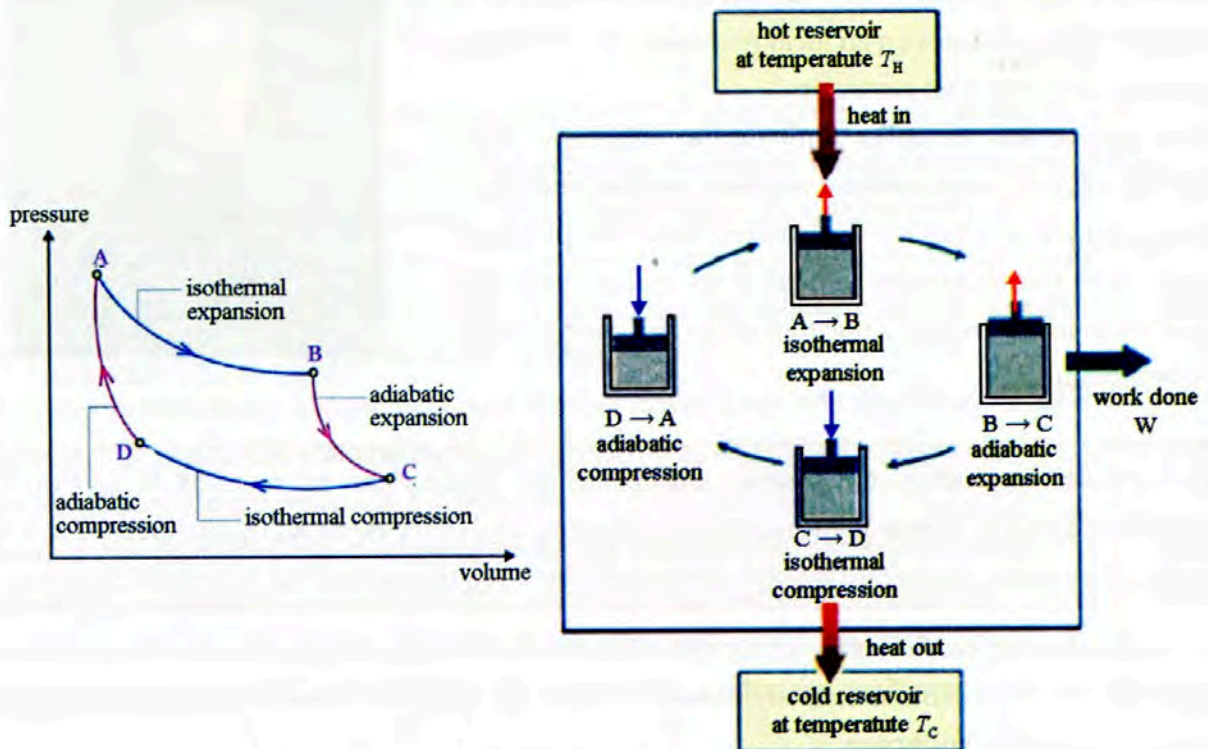


Figure 5.14 The four reversible processes of Carnot cycle

In Figure 5.14, the circle represents a reversible engine operating between two reservoirs at temperatures T_H and T_C , taking in heat Q_H from the reservoir at temperature T_H , rejecting heat Q_C to the reservoir at temperature T_C , and work $W = Q_H - Q_C$. Carnot analyzed the transformation of energy during one complete cycle of this engine's performance and determined the conditions for maximum efficiency.

Since the thermal efficiency of a heat engine η is defined as the ratio of the work output W to the heat input Q_H ,

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\eta = 1 - \frac{Q_C}{Q_H} \quad (5.16)$$

Kelvin proposed that we use a Carnot engine as a thermometer and define the ratio of two temperatures as the ratio of the heat absorbed by the engine to the heat rejected, when the engine is operated between reservoirs at these temperatures.

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

$$\eta_c = 1 - \frac{T_C}{T_H} \quad (5.17)$$

Although Carnot engine is not 100% efficient, but it is more efficient than other machine in transforming heat into work.

It should be noted that the temperatures of the reservoirs in this case are absolute temperature.

Carnot's theorem

Carnot's theorem states that no engine operating between two heat reservoirs can be more efficient than a Carnot engine operating between the same reservoirs.

$$\eta_c = 1 - \frac{T_C}{T_H}$$

Thus, this equation gives the maximum efficiency possible for any engine using the corresponding temperatures. A corollary to Carnot's theorem states that: all reversible engines operating between the same heat reservoirs are equally efficient.

Example 5.7 An engine takes in 9220 J and does 1750 J of work each cycle while operating between 689 °C and 397 °C. (a) What is its actual efficiency? (b) What is its maximum theoretical efficiency?

$$Q_H = 9220 \text{ J}, W = 1750 \text{ J}, T_H = 689 \text{ }^\circ\text{C} + 273 = 962 \text{ K}, T_C = 397 \text{ }^\circ\text{C} + 273 = 670 \text{ K}$$

Actual efficiency $\eta = \frac{W}{Q_H} = \frac{1750}{9220} = 0.19 = 19\%$

Maximum efficiency $\eta_C = 1 - \frac{T_C}{T_H}$
 $= 1 - \frac{670}{962} = 0.304 = 30.4\%$

Reviewed Exercise

- Is a temperature difference necessary to operate a heat engine? Explain why or why not.

Key Words: heat engine, diesel engine, gasoline engine, Carnot engine, Carnot cycle, stroke

SUMMARY

The rate of heat flow (**heat current**) H is the amount of heat energy transferred per unit time.

Heat conduction is one mode of energy transfer. The individual parts of a medium do not move as a whole in heat conduction.

Heat convection is the flow of heat through a fluid by movement of the fluid itself.

Heat radiation is a transfer of heat that does not require a material medium. Heat is transferred by thermal radiation which are electromagnetic waves that pass through medium or even vacuum. The object which can absorb all the electromagnetic radiations falling upon it is called a **blackbody** and the radiation emitted by a blackbody is called **blackbody radiation**.

Heat engine is a system that converts heat to mechanical energy, which can then be used to do mechanical work.

Thermal efficiency of a heat engine η is defined as the ratio of the work output W to the heat input Q_H .

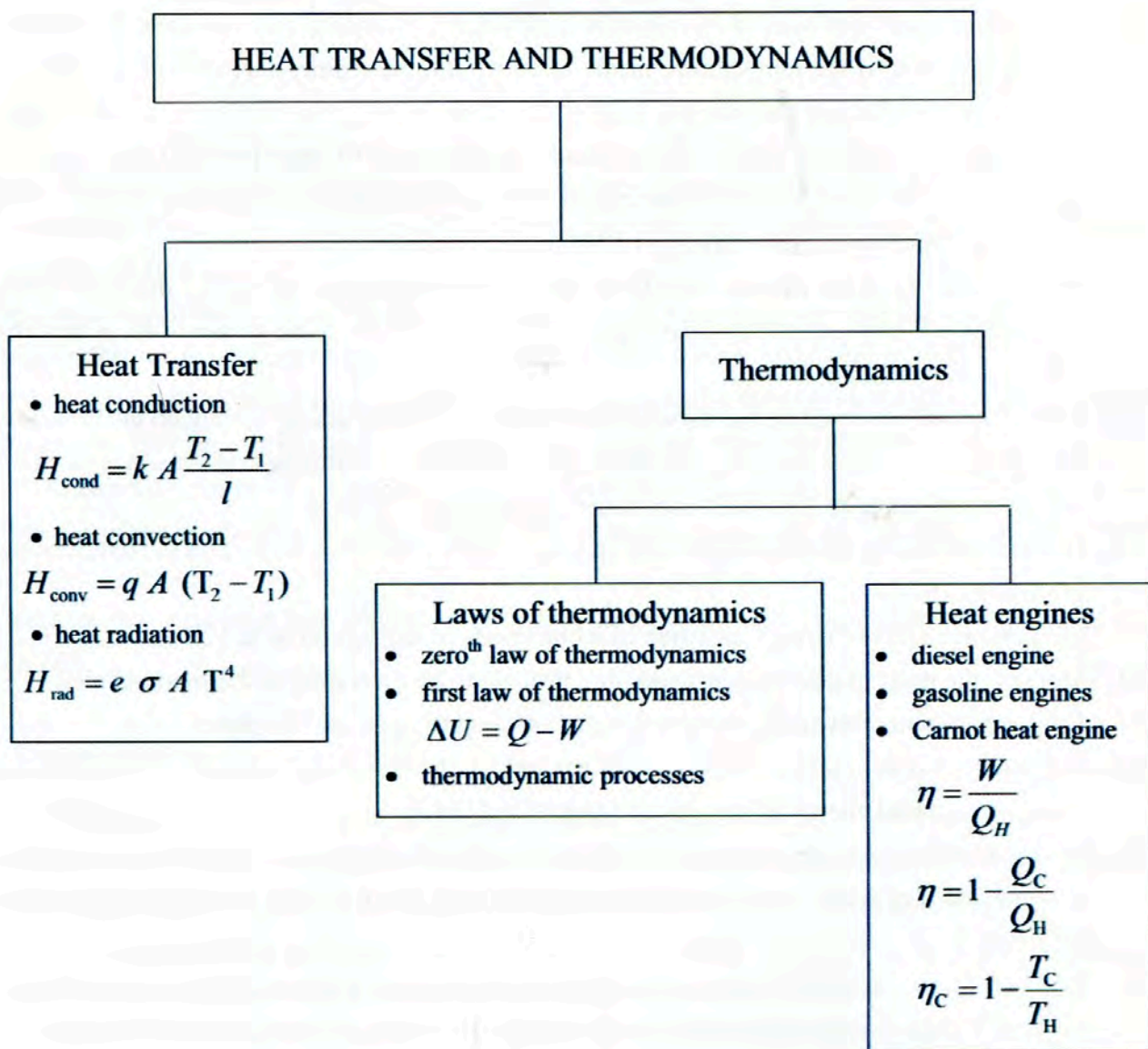
EXERCISES

1. What is thermal conductivity? Express its unit in SI system.
2. One end of a poker is placed in fire. After some time, the other end becomes hot. Explain how heat is transferred along the poker. Name the method of heat transfer in this case.
3. If a person wearing ordinary clothes travels out into space, the liquid in the body will boil. Why? Explain how a space suit can prevent this effect.
4. In cold regions it is seen that birds on the branches of trees often ruffle their feathers. Explain the reason why the birds feel warm by ruffling their feathers.
5. How does a blanket wrapped round your body keep you warm on a cold day?

6. Explain with a diagram why an air conditioner should be best positioned high, near the ceiling of a room.
7. Why do we generally wear dark clothes in the winter and light-colored clothes in the summer?
8. Can you warm a kitchen in winter by leaving the oven door open? Can you cool the kitchen on a hot summer day by leaving the refrigerator door open? Explain.
9. The area and thickness of a glass plate of a window are 0.25 m^2 and 4 mm respectively. The temperature of inside surface of glass plate is 25°C and its outside temperature is 26°C . Find the amount of heat that passes through the glass plate in one hour. The thermal conductivity of glass is $0.78 \text{ W m}^{-1} \text{ K}^{-1}$.
10. How much heat per second is conducted through a wooden wall of area 25 m^2 and thickness 0.04 m if the temperature inside is 20°C and the temperature outside is -10°C ? The thermal conductivity of wood is $8.37 \times 10^{-2} \text{ W m}^{-1} \text{ K}^{-1}$.
11. The filament of a 100 W electric bulb is made of tungsten. The emissivity of tungsten is 0.3 and its length is 0.2 m . Find the diameter of the filament, if its temperature is 3000 K when the bulb is switched on. ($\sigma = 5.685 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$)
12. The temperature of the filament is 2500 K when the bulb is switched on. The diameter of the filament is 0.1 mm and it is made of metal of emissivity 0.35 . If the power is 40 W find the length of the filament.
13. From calculations based on the radiation measurement of solar energy falling on the earth it is found that sun is radiating energy at a rate of 62.5 MW m^{-2} . Assuming that the sun is emitting energy as a blackbody, find the temperature of the surface of the sun.
14. If the rate of energy radiation from a black body of area 100 cm^2 is 42 W , find the temperature of that blackbody.
15. Compare the rates of energy radiation of a blackbody at temperatures 327°C and at 27°C .
16. Mention the most important condition for a process to be reversible in thermodynamics.
17. For a gas system, how much work is done in an isometric process? Explain.
18. An insulated system takes in 6.5 kcal of heat and has 2000 J of work done on it. What is the change in internal energy of the system? ($1 \text{ kcal} = 4184 \text{ J}$)
19. A heat engine undergoes a process in which its internal energy increases by 275 J while it is doing 360 J of work. How much heat is taken in (or given out) by the engine during this process?
20. One liter of air is cooled at constant pressure until its volume is halved, and then it is allowed to expand isothermally back to its original volume. Draw the process on a p - V diagram.
21. In an isothermal process (27°C), 2 kilomole of an ideal gas is compressed from a volume of 4 litre to 1 litre. Find the work done on the system. ($R = 8.3143 \text{ J mol}^{-1} \text{ K}^{-1}$)

22. An automobile engine has an efficiency of 20 % and produces an average of 23 000 J of mechanical work per second during operation. (i) How much heat input is required per second and (ii) how much heat is discharged as waste heat from this engine per second?

CONCEPT MAP



CHAPTER 6

EFFECTS AND CHARACTERISTICS OF SOUND

In this chapter, the range of audibility, infrasound and ultrasound together with their applications will be studied. Echo, reverberation and Doppler effect are also discussed.

Learning Outcomes

It is expected that students will

- examine the propagation of sound.
- identify audible frequencies.
- investigate the infrasound, ultrasound and their applications.
- describe the occurrence of echo.
- examine and explain the Doppler effect.
- differentiate between pitch and loudness of sound.

6.1 PROPAGATION OF SOUND IN A MEDIUM

Sound is a sequence of pressure waves which propagates through compressible media such as air or water. During their propagation, waves can be reflected, refracted or attenuated by the medium. The followings are the properties of media which affect the sound propagation.

- ♦ A relationship between density and pressure. This relationship, affected by temperature, determines the speed of sound within the medium.
- ♦ The viscosity of the medium. This determines the rate at which sound is attenuated. For many media, such as air or water, attenuation due to viscosity is negligible.

Speed of Sound

Speed of a sound wave can also be affected by the density, temperature, and elasticity of the medium through which the sound waves travel. Hence, the speed of sound will be different for different media.

In a solid medium, the speed of sound is given by,

$$v = \sqrt{\frac{Y}{\rho}} \quad (6.1)$$

where Y is Young's modulus and ρ is density of the solid material.

In liquids, the velocity of sound is given by a similar expression,

$$v = \sqrt{\frac{B}{\rho}} \quad (6.2)$$

where B is the bulk modulus and ρ is density of liquid.

In gases, the velocity of sound is given by,

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad (6.3)$$

where γ is the adiabatic constant of gas, p is pressure in gas and ρ is its density.

Longitudinal sound waves travel faster in solids than they do in liquids than they do in gases.

Young's modulus and the bulk modulus are elasticity properties of the medium. The SI units of

Young's modulus and the bulk modulus are N m^{-2} .

Example 6.1 What is the speed of sound in (i) a steel rod (ii) water?

Young's modulus of steel is $20 \times 10^{10} \text{ N m}^{-2}$ and density of steel is $7.8 \times 10^3 \text{ kg m}^{-3}$. The bulk modulus of water is $0.22 \times 10^{10} \text{ N m}^{-2}$ and density of water is 10^3 kg m^{-3} .

(i) Young's modulus of steel $Y = 20 \times 10^{10} \text{ N m}^{-2}$, density of steel $\rho = 7.8 \times 10^3 \text{ kg m}^{-3}$

The speed of sound in a steel rod $v = \sqrt{\frac{Y}{\rho}}$

$$\begin{aligned} v &= \sqrt{\frac{20 \times 10^{10}}{7.8 \times 10^3}} \\ &= 5.06 \times 10^3 \text{ m s}^{-1} \end{aligned}$$

(ii) The bulk modulus of water $B = 0.22 \times 10^{10} \text{ N m}^{-2}$, density of water $\rho = 10^3 \text{ kg m}^{-3}$.

The speed of sound in water $v = \sqrt{\frac{B}{\rho}}$

$$\begin{aligned} v &= \sqrt{\frac{0.22 \times 10^{10}}{10^3}} \\ &= 1.48 \times 10^3 \text{ m s}^{-1} \end{aligned}$$

Example 6.2 Calculate the velocity of sound in air at STP. The density of air at STP is 1.29 kg m^{-3} . Assume air to be diatomic with $\gamma = 1.4$. Also calculate the velocity of sound in air at 27°C .

At STP, atmospheric pressure $p = 1.01 \times 10^5 \text{ Pa}$, temperature $T = 273 \text{ K}$

Density of air $\rho = 1.29 \text{ kg m}^{-3}$, $\gamma = 1.4$

The velocity of sound in air is, $v = \sqrt{\frac{\gamma P}{\rho}}$

$$= \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.29}} = 331.07 \text{ m s}^{-1}$$

At 27 °C , $T_2 = 27 + 273 = 300$ K, $T_1 = 273$ K

Since $v \propto \sqrt{T}$,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

The velocity of sound in air at 27 °C is,

$$\frac{v_{300\text{K}}}{v_{273\text{K}}} = \sqrt{\frac{300}{273}}$$

$$\begin{aligned} v_{300\text{K}} &= 331.07 \sqrt{\frac{300}{273}} \\ &= 347.06 \text{ m s}^{-1} \end{aligned}$$

Intensity Level of Sound

The level of intensity of a sound is measured relative to a reference intensity value I_0 , taken as $10^{-12} \text{ W m}^{-2}$ which correspond to the minimum intensity of sound that can be heard (threshold of hearing or minimum audible intensity).

The decibel (dB) is the unit used for measuring sound intensity (intensity level of sound). The intensity level of sound β in dB is defined by Eq. (6.1).

$$\beta \text{ (dB)} = 10 \log_{10} \frac{I}{I_0} \quad (6.4)$$

where I is the intensity of sound and I_0 is the reference intensity. For every 10 dB added to the sound level, the loudness of the sound is increased by 10 times. That is 120 dB sound will be 100 times louder than 100 dB sound. Few examples of sound levels (representative average values) are listed in Table 6.1.

Table 6.1 Few examples of sound levels

Sources of sound	Intensity level of sound	
	(dB)	(W m^{-2})
jet plane taking off	120	1
loud music	100	10^{-2}
busy street traffic	70	10^{-5}
people talking	50	10^{-7}
whisper	20	10^{-10}
leaves falling	10	10^{-11}
threshold of hearing	0	10^{-12}

Example 6.3 At a busy street corner, the intensity level of sound is 70 dB. What is intensity of sound in W m^{-2} ?

Intensity level of sound $\beta = 70$ dB, threshold of hearing $I_0 = 10^{-12} \text{ W m}^{-2}$

$$\begin{aligned}\beta &= 10 \log_{10} \frac{I}{I_0} \\ 70 &= 10 \log_{10} \frac{I}{I_0} \\ 7 &= \log_{10} \frac{I}{I_0} \\ \frac{I}{I_0} &= 10^7\end{aligned}$$

$$I = 10^7 \times 10^{-12} = 10^{-5} \text{ W m}^{-2}$$

Example 6.4 What value of sound intensity in dB increases by a factor of 1000?

$$\frac{I}{I_0} = 1000$$

Intensity level of sound

$$\begin{aligned}\beta &= 10 \log_{10} \frac{I}{I_0} \\ &= 10 \log_{10} 10^3 = 30 \text{ dB}\end{aligned}$$

If the sound intensity is increased by a factor of 1000, the value of sound intensity increases 30 dB.

Sound intensity is a direct measure of the power reaching an eardrum measured in W m^{-2} .

Reviewed Exercise

1. Why is the speed of sound higher in solids than liquids and gases?
2. If the intensity of sound is doubled, by how many decibels does the sound level increase?

Key Words: air pressure, viscosity, Young's modulus, bulk modulus, adiabatic constant, elasticity,

6.2 ECHO

Echoes are produced where a sound wave is reflected from a large and hard surface. An echo is repetition of sound due to the reflection of sound. The echo or the reflected sound can be heard separately from the original sound if the source of the sound is much closer to the observer than to the reflecting surface. Reflection of sound obeys the laws of reflection.

To reduce the effect of echo in buildings, walls are covered up with padding and the floors are covered with rugs and carpets. These materials absorb the incident sound and reduce the amount of sound reflecting off wall and floor surfaces.

Echo can also be used to determine the distance d between a source of sound and the reflecting surface by using formula,

$$v = \frac{2d}{t} \quad (6.5)$$

where v is the speed of sound and t is the echo time.

Some animals such as bats are known to use echolocation to navigate and locate their prey as shown in Figure 6.1. Echolocation is the special ability of emitting sounds and interpreting the echoes.

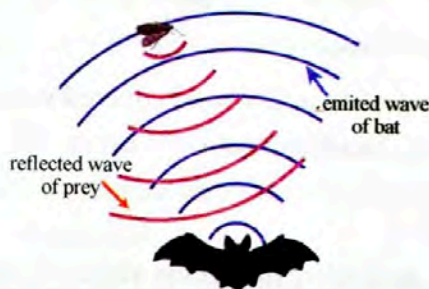


Figure 6.1 Echolocation of bat

Example 6.5 A bat flying at a speed of 20 m s^{-1} towards a stationary moth emits sound wave of 340 m s^{-1} . The bat hears the echoes 0.4 s later. (i) Calculate the distance travelled by the bat in 0.4 s . (ii) Calculate the distance travelled by the sound wave in 0.4 s . (iii) Determine the initial distance between the bat and the moth.

Speed of bat, $v_b = 20 \text{ m s}^{-1}$, speed of sound, $v_s = 340 \text{ m s}^{-1}$, $t = 0.4 \text{ s}$

(i) speed $v = \frac{d}{t}$
 distance travelled by the bat $d_b = v_b t$
 $= 20 \times 0.4 = 8 \text{ m}$

(ii) distance travelled by sound wave $d_s = v_s t$
 $= 340 \times 0.4 = 136 \text{ m}$

(iii) The sound wave will travel from the initial location of the flying bat to the stationary moth and bounce off as echoes. The echoes travel back to the bat at its position after 0.4 s . Hence, the total distance travelled by the bat and the sound wave is twice that of the initial distance between the bat and the moth.

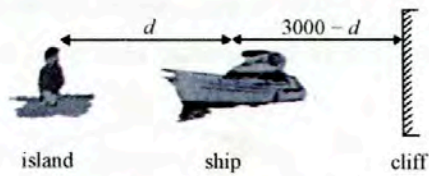
$$\text{Total distance} = 8 + 136 = 144 \text{ m}$$

$$\text{Initial distance between the bat and the moth} = \frac{\text{total distance}}{2} = \frac{144}{2} = 72 \text{ m}$$

A reverberation is a reflection of light or sound waves, or a wide reaching impact of an action. An example of a reverberation is the sound bouncing around in a large speaker.

Reviewed Exercise

- The sole survivor of a shipwreck swim to an island, which is 3000 m from a vertical cliff. He sees a ship anchored between the island and the cliff. A blast from the ship horn is heard twice with the time elapse of 4 s . Calculate distance between the ship and the survivor. (Speed of sound in air = 330 ms^{-1})



2. Why are the walls and roof of an auditorium covered with sound absorbent materials?

Key Words: echo, echolocation, reverberation

6.3 AUDIBILITY

Audible means able to be heard. Audibility means the ability to be heard or capability of detecting sounds in certain range of frequencies.

The human's ear is capable of detecting sounds only in a certain range of frequencies. The range of frequencies a person can hear is known as the range of audibility. For humans with normal hearing, the lower limit is 20 Hz and the upper limit is about 20 000 Hz. These limits are known as the limit of audibility. The audible range of different mammals are as shown in Figure 6.2.

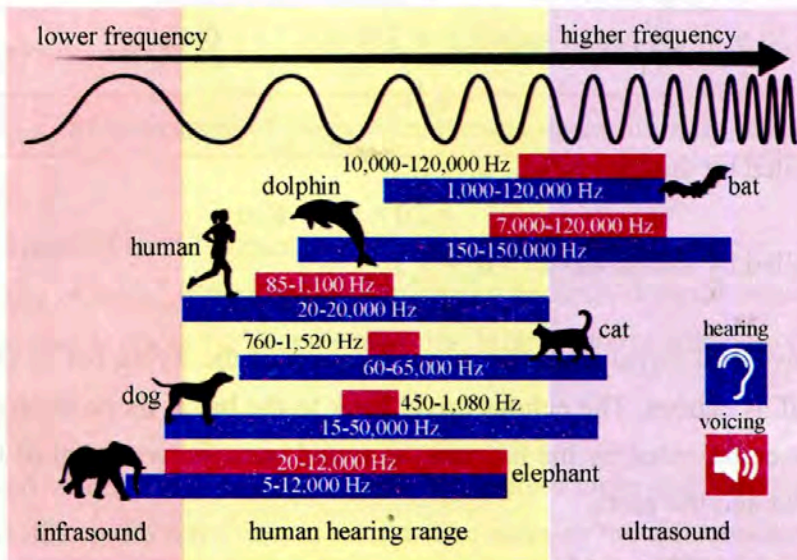


Figure 6.2 Audible range of different mammals [CREDIT: Source from the Internet]

Determination of the Audible Frequency Range by using Frequency Generator

By using frequency generator, the audible frequency range can be determined. Some portable sound frequency generator can create sound with frequencies 0 Hz to about 22 kHz. Tune the sound frequencies gradually from 0 Hz and record the frequency when start hearing the sound, i.e. the lower limit of audibility. Then increase the sound frequency up to about 20 kHz, and record the highest frequency above which the sound cannot be heard, i.e. the upper limit of audibility.



Infrasound

Low frequency sounds which are under the lower limit of audibility (20 Hz) are called infrasound and cannot be heard by human. A vibrating ruler can be seen but not heard its vibrating sound. This is because the sound produced is below the lower limit of audibility of human ear.

Ultrasound

High frequency sounds which are above the upper limit of audibility (20 000 Hz) are called ultrasound which cannot be heard by human ear. Bats and whales use ultrasound in echolocation and most sonar technologies use ultrasound.

If you blow a dog whistle, a dog may bark in response, even though you do not hear any sound. This is because the frequency of the sound produced by the whistle is above the upper limit of audibility of humans but within that of dogs.

Various Applications of Ultrasound

1. Quality Control

Manufacturers of concrete use ultrasound to check for cracks or cavities. It can also be used to inspect metal pipes and measure the thickness of the wooden boards.

2. Prenatal Ultrasound Test

A prenatal ultrasound test uses high-frequency sound waves, inaudible to the human ear, that are transmitted through the abdomen via a device called a transducer to look at the inside of the abdomen. With prenatal ultrasound, the echoes are recorded and transformed into video or photographic images of the baby.



3. Ultrasonic Scalers in Dentistry

Ultrasonic scalers use ultrasound to remove calculus (plaque) deposits from the teeth. They work mechanically using high frequency ultrasound that effectively blasts calculus. The plaque bacteria and calculus can be flushed from the teeth by a small jet of water or antibacterial mouthwash which is ejected from the top of the scaler.



4. Ultrasonic Cleaning

The transmission of high energy ultrasound may result in the certain of cavitation bubbles. Cavitation bubbles are created at sites of rarefaction. These cavitation bubbles may displace contaminant from surfaces. This effect also allows fresh chemicals to come into contact with the contaminant remaining on the surface to be removed. Ultrasonic cleaning is especially effective in the cleaning of irregular surfaces or internal cavities and passageways (such as medical ultrasonic cleaner, industrial ultrasonic cleaner).

5. Sonar

The sonar (sound navigation and ranging) is a type of technology that works based on echoes. It is used by the ships for navigation at sea and to detect the position of other vessels. For example, the depth of the sea or the position of the shoals of fish can be found by sending out a signal (a pulse of sound) from a transducer (transmitter) and noting the time interval for the reflected signal (the echo) to be received as shown in Figure 6.3.

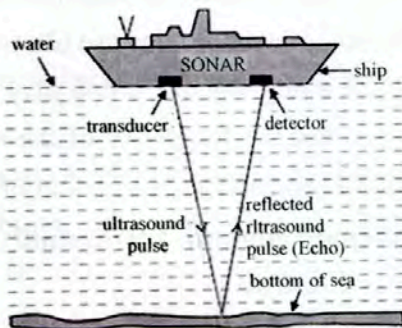
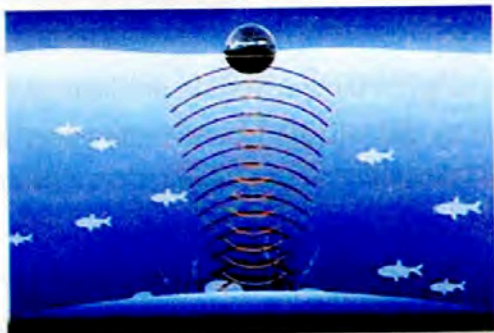


Figure 6.3 Illustration of sonar

Example 6.6 Elephants communicate over a long distance (up to 4 km) using sound with the fundamental frequencies as low as 15 Hz. A rhinoceros uses frequencies down to 10 Hz. (i) Why are elephants able to talk to one another over such a long distance? (ii) Can rhinoceros be able to communicate over long distances further than 4 km? Why?

- (i) Infrasound waves ($f < 20$ Hz) have long wavelengths, and are thus able to travel over long distances. So, elephants are able to talk using frequencies as low as 15 Hz over long distances (up to 4 km).
- (ii) Rhinoceros can be able to communicate over long distances further than 4 km because the lowest frequency they produced is 10 Hz which is less than 15 Hz and longer wavelength. Infrasound waves which have lower frequency and longer wavelengths are able to travel longer distances with less attenuation.

Reviewed Exercise

1. Why can infrasound not be heard by humans?
2. A piezoelectric transducer produces ultrasound at a frequency of 10 MHz. Calculate the wavelength of this ultrasound in blood if the speed of ultrasound in blood is 1560 ms^{-1} .

Key Words: range of audibility, infrasound, ultrasound

6.4 THE DOPPLER EFFECT

The Doppler effect or Doppler shift is the change in frequency of a wave related to an observer due to the relative motion between the source and observer. It is named after the Austrian physicist Christian Doppler, who described the phenomenon in 1842. Usually, only the motion of the source

and the observer directly towards or away from each other in the reference frame in which the wave medium is considered at rest. A common example of Doppler shift is the change of frequency (pitch) heard when a vehicle sounding a horn approach and recedes from an observer as shown in Figure 6.4.

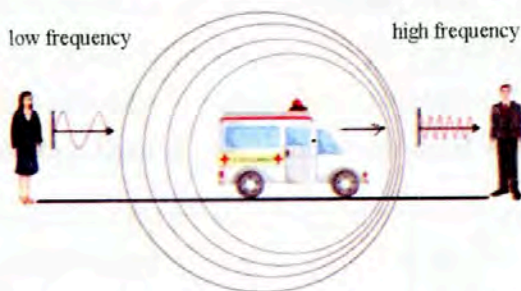


Figure 6.4 Illustration of Doppler effect

Doppler Equation

There are three cases we encounter in Doppler effect:

- (i) The observer is stationary and the source is moving towards or away from the observer.
- (ii) The source is stationary and the observer is moving towards or away from the source.
- (iii) Both the source and observer are moving relative to each other.

Without going into any detail, the Doppler equation can be expressed in general form as

$$f_o = f_s \left(\frac{v \pm v_o}{v \mp v_s} \right) \quad (6.6)$$

where f_o = frequency heard by the observer

f_s = frequency produced by the source

v = speed of sound in the medium

v_o = speed of the observer

v_s = speed of the source

Note that the appropriate signs must be taken into account in using this equation. Remember that when the source and observer are approaching relative to each other $f_o > f_s$; and when they are receding relative to each other $f_o < f_s$. Hence,

$+v_o$ is used when the observer moves towards the source,

$-v_o$ is used when the observer moves away from the source,

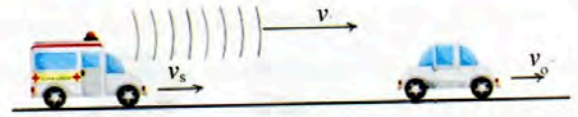
$-v_s$ is used when the source moves towards the observer, and

$+v_s$ is used when source the moves away from the observer.

When observer and source are stationary, $v_o = v_s = 0$.

Example 6.7 An ambulance travelling at 44 m s^{-1} approaches a car heading in a same direction at the speed of 28 m s^{-1} . The ambulance driver has a siren sounding at 550 Hz . At what frequency does the driver of the car hear the siren? (Speed of siren sound = $v = 330 \text{ m s}^{-1}$)

Speed of the car (observer) $v_o = 28 \text{ m s}^{-1}$
 Speed of the ambulance (source) $v_s = 44 \text{ m s}^{-1}$
 Speed of the siren sound $v = 330 \text{ m s}^{-1}$
 Frequency of the siren sound $f_s = 550 \text{ Hz}$



We have
$$f_o = f_s \left(\frac{v \pm v_o}{v \mp v_s} \right)$$

Since the observer moves away from the source, $-v_o$ is used and the source moves towards the observer, $-v_s$ is used.

Therefore,
$$f_o = f_s \left(\frac{v - v_o}{v - v_s} \right)$$

$$= 550 \left(\frac{330 - 28}{330 - 44} \right) = 580.8 \text{ Hz}$$

Application of Doppler Effect

Doppler effect is observed not only in sound but also in electromagnetic radiations (radio wave, microwave, light, etc.). Some applications of Doppler effect in a variety of fields is described below.

1. Red Shift

The wavelength of light from a star observed from the earth shifts due to Doppler effect towards the red end of the spectrum (lower frequency, longer wavelength) when the earth and the star are moving away from each other. This phenomenon is called red shift and the velocity of the astronomical body moving away from the earth can be determined from the fractional change in wavelength ($\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$). If the star were approaching the earth the spectral lines would shift towards blue end.

2. Radar

Radar (radio detection and ranging) is a form of echolocation that uses electromagnetic waves instead of sound but otherwise the concept is similar. The Doppler effect is used in some types of radar, to measure the velocity of detected objects. Some examples are police radar, air traffic control radar and weather radar as shown in Figure 6.5 (a), (b) and (c).

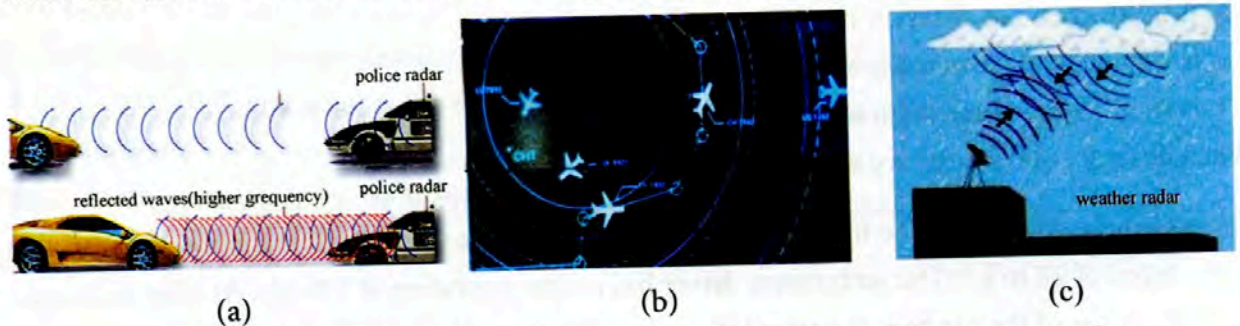


Figure 6.5 (a) Police radar, (b) air traffic control radar and (c) weather radar

3. Medical Applications

Doppler ultrasound is a technique that is used to examine blood flow. Blood flow rates can be found by measuring Doppler shift in frequency of ultrasound reflected by red blood cell (known as angiodynography). This technique can help reveal blockage to blood flow, show the formation of plaque in arteries and provide detailed information on the heartbeat of the fetus during labor and delivery. It can also be used to examine organ such as heart (echocardiography) and kidney.

Reviewed Exercise

1. In example 6.7, what will be the frequency of the siren heard by the driver of the car when the ambulance overtake his car?
2. A monorail approaches a platform at a speed of 50 m s^{-1} which it blows its whistle. A man standing on the platform hears the whistle with frequency 261 Hz . There is no wind and temperature is chilly 0°C . What is the observed frequency of the whistle when the train is at rest? (The speed of sound in air at 0°C is 332 m s^{-1})

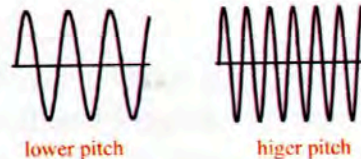


Key Words: Doppler effect, red shift, radar, Doppler ultrasound

6.5 PROPERTIES OF SOUND WAVES

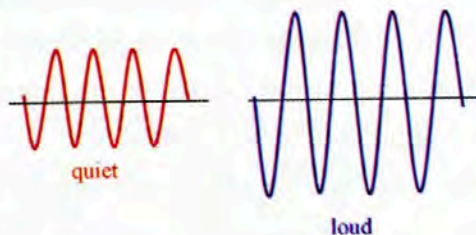
Pitch

The pitch of the note (how high it is) is determined by its frequency. The greater the frequency, the higher is the pitch.



Loudness

The loudness of the note depends upon the amplitude of the wave that produces it. The greater the amplitude, the louder the note, because more energy is used to produce a larger amplitude. The amplitude of the wave on the right is twice that of the wave on the left. So, the loudness is four times greater.



Quality or Timbre

Quality of a sound is the characteristic which distinguishes the two sounds of the same loudness and same pitch but emitted by two different instruments.

In Figure 6.6, the notes have the same frequency and hence the same pitch. However, they sound differently because of their different waveforms. That is, they differ in quality or timbre. The notes produced by the violin richer than others because it has more overtones. Notes of the same pitch

played upon different musical instruments are distinguished from each other by their quality. That is, the timbre of a sound depends on its waveform, which varies with the number of overtones, or harmonics.

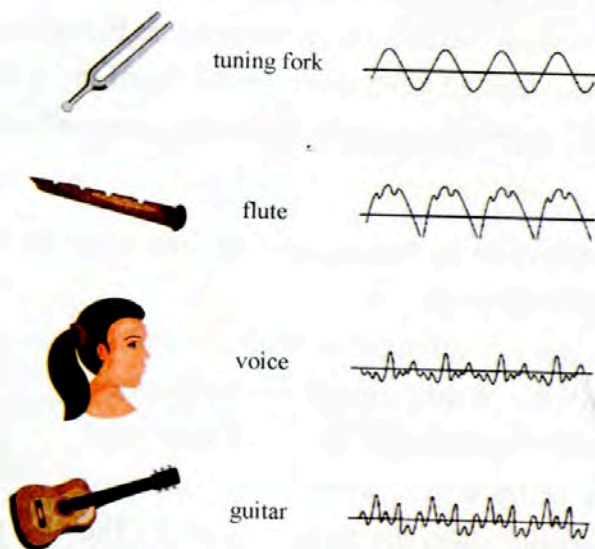


Figure 6.6 Different quality of sound produced by different musical instruments same pitch

Intensity and loudness of sound related to amplitude

The intensity of the sound at a place is the energy transmitted per second per unit area held normally to direction of sound and can be expressed as

$$I = \frac{1}{2} \rho v \omega^2 A^2 \quad (6.7)$$

where ρ = density of medium

v = velocity of sound wave

ω = angular frequency of sound wave

A = amplitude of sound wave

According to the Eq. (6.7), the intensity of a wave is directly proportional to the square of the amplitude of the wave ($I \propto A^2$).

Loudness is a sensation and hence, unlike intensity, it depends on the observer. To the human ears, the loudness of the sound is subjective. For a particular volume of sound, some may find it loud, while others finding it soft. Normally the greater the intensity, the greater the loudness of the sound is.

Example 6.8 Calculate the displacement of air molecules (amplitude) for a sound having a frequency of 1000 Hz at the threshold of hearing. The speed of sound in air is 343 m s^{-1} at 20°C and density of air is 1.29 kg m^{-3} .

Frequency of sound $f = 1000 \text{ Hz}$, density of air $\rho = 1.29 \text{ kg m}^{-3}$

Angular frequency $\omega = 2\pi f = 2 \times 3.142 \times 1000 = 6284 \text{ rad s}^{-1}$

Speed of sound in air $v = 343 \text{ m s}^{-1}$

At the threshold of hearing, intensity of sound $I = 1 \times 10^{-12} \text{ W m}^{-2}$

$$I = \frac{1}{2} \rho v \omega^2 A^2$$
$$10^{-12} = \frac{1}{2} \times 1.29 \times 343 \times (6284)^2 A^2$$

$$A = 1.07 \times 10^{-11} \text{ m}$$

Note that the amplitude of air molecules is very small.

Reviewed Exercise

- In terms of loudness and pitch, how would you compare the sounds made by a mosquito flying near your ear and the cracking of a toad?

Key Words: pitch, loudness, quality of sound (timbre)

SUMMARY

Audibility means the ability to be heard or capability of detecting sounds in certain range of frequencies.

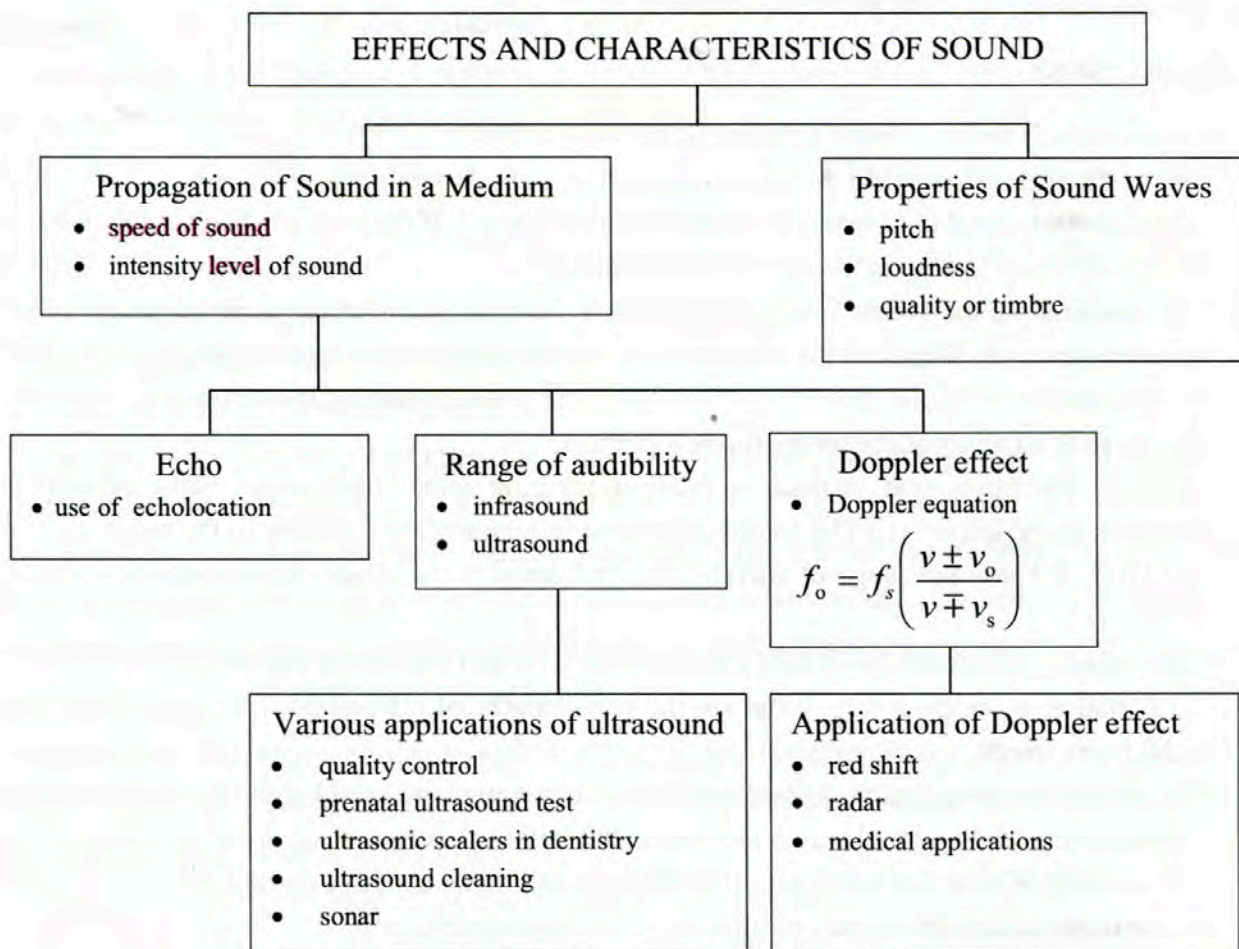
Timbre is the quality of sound which allows us to distinguish between different sound sources producing sound at the same pitch and loudness.

EXERCISES

1. Sound travels faster in moist (or) humid air. Why is this?
2. Name the factors on which the speed of sound in a gas depends.
3. The speed of sound in a particular liquid is $1.6 \times 10^3 \text{ m s}^{-1}$. If the bulk modulus of the liquid is $0.15 \times 10^{10} \text{ N m}^{-2}$, what is the density of the liquid?
4. An astronaut on the Moon's surface is unable to hear the sound of the rocket thrusters of his lunar lander craft. Which of the following is most likely to be the correct explanation for this?
A. His helmet is sound proof. B. The sound made by the thruster is very soft.
C. There is no atmosphere on the moon's surface.
5. (i) Why the ultrasound is used in medical imaging rather than sound wave of audible frequencies or lower? (ii) The frequencies used in imaging are typically in the range of 1 to 15 MHz. What is the range of wavelengths in human tissue where the velocity of sound is 1540 m s^{-1} ?
6. The velocity of ultrasound in lung (inclusive air), fat and skull bone are 600 m s^{-1} , 450 m s^{-1} and 4080 m s^{-1} respectively. What are the wavelengths of ultrasound if the sound waves at 2 MHz are used?
7. To get the better resolution, higher frequencies ultrasound are used but at the expense of less penetration because sound waves are attenuated within the distance of about 50λ in tissue. If the velocity of sound in soft tissue is 1500 m s^{-1} and mean depth of thyroid is 0.75 cm. What is the most suitable frequency of ultrasound to diagnose thyroid?

8. Define echo. Give its two applications.
9. Why is the intensity of an echo less than that of the original sound?
10. Elephants and whales are known to use infrasound to communicate over very large distances. What are the advantages of infrasound for long distance communication?
11. A sonar echo returns to a submarine 1.2 s after being emitted. What is the distance to the object creating the echo? (Assume that the submarine is in the ocean, not in fresh water and the velocity of sound in water is 1500 m s^{-1} .)
12. (i) What frequency is received by a person watching an oncoming ambulance moving at 110 km h^{-1} and emitting a steady 800 Hz sound from its siren? The speed of sound on this day is 345 m s^{-1} . (ii) What frequency does she receive after the ambulance has passed?
13. A 5000 Hz sound wave is emitted by a stationary source. This sound reflects from an object moving 3.5 m s^{-1} toward the source. What is the frequency of the wave reflected by the moving object as detected by a detector at rest near the source?
14. Is there a Doppler shift if the source and observer move in the same direction, with the same velocity? Explain.

CONCEPT MAP



APPLICATIONS OF LIGHT AND OPTICAL INSTRUMENTS

Many different types of lenses play an important part in our daily life. Spectacles, cameras, microscopes and telescopes are useful optical instruments employing lenses. The function and formation of images in these instruments are easily understandable with the help of ray diagrams. In this chapter, we will study optical instruments and their applications.

Learning Outcomes

It is expected that students will

- investigate the features of eye and camera.
- explain the function of microscopes and telescopes.
- describe the image created by these instruments and calculate their magnifications.
- study the application of light in instruments such as laser, fibre optics and spectrometer.
- understand the photometry optics.

7.1 THE CAMERA

A camera is an optical instrument that forms and records an image of an object. The image may be recorded on film (or) it may be detected by an electronic sensor that stores the image digitally. Regardless of how the image is recorded, all cameras form images in the same basic way. The essential features of a simple camera are shown in Figure 7.1.

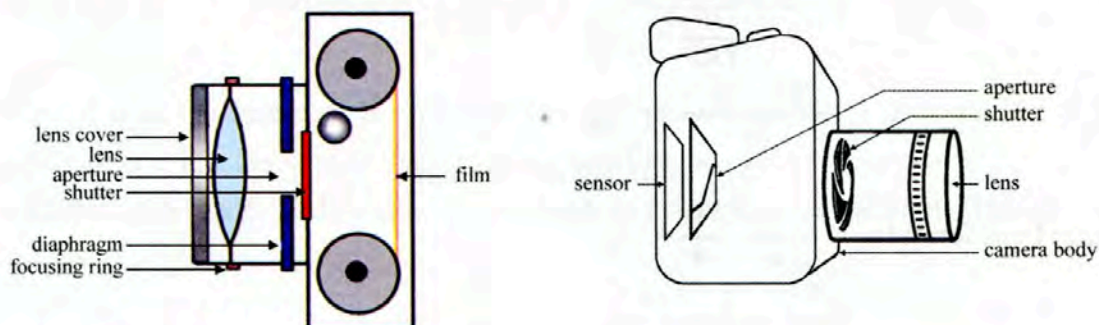


Figure 7.1(a) Camera with film (b) camera with sensor

The essential features of camera are expressed as follows.

Convex lens: A convex lens focuses the light from the object to form a real inverted image on the film or electronic sensor.

Diaphragm: A diaphragm is placed behind the convex lens to control the amount of light energy falling on the film.

Aperture: The aperture is a circular opening through which the light passes. It can be made wide or narrow by adjusting the diaphragm.

The size or the diameter of the aperture is expressed in terms of f -numbers. The f -number of a camera is the ratio of the focal length of the lens to the diameter of aperture. The smaller the f -number, the wider the aperture is. When taking a photograph in bright light a large f -number (narrow aperture) must be used and when taking a photograph in dim light a small f -number (wide aperture) must be used.

Shutter: A shutter controls the time during which light enters the camera through the lens. The shutter speed is expressed in terms of the exposure time. The shorter the exposure time, the higher the shutter speed is.

Focusing Ring: A camera consists of a focusing ring by which the distance between the lens and the film can be adjusted to obtain a sharp image on the film.

Film: The film at the back of the camera is actually a screen which receives the image. The film consists of a thin layer of chemical compounds of silver. Chemical reaction occurs when light energy falls on the film and as a result the image of an object is formed on the film. In a digital camera, the film is replaced by an electronic sensor. It is made up of millions of tiny pixels which response to light.

Example 7.1 A nature photographer wishes to photograph a 28 m tall tree from a distance of 58 m. What focal length of lens should be used if the image is to fill the 24 mm height of the film? Object height $OO' = 28$ m, object distance $u = 58$ m

Since the image formed by camera lens is real and inverted, image height $I'I'' = -24$ mm

$$\frac{I'I''}{OO'} = -\frac{v}{u}$$
$$\frac{-24 \times 10^{-3}}{28} = -\frac{v}{58}$$
$$v = 0.049 \text{ m} \approx 50 \text{ mm}$$

Using Lens formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{58} + \frac{1}{0.049} = \frac{1}{f}$$
$$f = 0.049 \text{ m} \approx 50 \text{ mm}$$

It is noted that for a distant object its image is formed at focus of the lens.

Reviewed Exercise

- Why must a camera lens be moved further from the film to focus on a closer object?

Key Words: convex lens, film, diaphragm, aperture, shutter, exposure time

7.2 HUMAN EYE

Human eye consists of sense organs which are capable of receiving visual images and sending the information to the brain. The human eye is very similar to a camera in many aspects. The optical features of the eye are shown in Figure 7.2.

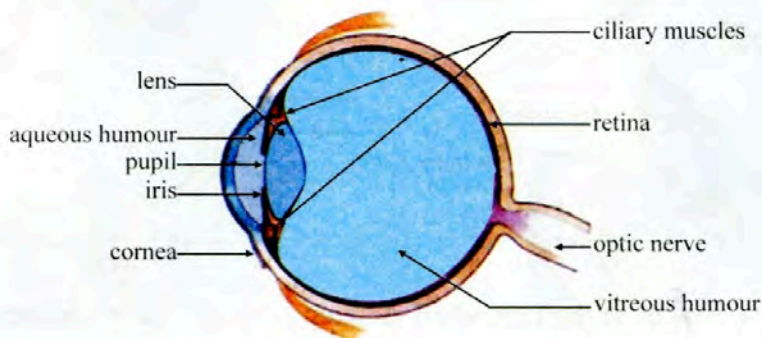


Figure 7.2 Optical features of the eye

The main features of human eye are described as follows.

Eyelid: Since the eyelid opens and closes the eye, it is similar to the lens cover of the camera.

Cornea: The front portion of the eye is covered by a tough, transparent membrane called the cornea.

Aqueous humour and vitreous humour: The region between the cornea and the eyelens is filled with a transparent liquid called the aqueous humour. The region between the eyelens and the retina contains a transparent liquid called the vitreous humour.

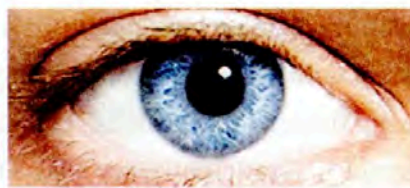
Iris: The iris which is the colored portion of the eye controls the amount of light energy entering the eye.

Pupil: The pupil is in the centre of the iris and the light passes through it. It is similar to the aperture in the camera. The iris adjusts the size of the pupil in the eye.

The pupil automatically dilates in dim light and contracts in bright light as shown in Figure 7.3 (a) and (b) respectively.



(a)



(b)

Figure 7.3 Behavior of the pupil in (a) dim light and (b) bright light

Ciliary muscles: The ciliary muscles are attached to the eyelens. The ciliary muscles can vary the thickness of the eyelens, thus alter the focal length of the lens to obtain a sharp image on the retina. When near objects are viewed, the ciliary muscles contract and permit the eyelens to form a more convex shape as shown in Figure 7.4 (a). When distant objects are viewed, the ciliary muscles relax and the eyelens becomes less convex as shown in Figure 7.4 (b).

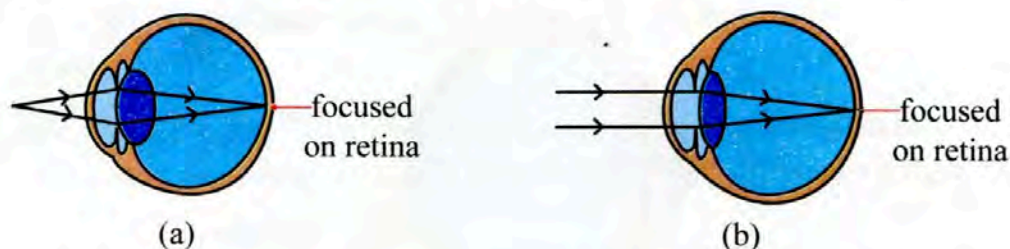


Figure 7.4 Shapes of the lens when viewing (a) near and (b) distant objects

Retina: The retina is the light sensitive area of the cells, at the back of the eye. The retina consists of rods and cones. The yellow spot which is the most light-sensitive spot on the retina is used for viewing very small objects. Since the yellow spot consists of cones exclusively, the small objects can be seen clearly. There is a spot on the retina which has no rods or cones. When an image is formed at this spot it cannot be seen at all. This spot is called the blind spot where the optic nerve enters the eye. The light energy falling on the retina is transmitted to the brain by means of the optic nerve.

The cornea, the aqueous humour, the eyelens and the vitreous humour are refracting substances. The image of an object formed on the retina by these four substances is real, inverted and smaller than the size of the object.

Common Defects of Vision

The common defects of vision are farsightedness (hyperopia) and nearsightedness (myopia). Farsightedness is a defect of vision in which distant objects are seen clearly, but objects nearby are blurry. On the other hand, nearsightedness is a defect of vision in which near objects are seen clearly, but objects far away are blurry. These defects are caused by the ciliary muscles which cannot contract or relax sufficiently or by wrongly shaped eyeballs.

For a person with farsightedness, the rays from near objects intersect at a point behind the retina after passing through the eyelens. This is due to an eyeball which is too short or due to the eyelens which cannot be sufficiently convex. This defect can be corrected by the use of spectacles with converging lens or convex lens as shown in Figure 7.5 (a).

For a person with nearsightedness, the rays from distant objects intersect at a point in front of the retina after passing through the eyelens. This is due to an eyeball which is too long or due to

the eyelens which is more convex than is necessary. This defect can be corrected by the use of spectacles with diverging lens or concave lens as shown in Figure 7.5 (b).

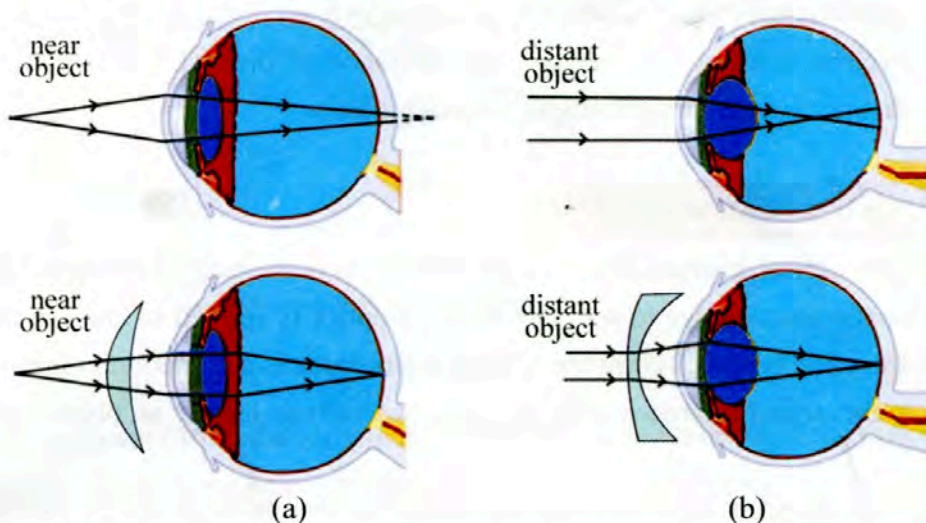


Figure 7.5 (a) Farsightedness and correction (b) nearsightedness and correction

People with normal vision can see an object in greatest detail when it is placed at a certain distance from the eye. This distance, which is known as the least distance of distinct vision is about 25 cm. People with normal vision can see clearly objects which are at infinity. Thus, for normal vision the maximum distance of distinct vision is infinity. When objects at infinity are viewed, the eyes are relaxed and such objects can be viewed conveniently for a long time.

Example 7.2 Calculate the power of the eyelens for viewing object at the longest and shortest distances possible with normal vision, assuming eyelens to retina distance of 2 cm.

For normal vision, the image must be on the retina, $v = +2 \text{ cm} = +0.02 \text{ m}$ (The image formed on the retina is real.)

(i) Object at the maximum (longest) distance of distinct vision, $u \approx \infty$

Using lens formula,
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Since, $P = \frac{1}{f(\text{m})}$,
$$P = \frac{1}{u} + \frac{1}{v}$$

$$= \frac{1}{\infty} + \frac{1}{0.02} = 50 \text{ D}$$

(ii) Object at the least (shortest) distance of distinct vision, $u = 25 \text{ cm} = 0.25 \text{ m}$

$$P = \frac{1}{u} + \frac{1}{v}$$

$$= \frac{1}{0.25} + \frac{1}{0.02} = 54 \text{ D}$$

Reviewed Exercise

1. What are meant by farsightedness and nearsightedness and how can they be corrected?
2. Briefly explain the optical features of the human eye with labeled diagram.

Key Words: eyelens, normal vision, maximum distance of distinct vision, least distance of distinct vision, farsightedness, nearsightedness

7.3 COMPOUND MICROSCOPE

A microscope is an optical instrument which uses lenses to make enlarged images of objects that are too small for the unaided eye to see. Geometrical optics is applied to analyse the formation of an image in the microscope. A common type of microscope is a compound microscope shown in Figure 7.6. A compound microscope has at least two convex lenses: an objective lens and an eyepiece lens.

The lens nearer the object is called the objective and the lens through which the final image is viewed is called the eyepiece. Figure 7.7 shows the formation of magnified image of a very small object by a compound microscope. O represents the objective of short focal length. When the compound microscope is used, the object should be placed between F_o and $2F_o$, where F_o is the focus of the objective. The objective produces an image which is real, inverted and larger than the size of the object. This image is between the eyepiece E and its focus F_e . The eyepiece magnifies this image and the final image formed is virtual and on the same side as the object. The final image can be seen clearly when the eye is very close to the eyepiece.

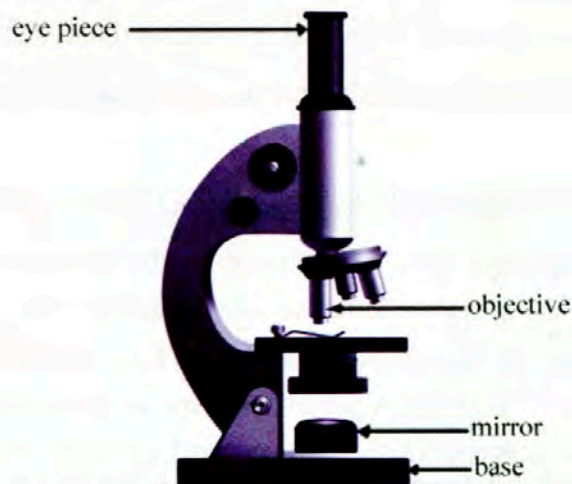


Figure 7.6 Compound microscope

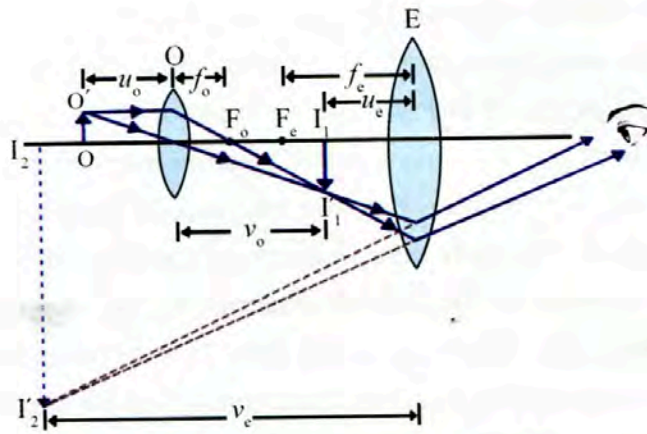


Figure 7.7 Image formation by compound microscope

The magnifying power (MP) of a compound microscope is the product of the magnification of the objective and the magnifying power of the eyepiece.

$$MP = \frac{v_o}{u_o} \times \frac{25}{u_e} \quad (7.1)$$

The separation of the lenses is, $L = v_o + u_e$

(i) When the final image is formed at infinity (for normal setting), $u_e = f_e$.

$$MP = \frac{v_o}{u_o} \times \frac{25}{f_e} \quad (7.2)$$

(ii) When the final image is formed at a distance of 25 cm from the eyepiece,

$$u_e = \frac{25 f_e}{25 + f_e}$$

Therefore,

$$MP = \frac{v_o}{u_o} \left(\frac{25}{f_e} + 1 \right) \quad (7.3)$$

For a large magnifying power of a compound microscope both the objective and the eyepiece should have short focal lengths. Also, the focal length of the objective should be shorter than that of the eyepiece. Compound microscopes have the magnifying power of several hundreds. As the magnifying power increases, the sharpness of images is reduced. Compound microscopes are used to study living cells and for regular use when magnification is enough.

Electron Microscope

Electron microscope is a microscope with high magnification and resolution, employing electron beams in place of light and using electron lenses. Optical microscopes have a maximum magnifying power of 1000 compared to the better magnification of the electron microscope that can reach 1 000 000. Electron microscopes deliver a more detailed image compared to optical microscopes.

Optical microscopes use simple lenses, whereas electron microscopes use electrostatic or

electromagnetic lenses. Optical microscopes use photons or light energy, while electron microscopes use electrons, which have shorter wavelengths that allows greater magnification.

In an electron microscope, as shown in Figure 7.8, a beam of accelerated electrons is used as a source of illumination. There are two main types of electron microscope: the Scanning Electron Microscope (SEM) and the Transmission Electron Microscope (TEM) as shown in Figure 7.8. They use an electron beam to illuminate the specimen and create an image.

The accelerated electron beam is focused by electrostatic or electromagnetic lenses, and transmitted through the specimen. When it emerges from the specimen, the electron beam carries information about the structure of the specimen that is magnified by the objective lens system of the microscope. The image of the specimen may be viewed by projecting the magnified electron image onto a fluorescent viewing screen. Alternatively, the image can be recorded by a photographic film or by a digital camera. Electron microscopes are used to investigate the ultrastructure of a wide range of biological and inorganic specimens, metals, and crystals.

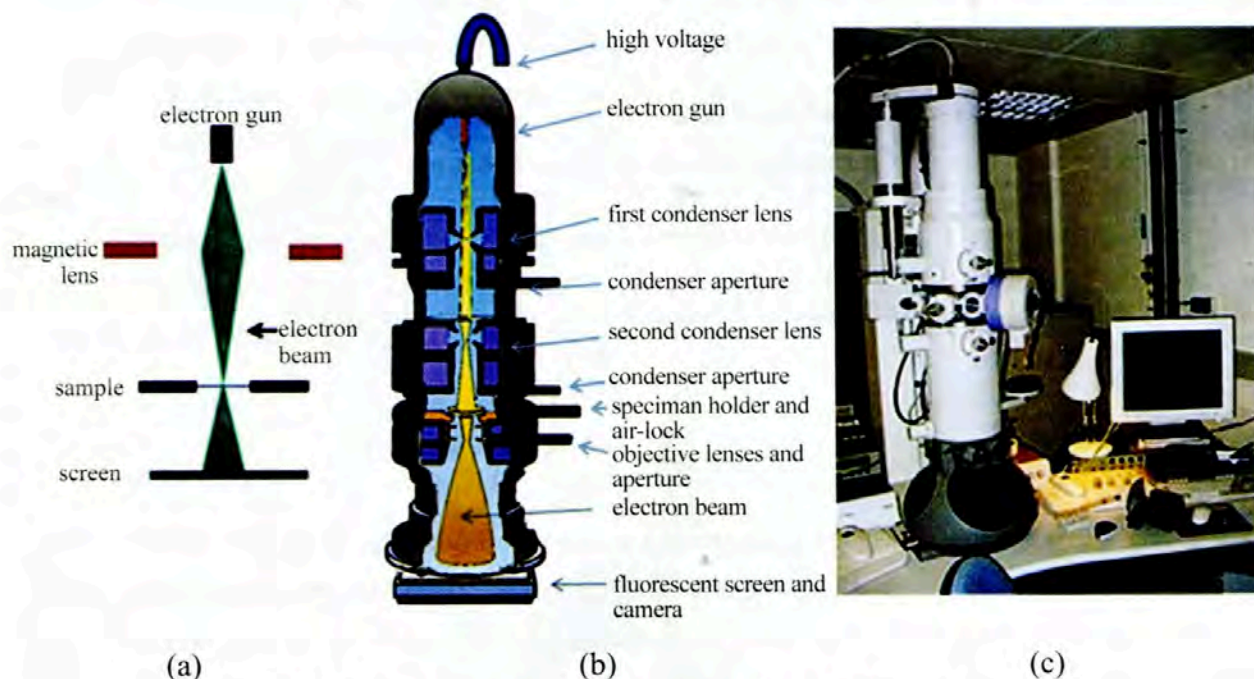


Figure 7.8 Transmission electron microscope (a) basic principle (b) internal structure (c) photograph

Example 7.3 A compound microscope consists of two thin converging lenses. The focal length of the objective is 10 mm and that of the eyepiece is 20 mm. If an object is placed 11 mm from the objective, the instrument produces an image at infinity. Calculate the separation of the lenses and the magnifying power of the instrument.

Object distance from the objective, $u_o = 11$ mm

Focal length of the objective, $f_o = 10$ mm, focal length of the eyepiece, $f_e = 20$ mm

Distance of image v_o produced by the objective is,

$$\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$$

$$\frac{1}{11} + \frac{1}{v_o} = \frac{1}{10}$$

$$v_o = 110 \text{ mm}$$

Since the final image is at infinity, the first image I' must be at the focus of the eyepiece.

Therefore $u_e = f_e = 20 \text{ mm}$

The separation of the lenses is $L = v_o + f_e$
 $= 110 + 20 = 130 \text{ mm}$

Since the final image is at infinity, the magnifying power is

$$MP = \frac{v_o}{u_o} \times \frac{25}{f_e}$$

$$= \frac{110}{11} \times \frac{250}{20} = 125$$

Reviewed Exercise

1. The final image produced by the compound microscope cannot be projected. Why?
2. A compound microscope 17 cm long has an eyepiece with a focal length of 2.5 cm and objective with focal length of 0.25 cm. What is the magnifying power for normal setting? (Hint: The magnified image of the objective lens forms at the focal point of the eyepiece for normal setting)

Key Words: electron microscope, objective, magnifying power, eyepiece, scanning electron microscope, transmission electron microscope

7.4 TELESCOPE

An optical instrument used for viewing distant objects is called a telescope. Two types of telescope are astronomical telescope and terrestrial telescope.

Astronomical Telescope

The refracting astronomical telescope consists of an objective of long focal length and an eyepiece of short focal length. Both lenses are convex lenses. Galileo invented a refracting telescope.

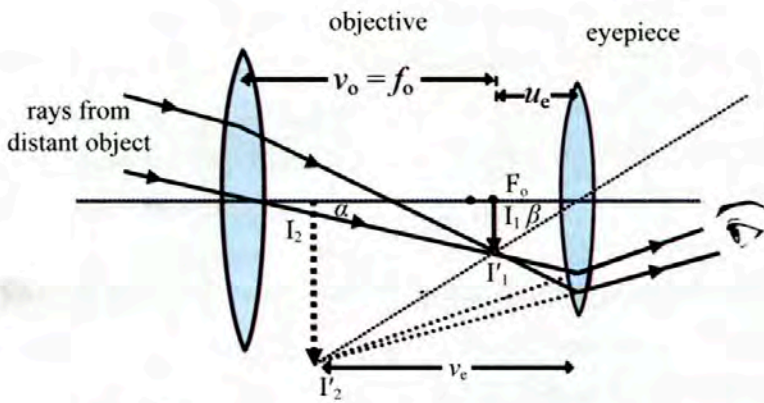


Figure 7.9 Ray diagram of an astronomical telescope

A real and inverted image of a distant object is formed at the focus of the objective. This image is within the focal length of the eyepiece. Hence, the final image formed by the eyepiece is virtual, larger than the first image and inverted with respect to the distant object as shown in Figure 7.9. However, this does not cause any problem in viewing planets, stars and moons.

The magnifying power (MP) of the refracting astronomical telescope is defined as the ratio of the angle β subtended by the image seen by the eye through the optical instrument to the angle α subtended by the object as seen without instrument.

$$MP = \frac{\beta}{\alpha} \quad (7.4)$$

From Figure 7.9,

$$\beta = \tan \beta = \frac{I_1'}{u_e}, \quad \alpha = \tan \alpha = \frac{I_1'}{v_o}$$

Therefore,

$$MP = \frac{v_o}{u_e} \quad (7.5)$$

When the final image is formed at infinity, $v_o = f_o$, $u_e = f_e$

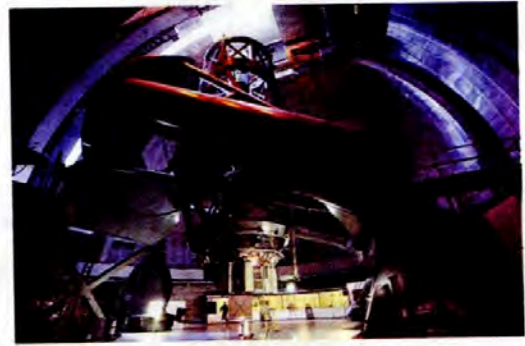
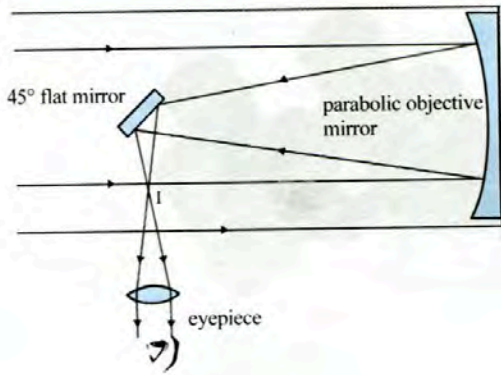
$$MP = \frac{f_o}{f_e} \quad (7.6)$$

The separation of the lenses $L = f_o + f_e$

According to E.q (7.6) for a high magnifying power of the refracting astronomical telescope, an objective of long focal length and an eyepiece of short focal length are required. To obtain the sharp images of distant objects the objective should have a large diameter. In practice, it is difficult to make a perfect convex lens of large diameter free from defects so that a concave mirror instead of a convex lens is used as an objective. Most telescopes used at the astronomical observatories in the world are reflecting telescopes with concave mirror used as the objective.

The parallel rays from a distant object are reflected from the concave mirror and they are allowed to fall on a plane mirror. Then, the reflected rays from the plane mirror enter the eye after refraction through the eyepiece as shown in Figure 7.10 (a). The eyepiece is, in fact, a magnifying glass which magnifies the image.

The reflecting telescope was first suggested by Newton and this arrangement is called the Newtonian design (Hale telescope).



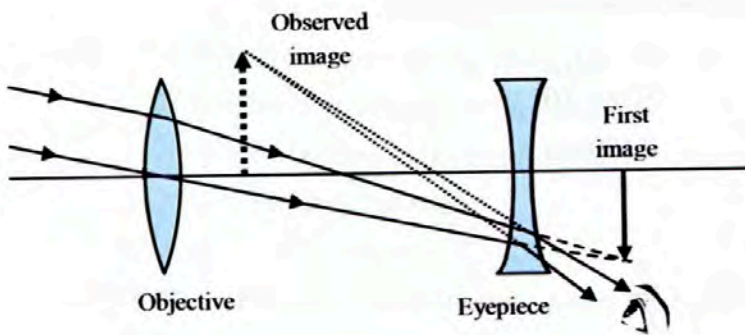
(a)

(b)

Figure 7.10 (a) Ray diagram of reflecting astronomical telescope
(b) Hale telescope high resolution stock

Terrestrial Telescope

It is not convenient to use the astronomical telescope for viewing objects on the earth since the final image produced is inverted. The astronomical telescope should be modified to produce a final image which is erect. In the terrestrial telescope a concave lens, instead of a convex lens, is used as an eyepiece to produce a final image which is erect as shown in Figure 7.11(a).



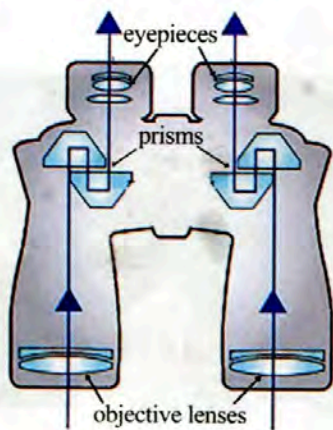
(a)

(b)

Figure 7.11 Terrestrial telescope (a) ray diagram (b) photograph

Since the terrestrial telescope of this type is very long as shown in Figure 7.11(b), it cannot be used conveniently. Total reflecting prisms ($90^\circ - 45^\circ - 45^\circ$ prisms) are used in binoculars to reduce the length of the terrestrial telescope as shown in Figure 7.12. A binoculars is an optical instrument which consists of a pair of telescopes one for each eye, and those telescopes employ the total reflecting prisms. Because of the successive total internal reflections in the prisms, the length of

the telescope is greatly reduced. Binoculars, therefore, are widely used as terrestrial telescopes.



(a)

(b)

Figure 7.12 Binoculars (a) ray diagram (b) photograph

Example 7.4 A telescope consists of two converging lenses: an objective of focal length 500 mm and an eyepiece of focal length 50 mm. When the telescope is in normal adjustment: (i) What is the separation of the lenses? (ii) Where is the final image located? (iii) Is the image erect or inverted? (iv) What is the magnifying power?

Focal length of objective, $f_o = 500$ mm, focal length of eyepiece, $f_e = 50$ mm

When the telescope is in normal adjustment the final image of a distant object is formed at infinity.

Since the object to be viewed is at infinity, its image I' appears at the focus of the objective.

Since the final image is at infinity, I' is at the focus of the eyepiece.

(i) The separation of the lenses

$$L = f_o + f_e$$

$$= 500 + 50$$

$$= 550 \text{ mm}$$

(ii) The final image is located at infinity.

(iii) The image is inverted.

(iv) The magnifying power

$$MP = \frac{f_o}{f_e}$$

$$= \frac{500}{50} = 10$$

Reviewed Exercise

- An astronomical telescope has its two-lens space 75.2 cm apart. If the objective lens has a focal length of 70 cm, what is the magnifying power of this telescope when viewing with a relaxed eye?

Key Words: refracting telescope, reflecting telescope, terrestrial telescope, binoculars

7.5 LASER

Optics is the branch of physics that studies the nature and properties of light. Light and optics provide various applications in our daily life such as the reflection and refraction of light in mirrors and lenses, and other real-world instruments such as microscope, telescope, lasers, spectrometer and fibre optics.

A laser, acronym for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation, is a source of a narrow beam of intense, monochromatic, coherent light in the visible, infrared or ultraviolet part of the electromagnetic radiation. It produces stimulated emission of photons from excited atoms or molecules. The power in a continuous beam can range from a fraction of a milliwatt to around 20 kilowatts in commercial lasers and up to more than a megawatt in special military lasers.

Lasers are classified into five main types:

- ◆ gas lasers
- ◆ solid-state lasers
- ◆ fiber lasers
- ◆ liquid lasers (dye lasers)
- ◆ semiconductor lasers (laser diodes)

In practice, a laser contains three essential elements: laser medium, power supply (for pumping process) and optical cavity as shown in Figure 7.13.

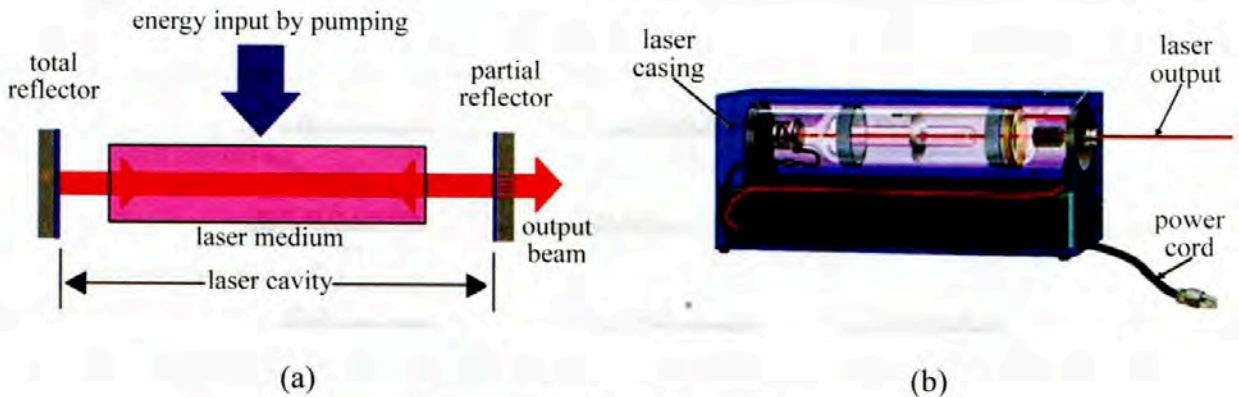


Figure 7.13 A laser system

The laser medium (gain medium) is the material emitting the laser beam. It can be a gas, liquid, crystalline solid or semiconductor crystal. The power supply delivers energy to the laser medium in the form needed to excite it to emit light. The energy is supplied as an electric current, a flash lamp or another laser. This process is called pumping.

The optical cavity (or resonator) concentrates the light to stimulate the emission of laser radiation. Photons are reflected back and forth in the optical cavity increasing the probability of stimulated emission; and hence, sustain laser action. The optical cavity consists of two highly reflective mirrors. One mirror with reflectivity of almost 100% (perfect reflector) completely reflects the

light. The other mirror with a reflectivity of about 80 to 90 % (partial reflector) reflects most part of the light, but allows some part of the light to come out as a laser beam.

Two essential processes to initiate a laser action are (i) stimulated emission and (ii) population inversion. In stimulated emission, the transition occurs when the atom in its excited state interacts with an incident photon (quantum of light energy) at the transition frequency as shown in Figure 7.14. Atoms undergoing stimulated emission radiate photons of the same frequency in the same direction and in phase with the stimulating radiation.

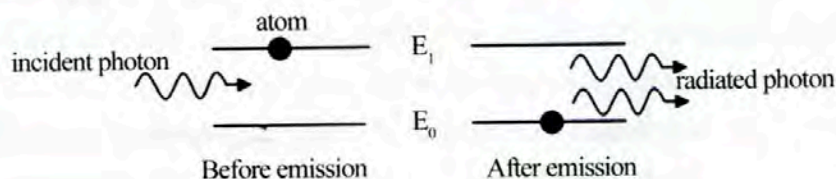


Figure 7.14 Stimulated emission

For stimulated emission to occur, a population inversion would be needed. That means the population of the upper level of a transition would have to be higher than that of the lower level. Then a photon of the transition energy would be more likely to stimulate emission from the excited state than to be absorbed by the lower state. The result is laser gain or amplification, a net increase in the number of photons with the transition energy. To sustain a laser action the lower level must be depopulated while the upper level is populated, otherwise the population inversion will end as shown in Figure 7.15.

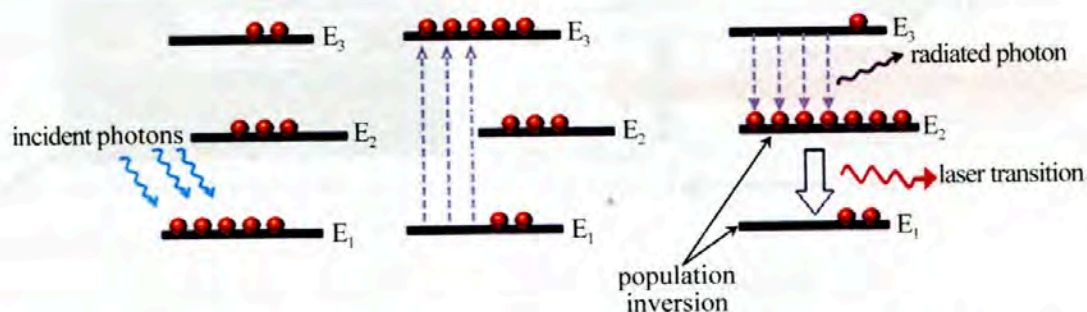


Figure 7.15 Population inversion obtained by pumping with flash lamp

Characteristics of Laser Light

The output lasers beam is narrow, the emission is coherent and the light is monochromatic. A typical beam divergence is around a milliradian, i.e, it will spread only to 1 m in diameter after travelling 1 km. Thus, a modest laser power can produce a high intensity inside the small area of laser beam. For example, 1 mW He-Ne laser beam is much brighter than the sunlight. One of the unique properties of a laser beam is coherence. This means that the light waves it contains are in phase with one another. This allows the wide use of lasers in holography.

Applications of Laser

Lasers are used in various applications; bloodless surgery, destroying kidney stones, eyelens curvature corrections, cancer diagnosis and therapy, fibre-optic endoscopy to detect ulcers in the intestines, in dentistry and cosmetic treatments.

- ◆ In communication, laser light is used in optical fibre communications to send information over large distances and also used in space communication such as radars and satellites.
- ◆ Laser range finders are used to determine the distance to an object.
- ◆ In industries, high power lasers are used to cut metals, drill aerosol nozzles and orifices with the required precision.
- ◆ In shops and business establishments, laser light is used to collect the information from the bar code printed on the product.
- ◆ Lasers are used in the semiconductor industries for photolithography, in computer printers and to store and retrieve information in compact disc (CD).

Example 7.5 In a laser unit, laser beam of wavelength 6328 \AA is emitted. How many photons are released per second if the output power is 1 mW ? (Plank constant $h = 6.62 \times 10^{-34} \text{ J s}$)

$$\lambda = 6328 \text{ \AA} = 6328 \times 10^{-10} \text{ m}, P = 1 \text{ mW} = 10^{-3} \text{ W}$$

$$\begin{aligned} \text{Energy of a photon in laser beam, } E &= \frac{hc}{\lambda} \\ &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6328 \times 10^{-10}} = 3.14 \times 10^{-19} \text{ J} \end{aligned}$$

Since output power is 1 mW , the output energy in 1 s is 1 mJ .

$$\begin{aligned} \text{Number of photons per second} &= \frac{\text{Output energy per second}}{\text{Energy of a photon}} \\ &= \frac{10^{-3}}{3.14 \times 10^{-19}} = 3.18 \times 10^{15} \end{aligned}$$

Reviewed Exercise

- How many photons are emitted in 1 s by a 7.5 mW of CO_2 laser with the wavelength of 10.6 \mu m ? (Plank constant $h = 6.62 \times 10^{-34} \text{ J s}$)

Key Words: population inversion, coherence, pumping, monochromaticity

7.6 FIBRE OPTICS

The fibre optics works on the principle of total internal reflection. Fibre optics refers to the technology that transmits information as light pulses along a glass or plastic fibre. A fibre optic cable can contain a varying number of glass fibres from a few up to hundreds.

An optical fibre is composed of two concentric layers, called the core and cladding which surrounds the glass fibre core. The cladding is surrounded by a layer called sheath (plastic buffer)

which protects the optical fibre as shown in Figure 7.16 (c). The core and cladding have different refractive indices. The refractive index of core will be slightly greater than that of cladding. If the light ray is injected and strikes the core to cladding interface at an angle greater than the critical angle, it is total internally reflected back into the core as shown in Figure 7.16 (a) and (b). The light ray then continues bouncing down the length of the fibre cable. However, if the angle of incidence at the core to cladding interface is less than the critical angle, both reflection and refraction take place. Because of refraction and absorption, the light beam attenuates and dies off over a certain distance.

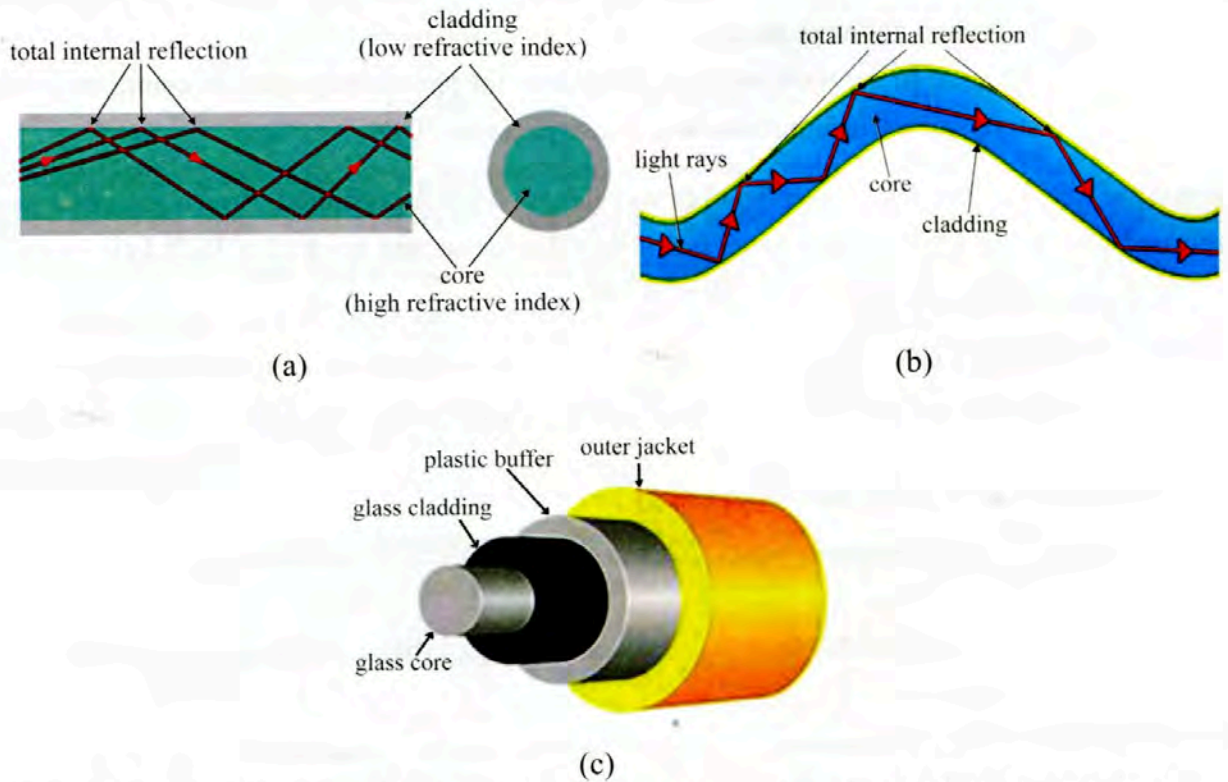


Figure 7.16 (a) Principle of total internal reflection in a section (b) cable of an optical fibre (c) structure of an optical fibre

A useful application of fibre optics is in endoscopy. In medicine, endoscopy is a non-surgical procedure to examine the interior of a hollow organ or cavity of the human body. Endoscopes use optical fibres to observe an image inside the body. A bundle of optical fibres is inserted into the body to clearly monitor the interior organs of the body as shown in Figure 7.17.

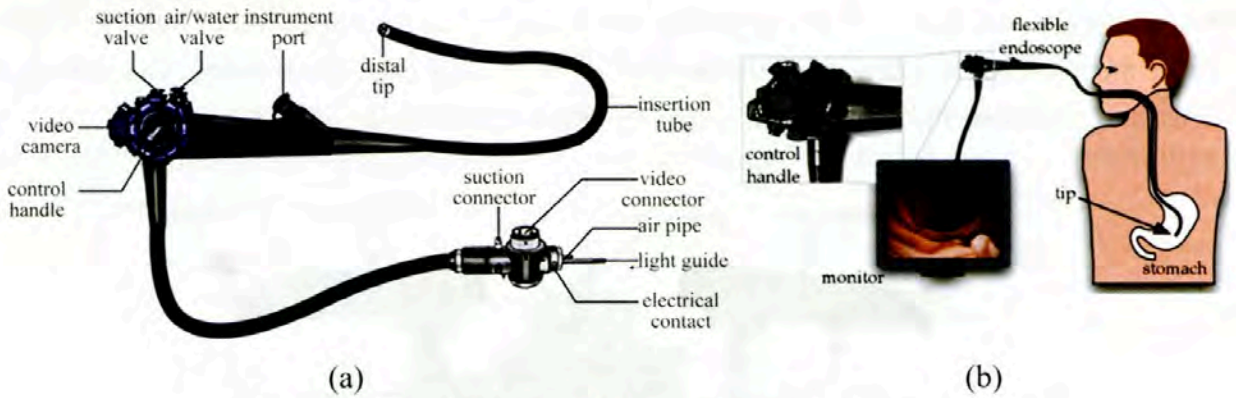


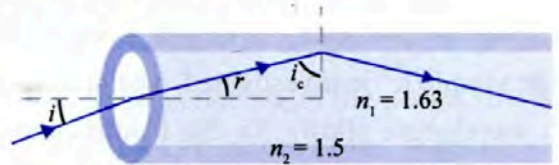
Figure 7.17(a) Endoscope (b) endoscopy of stomach

Example 7.6 The core of an optical fibre has a diameter $2\ \mu\text{m}$ and refractive index of 1.63. The refractive index of the cladding is 1.5. Determine the maximum angle for which the light rays incident on the end of fibre rod to total internal reflection at the core to cladding interface.

Refractive index of core $n_1 = 1.63$ and that of cladding $n_2 = 1.5$

According to Snell's law, the critical angle for core to cladding interface

$$\begin{aligned} \sin i_c &= \frac{n_2}{n_1} \\ &= \frac{1.5}{1.63} \\ i_c &= 67^\circ \end{aligned}$$



From the figure, $r = 90^\circ - i_c = 90^\circ - 67^\circ = 23^\circ$

Refraction at air-core interface, $n_1 = \frac{\sin i}{\sin r}$

$$\begin{aligned} \sin i &= 1.63 \times \sin 23^\circ \\ i &= 39.6^\circ \end{aligned}$$

Reviewed Exercise

- What is the function of cladding in an optical fibre?

Key Words: total internal reflection, cladding, endoscopy

7.7 SPECTROMETER

A spectrometer is a scientific instrument used to separate and measure spectral components of a physical phenomenon. It is a widely used scientific tool for many disciplines, including biology, chemistry, agriculture and more.

The optical spectrometer, shown in Figure 7.18, consists of light source, collimator, dispersion

elements and a telescope. The collimated beam from a specific light source is dispersed by dispersion element such as prism or grating. When observing the dispersion of light, the user can rotate the telescope and determine the angle of dispersion for each spectral line. This information is used to determine the wavelength; and hence, the chemical make up of the source.

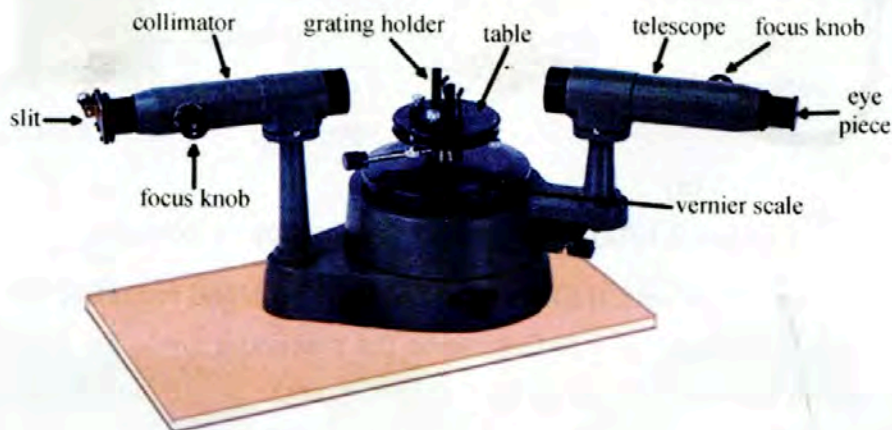


Figure 7.18 Optical spectrometer

In a spectrophotometer, dispersion element separates light into spectrum which is allowed to fall on the sample to be investigated. The change in the absorption and emission of the light intensity with wavelength allows for the identification of the materials as shown in Figure 7.19. Three of the most common optical spectrometers are spectrophotometers (UV-Vis spectrometer), spectrofluorometers and Raman spectrometers.

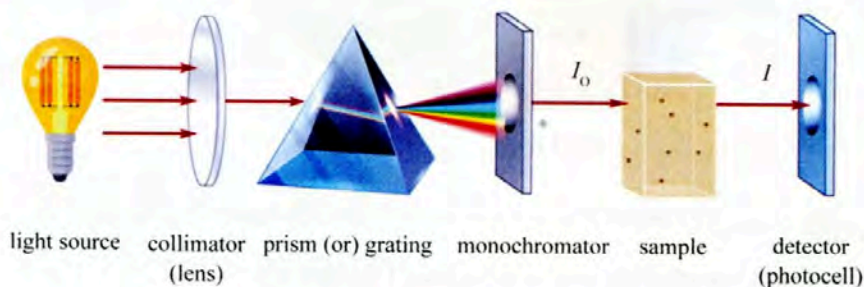


Figure 7.19 Schematics of a spectrophotometer

Reviewed Exercise

1. Name the dispersion elements used in the optical spectrometer.
2. Mention the different types of spectrometer.

Key Words: spectral line, spectrophotometers, spectrofluorometers, Raman spectrometers

7.8 PHOTOMETRY

Photometry is the science of measuring visible light in units that are weighted according to the sensitivity of human eye. The human eye responds only to light having wavelengths between about 400 nm and 700 nm. As shown by the sensitivity curve in Figure 7.20, human eye is more sensitive to green light of 555 nm compared to other colors.

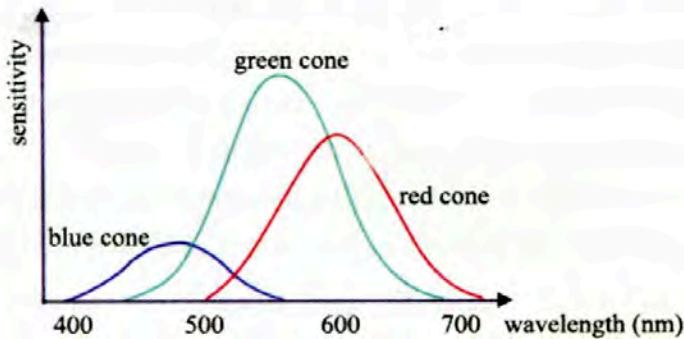


Figure 7. 20 Sensitivity response of human eye depending on wavelength

In photometry, there are four main photometric quantities. They are luminous flux, luminous intensity, luminance, and illuminance.

Luminous flux: Luminous flux (luminous power) Φ is a measurement of the power of visible light produced by a light source. It is measured in lumen (lm). One lumen (1 lm) is equivalent to 1/683 watt of light with a wavelength of 555 nm.

Luminous intensity: Luminous intensity I_v is a measure of the luminous flux emitted by a light source in a direction per unit solid angle shown in Figure 7.21.

$$I_v = \frac{\Phi}{\Omega} \quad (7.7)$$

where I_v is luminous intensity, Φ is luminous flux and Ω is the solid angle.

Luminous intensity is measured in candela (cd) or lumen per steradian. The candela is the SI basic unit.

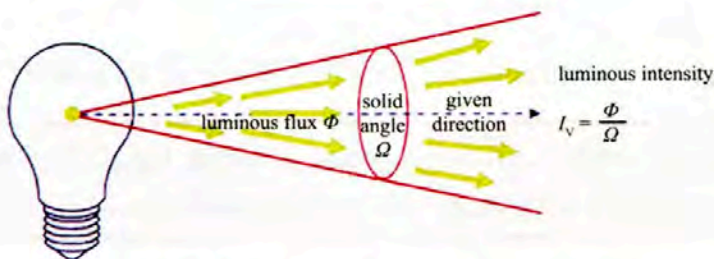


Figure 7.21 Luminous flux and luminous intensity

A solid angle Ω is a measure of the amount of the field of view from some particular point that a given object covers as shown in Figure 7.21.

$$\Omega = \frac{A}{r^2} \quad (7.8)$$

where Ω is the solid angle, A is the surface area of the spherical cap and r is the radius of the sphere.

In SI system, a solid angle is expressed in steradian (sr). A steradian can be defined as the solid angle subtended at the centre of a unit sphere by a unit area on its surface. The solid angle subtended at the centre by the whole surface area of the sphere is 4π steradian.

Luminance: Luminance L_v is the luminous intensity per unit area of light travelling in a given direction. The SI unit for luminance is candela per metre squared (cd m^{-2}).

Illuminance: Illuminance E_v is the total luminous flux incident on a surface per unit area. It is a measure of how much the incident light illuminates the surface as shown in Figure 7.22.

$$E_v = \frac{\Phi}{A} = \frac{I_v \Omega}{A} = \frac{I_v A}{Ar^2}$$

where A is illuminated surface area, luminous flux of source Φ is $I_v \Omega$, and Ω is solid angle.

$$E_v = \frac{I_v}{r^2} \quad (7.9)$$

The SI unit for illuminance is lux (lx) ($1 \text{ lx} = 1 \text{ lm m}^{-2}$).

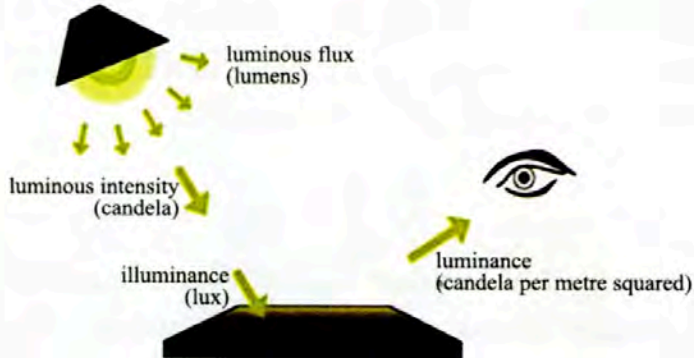


Figure 7. 22 Illustration of four quantities of photometry

Reading areas in school study halls need potent working lights. The minimum average illuminance should be 500 lux with a high uniformity and low glare.

The Inverse Square Law of Illuminance

In Figure 7.23, the illuminance E_v on a surface of distance r away from the source of light of luminous intensity I_v is

$$E_v = \frac{I_v \cos \theta}{r^2} \quad (7.10)$$

where θ is angle between the direction of light and normal to the illuminated surface.

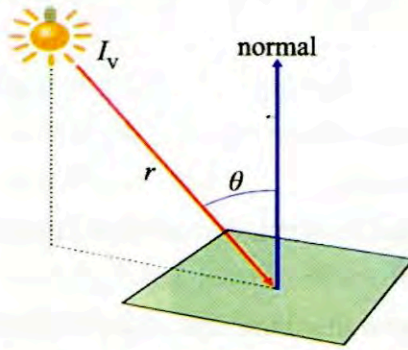


Figure 7.23 Illuminance E_v on a surface of distance r away from the source of light

Therefore, the illuminance E_v at any point on a surface is inversely proportional to the square of the distance r between the source and surface. This is known as inverse square law of illuminance.

Example 7.7 A 10 W fluorescent lamp has a luminous intensity of 35 cd. Find luminous flux it emits.

$$I_v = 35 \text{ cd}, \text{ solid angle for a sphere} = 4\pi \text{ sr}$$

$$\text{luminous flux emitted from the lamp } \Phi = I_v \Omega$$

$$= 35 \times 4\pi = 439.8 \text{ lm}$$

Example 7.8 The luminous intensity of a source is 200 cd. The mounting height of the lamp is 6 m from the ground. Find the illuminance on the ground E_v : (i) just below the lamp, (ii) 3 m horizontally away from the lamp on the ground.

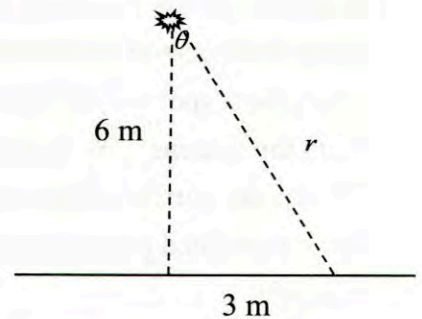
$$\text{luminous intensity of S source } I_v = 200 \text{ cd},$$

$$\text{The illuminance on a surface } E_v = \frac{I_v \cos \theta}{r^2}$$

(i) The illuminance on the ground just below the lamp.

From the figure, $r = 6 \text{ m}$, $\theta = 0$,

$$E_v = \frac{200 \times \cos 0}{6^2} = 5.55 \text{ lx}$$



(ii) The illuminance on the ground 3 m horizontally away from the lamp.

$$\text{From the figure, } r = \sqrt{6^2 + 3^2} = 6.708 \text{ m}, \cos \theta = \frac{6}{6.708}$$

$$E_v = \frac{200}{(6.708)^2} \times \frac{6}{6.708} = 3.98 \text{ lx}$$

Reviewed Exercise

- The luminous intensity of a light source is 500 cd. Find the illuminance of a surface which is 10 m from source if light falls normally on it.

Key Words: photometry, luminous flux, lumen, luminous intensity, candela, illuminance, lux, luminance

SUMMARY

Farsightedness is a defect of vision in which distant objects are seen clearly, but objects nearby are blurry.

Nearsightedness is a defect of vision in which near objects are seen clearly, but objects far away are blurry.

Microscope uses lenses to make enlarged images of objects that are too small for the unaided eye to see.

The optical instrument used for viewing distant objects is called a **telescope**.

A **laser**, acronym for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation, can be considered as a source of a narrow beam of intense, monochromatic, coherent light in the visible, infrared or ultraviolet part of the electromagnetic radiation.

EXERCISES

1. Name the essential parts of a human eye.
2. Discuss the important role of ciliary muscles for the eye.
3. State two defects of vision. Mention of the causes of the defects and corrections to these defects.
4. Choose the correct answer from the following.
The yellow spot which is the most light-sensitive spot is
- A. in the eyelens B. between the eyelens and the retina C. on the retina
5. Choose the correct answer from the following.
For a farsighted person the rays from the near objects pass through the eyelens and intersect at a point
- A. in front of the retina B. on the retina C. behind the retina
6. Draw a ray diagram showing how the rays of light travel in a compound microscope.
7. A concave mirror instead of a lens is used as an objective in an astronomical telescope. Why?
8. How is a terrestrial telescope constructed to obtain the final image which is erect?

9. Choose the correct answer.

In an optical fibre, the light is transmitted through the

A. core B. cladding C. jacket

10. Choose the correct answer.

The core of a fibre-optic cable is made of.....

A. copper wire B. thin strand of glass C. lead wire

11. A compound microscope consists of two thin lenses, an objective of focal length 20 mm and an eyepiece of focal length 50 mm, placed 220 mm apart. If the final image is viewed at infinity, calculate the distance of the object from the objective.

12. A telescope consists of two converging lenses. When in normal adjustment, so that the image of a distant object is formed at infinity, the lenses are 450 mm apart, and the magnifying power is 8. What are the focal lengths of the two lenses? Is the image erect or inverted?

13. A telescope consists of two thin converging lenses of focal lengths 100 cm and 10 cm respectively. It is used to view an object 2000 cm from the objective. What is the separation of the lenses if the final image is 25 cm from the eyelens? Determine the magnifying power for an observer whose eye is close to the eyelens.

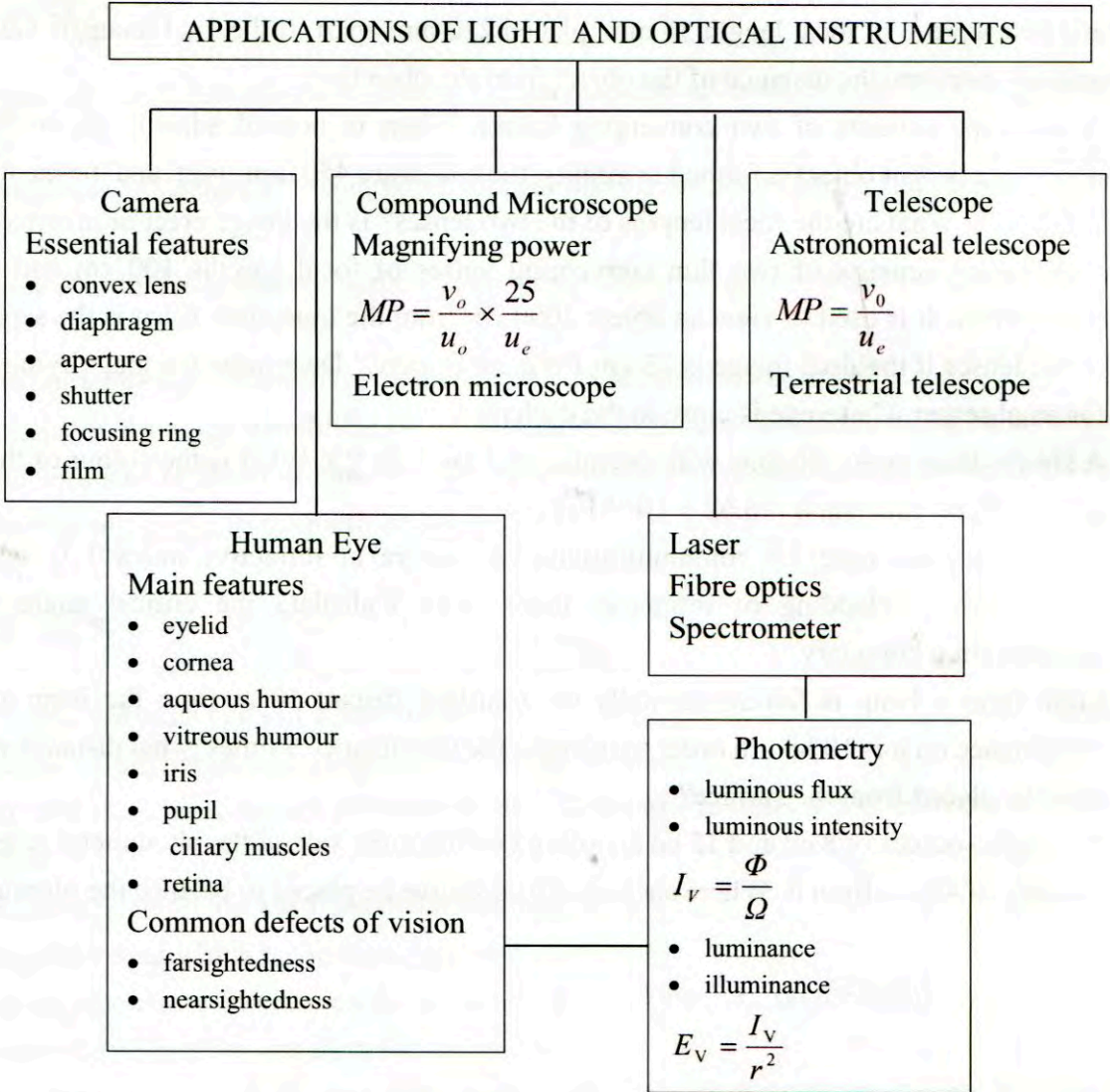
14. A He-Ne laser emits photons with energies of 3.14×10^{-19} J. What is the colour of the laser light? (Plank constant $h = 6.62 \times 10^{-34}$ J s)

15. An optical fibre used for communications has a core of refractive index 1.55 which is surrounded by cladding of refractive index 1.45. Calculate the critical angle of the core-cladding boundary.

16. Light from a lamp is falling normally on a surface distant 10 m from the lamp and the illuminance on it is 10 lux. In order to increase the illuminance 9 times, what distance will the lamp be placed from the surface?

17. Two light sources of 8 cd and 12 cd are placed on the same side of the photometer screen at a distance of 40 cm from it. Where should a 80 cd source be placed to balance the illuminance?

CONCEPT MAP



CHAPTER 8

INTERFERENCE AND DIFFRACTION OF LIGHT

In this chapter, interference and diffraction of light will be discussed. The study of interference and diffraction, for which the ray approximation of geometrical optics is not valid, are dealt with wave optics (physical optics). Wave optics includes the wave characteristics of light. In wave optics, Huygens' principle explained how the light propagates from a light source.

Learning Outcomes

It is expected that students will

- investigate the Huygens' principle of wave propagation.
- understand the interference of light.
- examine the diffraction of light.
- discuss the pattern obtained from diffraction gratings.
- explain diffraction grating effects.

8.1 HUYGENS' PRINCIPLE

In 1678, the Dutch physicist Christiaan Huygens introduced the wave theory of light. Huygens' principle states that every point on a wavefront may be considered as a source of secondary wavelets. The new position of wavefront is the envelope of the wavelets as shown in Figure 8.1.

If the propagating wave with a frequency f is transmitted through the medium at a speed v , then the secondary wavelets will have the same frequency and speed. These secondary wavelets spread out in all directions with the speed v .



Christiaan Huygens
(1629-1695)

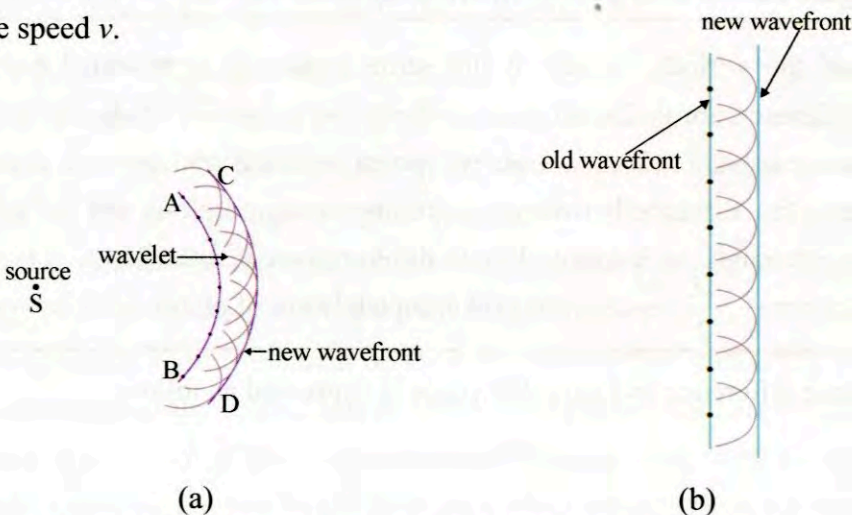


Figure 8.1 Propagation of (a) spherical wavefront and (b) plane wavefront

Wavefront

A wavefront is defined as a surface associated with a propagating wave which is a locus of all points in the wave that have the same phase. On a particular wavefront, at a given moment of time, all particles of the medium are undergoing the same phase. Wavefront is always perpendicular to the direction of propagation (or) rays. There are two types of wavefront; plane wavefront and spherical wavefront. The spherical wavefront from a source transforms into plane wavefront when it is far away from the source.

Phase (Phase Angle)

The phase angle is the position of a point on a reference circle of a waveform at an instant as shown in Figure 8.2. The angular position of a point of a periodic wave is known as the phase angle. The phase angle is one of the important characteristics of the periodic wave. In Figure 8.2, phase angles of point A and point B are 0 and $\frac{\pi}{2}$ respectively.

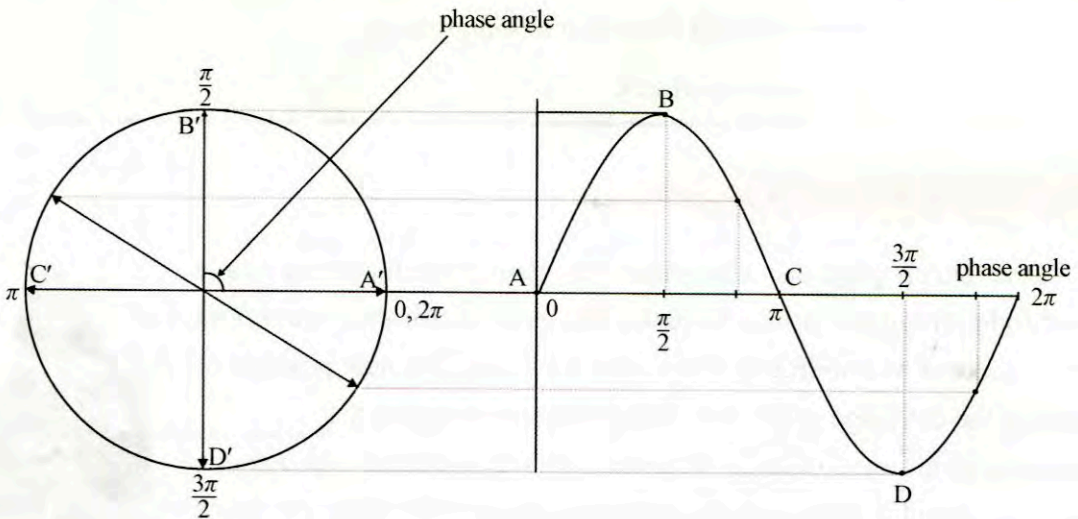


Figure 8.2 Phase angle for a point on a reference circle

The different states of the periodic waves at the same instant is represented by phase difference. If phase difference are 0, 2π , 4π , 6π , ..., these two positions are in phase (same phase). If phase difference are π , 3π , 5π , ..., these two positions are completely out of phase. Path difference indicates the distance between two points measured along the direction of propagation of the wave through the medium. If path difference are 0, λ , 2λ , 3λ , ..., it is said in phase. If path difference are $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$, ..., it is said completely out of phase.

The relation between phase difference and path difference is expressed as follow.

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x \quad (8.1)$$

where $\Delta\phi$ is phase difference between two waves of the same wavelength λ and Δx is the path difference of these waves.

Example 8.1 What is the phase difference for two waves of the same frequency arriving at a point with the respective path lengths of 6λ and 4.5λ ?

The path difference of two waves $\Delta x = 6\lambda - 4.5\lambda = 1.5\lambda$

The relation between phase difference and path difference is,

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times 1.5\lambda = 3\pi$$

Thus, the two waves are out of phase.

Reviewed Exercise

- How is the phase difference of two waves related to their path difference?

Key Words: wavefront, wavelet, phase angle, wavelength, periodic wave, phase difference, path difference, same phase, out of phase

8.2 INTERFERENCE OF LIGHT

Wave interference is the phenomenon that occurs when two waves meet while traveling along the same medium. Interference of light waves is defined as the modification in the distribution of light energy (intensity) when two or more waves superimpose each other.

If light waves interfere with one another, the maximum and the minimum intensity of light are obtained alternatively. To observe interference of light waves, the two sources of light must be coherent with respect to each other. If the waves emitted from two sources have the same frequency and constant phase difference, the sources are coherent sources. This means that the light sources must maintain a constant phase relationship.

Young's Double-Slit Experiment

In 1801, Thomas Young, English physician and physicist, obtained convincing evidence for the wave nature of light and was able to measure wavelengths for visible light. Figure 8.3 shows a schematic diagram of Young's double-slit experiment. Monochromatic light (a particular wavelength) from a single source are used in his experiment.



Thomas Young
(1773–1829)

A monochromatic light is incident on the first screen which contains a narrow slit. The emerging light then arrives at the second screen which has two parallel narrow slits S_1 and S_2 separated by a distance d . A screen is placed at a large distance D away from the slits shown in Figure 8.3. We might expect to see two bright lines (maximum intensity) on a screen, but instead a series of

alternate bright and dark lines are seen on the screen. Young was able to explain this result as interference phenomenon.

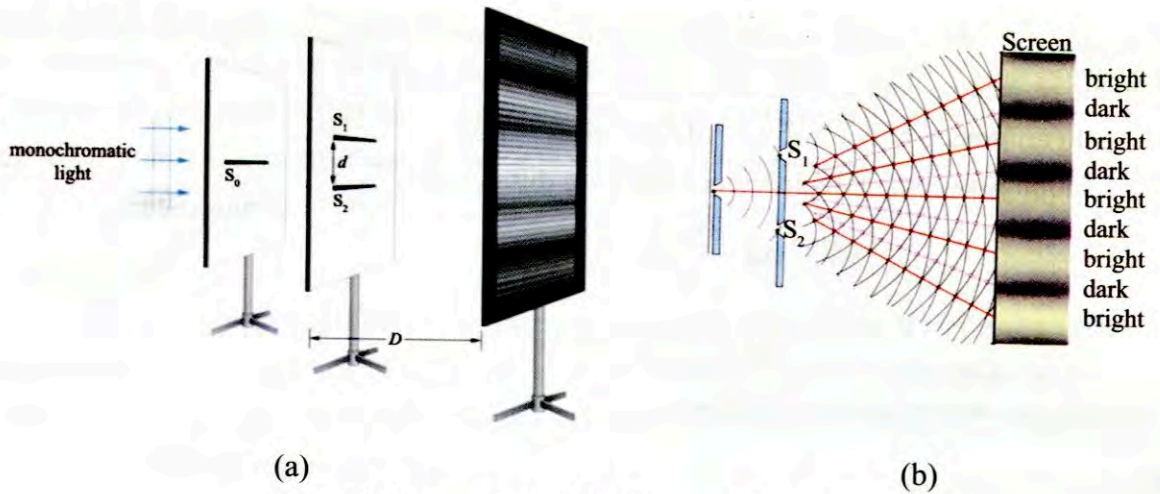


Figure 8.3 Young's double slits experiment

When a plane wave passes through a slit, a circular wavelet emerges from a slit because the circular wavelet is emitted from a point on the plane wave according to Huygens' principle. When this circular wave passes through two slits, each slit acts as a secondary source of circular waves. At any time, two waves emitted from each slit are of same frequency, same amplitude and are in phase. That is the two waves are coherent. Two waves from each source will interfere and as a result, an interference pattern will be displayed on the screen. The interference pattern consists of a series of bright and dark parallel lines called fringes.

The formation of interference can be explained as follows. The waves spread out in all directions after passing through two slits. Figure 8.4 (a), (b) and (c), show only for three different bending angles θ of waves. In Figure 8.4 (a), the waves reaching the centre of the screen ($\theta = 0$) travel the same distance, so they are in phase. A crest (or trough) of one wave arrives at the same time as a crest (or trough) of the other wave. Hence, constructive interference occurs at the centre of the screen and the bright fringe appears. The bright fringe at centre of the screen is called central bright line.

In Figure 8.4 (b) and (c) light waves arriving the point P from S_1 and S_2 will travel different distances. Light waves from the two sources S_1 and S_2 are initially in phase. When arriving at point P, they may be out of phase or in phase according to the path difference ($S_2P - S_1P$).

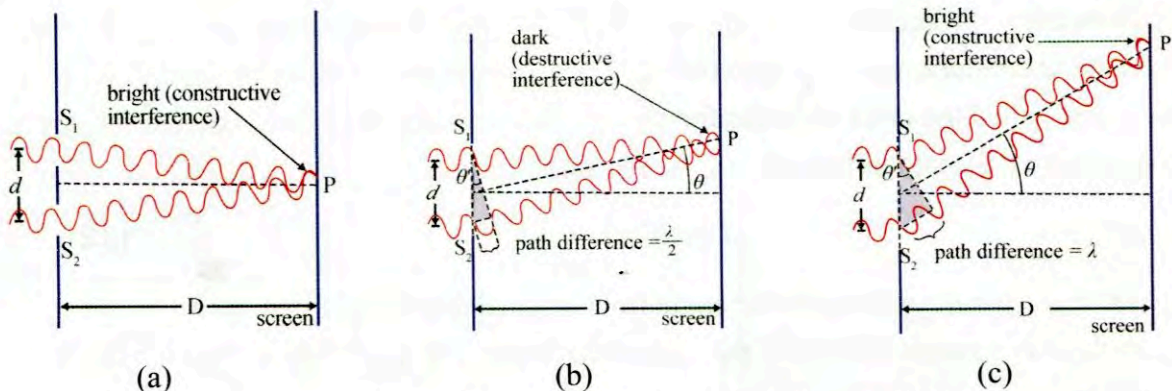


Figure 8.4 (a) At the centre of screen the waves from the two slits travel the same distance and are in phase
 (b) the lower wave travels an extra distance of one half wavelength and the waves from two slits are completely out of phase
 (c) the lower wave travels an extra distance of one whole wavelength and the waves from two slits are in phase

If the path difference is equal to an integral number of wavelengths ($0, \lambda, 2\lambda, \dots$) the waves will interfere constructively, leading to a bright fringe on the screen.

If the path difference is equal to a half-integral number of wavelengths ($\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots$) the waves will interfere destructively, leading to a dark fringe on the screen.

Constructive interference results in bright fringes and destructive interference results in a dark fringe.

Position of Fringes in Young's Double Slit Experiment

The position of particular bright and dark fringes on the screen is given by the distance from the central bright fringe, denoted y as shown in Figure 8.5:

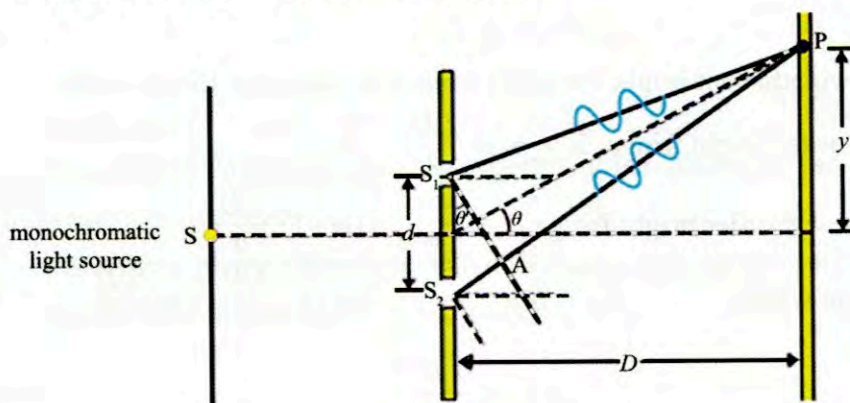


Figure 8.5 Position of fringes

For bright fringe to be formed at P, the path difference is equal to an integral number of wavelengths.

$$\text{Path difference} = m \lambda \quad (m = 0, 1, 2, 3, \dots)$$

where m is called the order of bright fringe.

From Figure 8.5 path difference $(S_2 A) = d \sin \theta'$

$$\text{Therefore,} \quad d \sin \theta' = m \lambda \quad (8.2)$$

For $D \gg d$, $\theta \approx \theta'$, where θ is the angle from original direction of light wave.

Under these conditions θ is small, the approximation can be used $\theta \text{ (rad)} \approx \sin \theta \approx \tan \theta \approx \frac{y}{D}$

$$\text{Therefore,} \quad d \sin \theta = m \lambda$$

$$d \frac{y}{D} = m \lambda$$

$$y = m \frac{\lambda D}{d} \quad (8.3)$$

where y is distance of the m^{th} bright fringe from the central bright line.

The zeroth order ($m = 0$) bright corresponds to the central bright line at $\theta = 0$, and the first-order brights ($m = 1$) can be seen on either side of the central line.

Similarly, the condition for dark fringe at P, when the path difference is a multiple of half wavelengths, is given by,

$$\text{For dark fringes,} \quad d \sin \theta = \left(m + \frac{1}{2}\right) \lambda \quad (m = 0, 1, 2, 3, \dots)$$

$$d \frac{y}{D} = \left(m + \frac{1}{2}\right) \lambda$$

$$y = \left(m + \frac{1}{2}\right) \frac{\lambda D}{d} \quad (8.4)$$

where y is distance of the m^{th} dark fringe from the central bright line.

Fringe width

Distance between two adjacent bright (or dark) fringes is called the fringe width.

The distance of m^{th} order bright fringe is $y_m = m \frac{\lambda D}{d}$

The distance of $(m + 1)^{\text{th}}$ order bright fringe is $y_{(m+1)} = (m+1) \frac{\lambda D}{d}$

$$\text{Fringe width} \quad \Delta y = \Delta y_{(m+1)} - y_m = (m+1) \frac{\lambda D}{d} - m \frac{\lambda D}{d}$$

$$\Delta y_m = \frac{\lambda D}{d} \quad (8.5)$$

The position of fringes and fringe width depend on the wavelength of light source.

Intensity Distribution of Fringes

In Young's double slit experiment, Figure 8.6 shows graphically how intensity of interference fringes on the screen varies with path difference. Bright and dark fringes are formed alternatively where bright fringe has maximum intensity and intensity of dark fringe is zero.

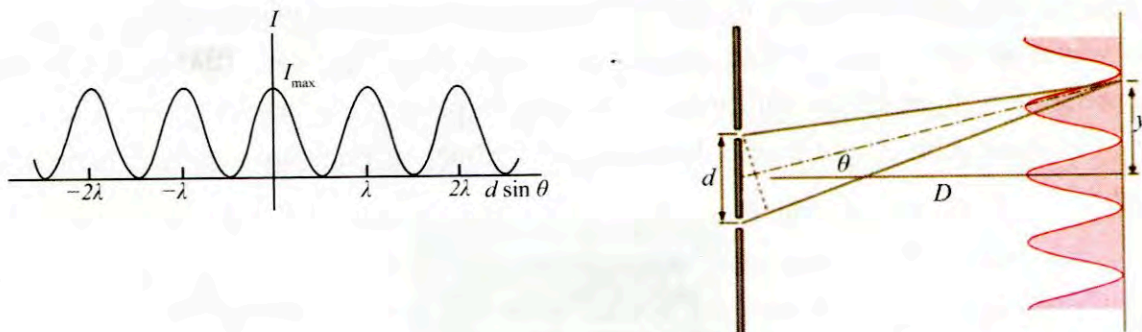


Figure 8.6 Intensity distribution of interference fringes

Example 8.2 In a double slit interference experiment, the distance between the slits is 5 mm and the screen is 2 m from the slits. Yellow light from a sodium lamp is used and it has a wavelength of 5.89×10^{-7} m. Find the distance between the first and second order bright fringes on the screen.

Distance between the slits $d = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$

Distance between the slits and the screen $D = 2 \text{ m}$

Wavelength $\lambda = 5.89 \times 10^{-7} \text{ m}$

The distance between the first and second order bright fringes = fringe width

$$\Delta y = \frac{\lambda D}{d}$$

$$\begin{aligned} \Delta y &= \frac{5.89 \times 10^{-7} \times 2}{5 \times 10^{-3}} \\ &= 2.356 \times 10^{-4} \text{ m} \end{aligned}$$

Reviewed Exercise

- If the violet light in Young's double slit experiment is replaced by red light, will the fringe width decrease or increase?

Key Words: wave front, phase difference, path difference, constructive interference, destructive interference, fringe width

8.3 DIFFRACTION OF LIGHT

Diffraction is defined as the bending of waves around the corners of an obstacle (or) through an aperture into the region of geometrical shadow of the obstacle (or) aperture. The amount of bending depends on the wavelength of light relative to the size of the aperture. If the aperture is much larger than the wavelength of light, the bending will be almost unnoticeable.

Light waves diffract (bend) around objects and create bright and dark fringes. A very simple demonstration of light diffraction can be conducted by placing the sharp edges of a razor blade in front of intense blue light from a laser source. Diffraction rings are clearly demonstrated in Figure 8.7.

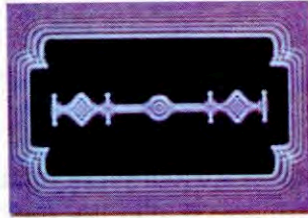


Figure 8.7 Diffraction of light at the edges of a razor blade

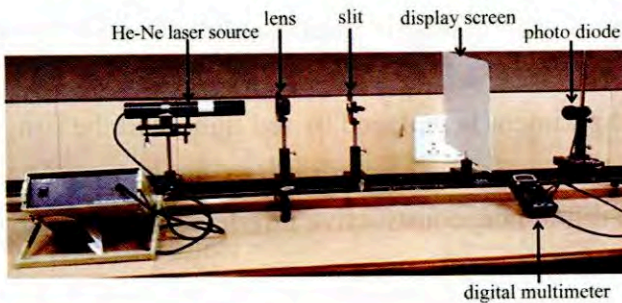
There are two main classes of diffraction, which are known as Fresnel diffraction and Fraunhofer diffraction.

Fresnel diffraction: The source of light and the screen are at a finite distance from the obstacle causing the diffraction.

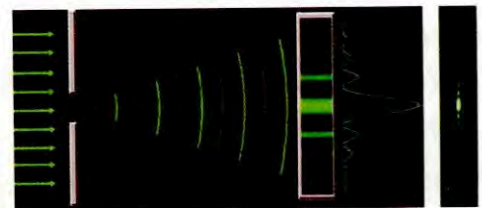
Fraunhofer diffraction: There is an infinite distance between the source and the screen from the obstacle. In this section Fraunhofer diffraction will be discussed.

Diffraction by a Single Slit (Fraunhofer diffraction)

The experimental arrangement for Fraunhofer diffraction is shown in Figure 8.8. When light passes through a single slit whose width a is of the order of the wavelength of the light, then the diffraction pattern is focussed on a screen that is a distance $D \gg a$ away from the slit.



(a)



(b)

Figure 8.8 The experimental arrangement for a single slit diffraction

The diffraction pattern consists of a bright central portion (the central maximum). On either side there are alternating bright and dark fringes shown in Figure 8.8 (b). These bright and dark fringes are created by interference of diffracted light waves. Each successive bright fringes becomes less intense proceeding outward, away from the central maximum.

Consider all the secondary wavelets that can be sent out from any two corresponding points of the upper and lower halves of the slit. They reach the point P_0 in the same phase and constructively interfere to produce a central bright maximum.

Now we will explain how the first minimum is formed. Consider the slit to be divided by two parts. The angular position θ of any point on the screen is measured from centre of the slit. The path difference between two waves from the centre and top (or) the centre and bottom edge of the slit is $\frac{\lambda}{2}$ and these two waves are exactly out of phase with one another. When they arrive at the screen, they interfere destructively and produces a dark fringe (minimum intensity) called the first minimum. The first minima are seen on the either side central bright maximum.

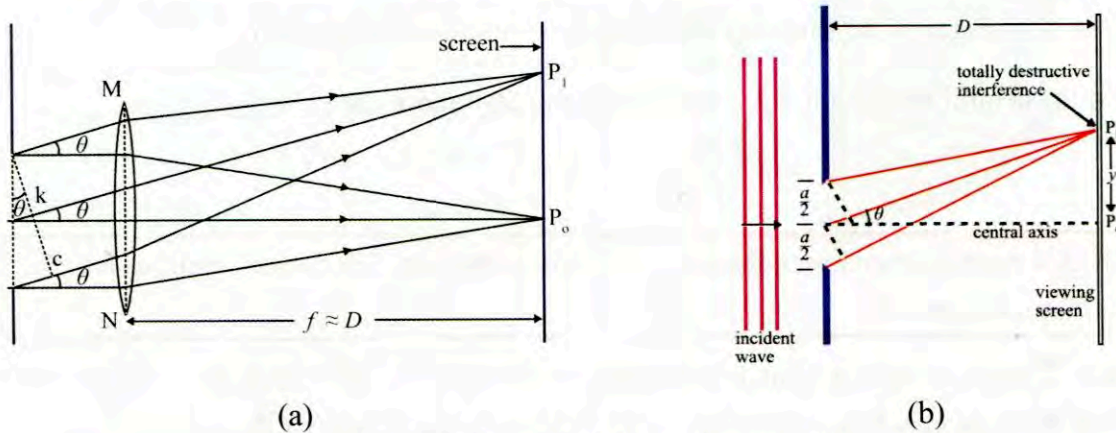


Figure 8.9 (a) Central bright maximum at P_0

(b) Path difference for two waves from the centre and top (or) from the centre and bottom

In Figure 8.9 (b), to get the first minimum (dark) the path difference between two waves from the centre and top (or) the centre and bottom edge of the slit is $\frac{a}{2} \sin \theta$.

$$\text{Thus, for the first minimum } \frac{a}{2} \sin \theta = \frac{\lambda}{2}$$

$$a \sin \theta = \lambda$$

For the second minimum, the slit is divided into four parts. The path difference between two waves from the corresponding points from each quarter of the slit is $\frac{a}{4} \sin \theta$.

$$\text{Thus, for the second minimum } \frac{a}{4} \sin \theta = \frac{\lambda}{2}$$

$$a \sin \theta = 2\lambda$$

Generally for the m^{th} minimum, $a \sin \theta = m \lambda$ where $m = 1, 2, 3, \dots$ (8.6)

The position of m^{th} minimum y is the distance from the central maximum.

From Figure 8.9, $y = D \tan \theta \approx D \sin \theta \approx m \frac{\lambda D}{a}$

The intensity distribution for single-slit diffraction is shown in Figure 8.10.

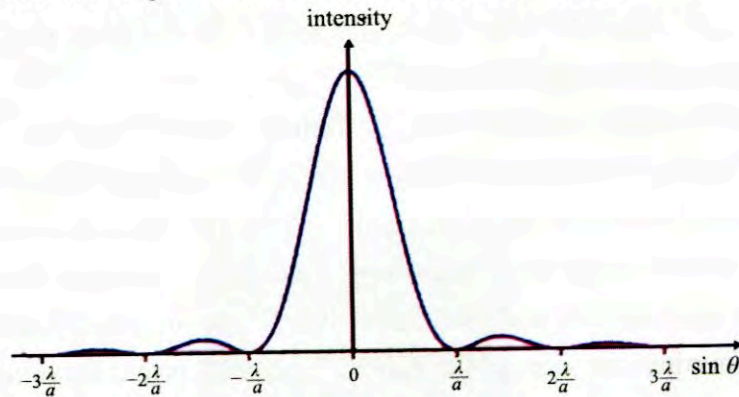


Figure 8.10 Intensity distribution of single-slit diffraction

The width of the central maximum = $2 \times$ position of the first minimum

$$= 2 \frac{\lambda D}{a}$$

The width of the central maximum depends on width of the slit. The narrower slit causes the wider central maximum.

Example 8.3 A beam of green light from a laser light source is diffracted by a slit of width 0.55 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of the first minima on both sides of central bright fringe is 4.1 mm. Calculate the wavelength of the laser light.

Width of the slits $a = 0.55 \text{ mm} = 0.55 \times 10^{-3} \text{ m}$

Distance between the slits and the wall $D = 2.06 \text{ m}$

The width of the central maximum, $2 \frac{\lambda D}{a} = 4.1 \text{ mm}$

$$\lambda = \frac{4.1 \times 10^{-3} a}{2 D}$$

$$\lambda = \frac{4.1 \times 10^{-3} \times 0.55 \times 10^{-3}}{2 \times 2.06}$$

$$= 0.5473 \times 10^{-6} \text{ m} = 547.3 \text{ nm}$$

Reviewed Exercise

1. When the slit width in a single slit experiment is reduced, what will be affect on the diffraction pattern?

2. What is the difference between Fresnel diffraction and Fraunhofer diffraction?

Key Words: Fraunhofer diffraction, diffraction pattern, central maximum, minimum intensity

8.4 DIFFRACTION GRATING

Most modern-day applications of slit interference use not just two slits but many, approaching several thousand per centimetre for practical purposes. This optical instrument is called the diffraction grating, an important tool in optical analysis.

A diffraction grating is an optical device which consists of a large number of parallel slits of same width separated by equal opaque spaces. A diffraction grating can disperse light into its wavelength components. Therefore, it is used for the purpose of resolving light into spectra. Diffraction grating is thus widely used as dispersive elements in spectrographic instruments and a crucial tool in the study of atomic and molecular compositions.

Two types of grating are transmission grating and reflection grating. Figure shows a diffraction grating which can be manufactured by carving glass with a sharp tool in a large number of precisely positioned parallel lines. The number of lines (or slits) per unit length (lines per cm) is called the grating constant.

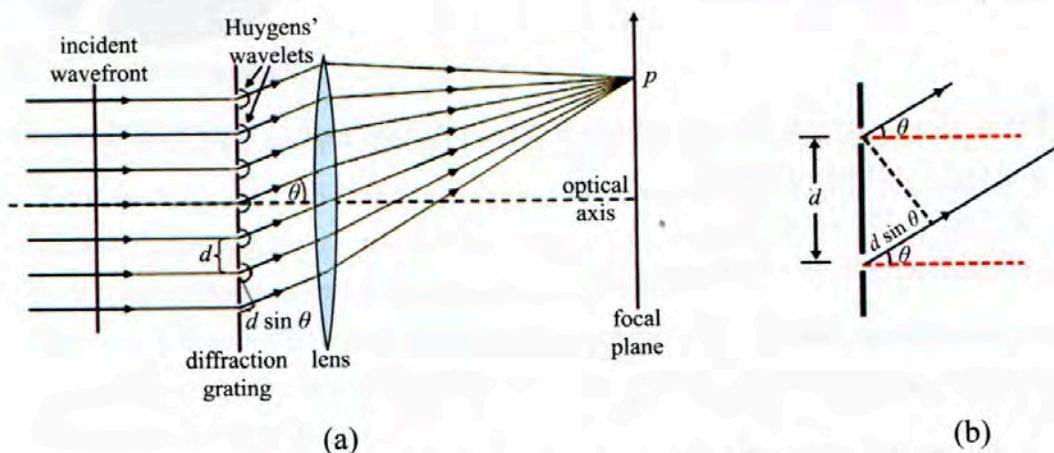
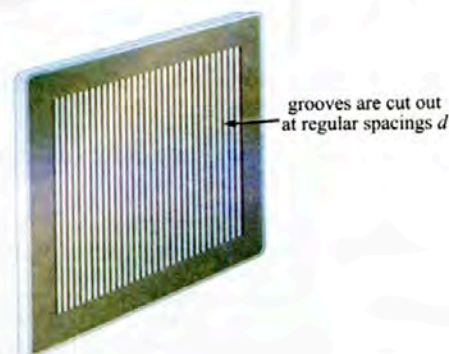


Figure 8.11 Diffraction of light in each slit of diffraction grating

The condition for maximum intensity is the same as that for a Young's double slit. For maximum intensity to be formed at P, the path difference of two waves from adjacent slits is equal to an integral number of wavelengths.

$$\text{Path difference} = m \lambda \quad (m = 0, 1, 2, 3, \dots)$$

From Figure 8.11 Path difference = $d \sin \theta$

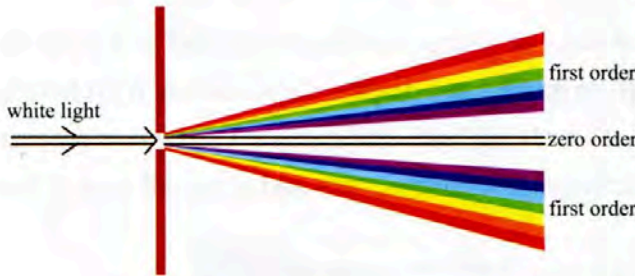
Therefore, $d \sin \theta = m \lambda$

where d is the distance between the adjacent slits, θ is the angle of diffraction, λ is the wavelength of the light and m is the order of maxima.

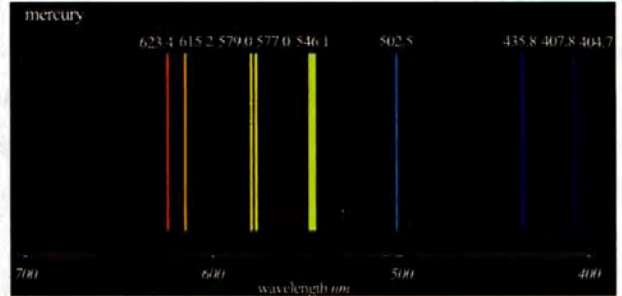
If the number of slits per unit length is N , d is equal to $\frac{1}{N}$.

In Figure 8.12, there is no dispersion of incident white light at the centre (i.e., $m = 0$).

In the first order diffraction spectrum, longer wavelength (red light) diffracts at a greater angle than shorter wavelength (violet light).



(a)



(b)

Figure 8.12 (a) Spectrum for white light (b) mercury spectrum

A compact disc is an example of reflection gratings. A reflection grating is a shiny surface having many fine grooves. The tracks of a compact disc act as a diffraction grating, producing a separation of the colors of white light shown in Figure.



Spectrum due to compact disc

Example 8.4 Light of wavelength 457 nm illuminates a diffraction grating with 5000 lines cm^{-1} . What is the second order diffracted angle?

Wavelength $\lambda = 457 \text{ nm} = 457 \times 10^{-9} \text{ m}$

Number of slits per unit length $N = 5000 \text{ lines cm}^{-1}$

Distance between the adjacent slits $d = \frac{1}{N} = \frac{1}{5000} = 2 \times 10^{-4} \text{ cm} = 2 \times 10^{-6} \text{ m}$

Diffracted angle θ for m^{th} order, $d \sin \theta = m \lambda$

for second order $m = 2$

$$2 \times 10^{-6} \sin \theta = 2 \times 457 \times 10^{-9}$$

$$\theta = 27.19^\circ$$

Reviewed Exercise

1. Why can a grating with smaller grating constant produce the wider the diffraction pattern?
2. Why does the compact disc show the diffraction pattern?
3. What are the differences between the interference and diffraction?
4. What is the difference between the diffraction grating and prism?

SUMMARY

A **wave front** is defined as a surface associated with a propagating wave which is a locus of all points having the same phase in the wave.

The angular position of a periodic wave is known as the **phase angle**.

If the waves emitted from two sources have the same frequency and constant phase difference, the sources are **coherent sources**.

Interference of light waves is defined as the modification in the distribution of light energy (intensity) when two or more waves superimpose each other.

The condition for **constructive interference** is that the path difference should be equal to an integral multiple of the wavelength.

The condition for **destructive interference** is that the path difference should be equal to odd integral multiple of half wavelength.

Monochromatic light means that it has a single wavelength.

Diffraction of light is defined as the bending of waves around the corners of an obstacle (or) through an aperture into the region of geometrical shadow of the obstacle (or) aperture.

A **diffraction grating** is a component of optical devices and consists of a large number of parallel slits of same width and separated by equal opaque spaces.

EXERCISES

1. Why is monochromatic coherent light used in the double slits experiment?
2. What are the conditions for constructive interference and destructive interference?
3. What will happen on the interference pattern of a double slit if, (i) separation between the two slits is decreased? (ii) wavelength of light increased?
4. How are diffraction gratings used in spectrometers?
5. At what angle is the first-order maximum for 450 nm wavelength blue light falling on double slits separated by 0.05 mm?
6. A beam of 650 nm light is directed upon two slits that are separated by a distance of 0.750 mm. A screen is placed 1.50 m away to capture the interference pattern. What is the distance from the 1st order maximum to the 3rd order minimum?
7. Find the distance between adjacent dark spots from a double slits interference pattern if the wavelength of light is 500 nm, the distance between the slits is 1 mm, and the distance from the slit to the screen is 2 m.

8. Light of wavelength 580 nm is incident on a slit of width 0.3 mm. The observing screen is placed 2 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringes.
9. If the distance between two slits is 0.05 mm and the distance to a screen is 2.5 m, find the spacing between the first- and second-order bright fringes for yellow light of 580 nm wavelength.
10. A grating containing 4000 slits per centimetre is illuminated with a monochromatic light and produces the second-order bright line at a 30° angle. What is the wavelength of the light used?
11. A diffraction grating has 2000 lines per centimetre. At what angle will the first-order maximum be for 520 nm wavelength green light?
12. How many lines per centimetre are there on a diffraction grating that gives a first-order maximum for 470 nm blue light at an angle of 25° ?

CONCEPT MAP

